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Fluid Induced Piping Vibration with Elastically Restrained Different End Supports*

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***Abstract*—The dynamic stability of elastically restrained pipe conveying fluid is investigated in this study. The frequency expression is derived for classical boundary conditions by considering the supports as compliant material with linear and rotational stiffness. A new transcendental frequency equation is developed by using Euler-Bernoulli beam theory; the equation of motion is derived from energy expressions using the Hamilton's Principle. The natural frequencies are presented for a wide range of restraint parameters. Cases are studied for different boundary conditions- Linearly and Rotationally Restrained, Rotational-Linearly & Linearly Restrained, Rotational-Linearly and Fixed, Rotationally Restrained –Guided, are computed and it is noted that as the flow velocity increases the first mode frequency decreases and by varying the mass ratio the frequency increases.**

Keywords—Elastically Restrained, Frequency, Linearly Restrained, Pipe, Guided Support

I. INTRODUCTION

The vibration analysis of piping systems is important from the view point of safeguarding the equipment's and pipelines from damage which is mostly applicable in chemical, petrochemical and other allied industries. It is well known that pipeline systems may undergo divergence and flutter type of instability due to fluid-structure interaction.

The dynamic behavior of fluid conveying pipes was predicted first by Ashley and Haviland in 1950 [1]. and later by Housner in 1952. Housner considered a simply supported beam model for the pipeline and analysed it using a series solution approach which showed that critical flow velocity could cause buckling [2]. S. S Rao developed a mathematical model for transverse vibration for elastically restrained conditions for beams [3]. Naguleswaran and Williams developed solutions for natural frequencies in axial mode for Hinged-Hinged, Fixed-Hinged and Fixed-Fixed boundary conditions [4]. Chen and Paidoussis developed dynamic stiffness matrix for coupled fluid structure interaction [5], [6]. Huang Yi-min considered Galerkin's method and obtained natural frequencies for fluid conveying pipeline with different boundary conditions [7]. R.A. Stein and M.W.Tobriner discussed Vibration of pipes containing flowing fluid, in which the effects of foundation modulus, flow velocity and internal pressure on the dynamic stability, frequency response and wave propagation characteristics of an un-damped system was studied [8]. Wang Shizhong, Liu Yulan, Huang Wenhu, had conducted research on solid liquid coupling dynamics of pipe conveying fluid, where they studied the influence of flowing velocity, pressure, solid-liquid coupling damping and solid-liquid coupling stiffness on natural frequency for simply supported ends [9]. Weaver D.S and Unny T.E. studied the dynamic stability of finite length of pipe conveying fluid using Flugge-Kempner equation to find the critical flow velocities [10].

In most of the cases, the differential equation of motion of fluid-conveyed pipe is deduced using the Galerkin's method in Lagrange system. Subsequently, the solution of the differential equation is obtained by considering many numerical methods such as transfer matrix, finite element, perturbation, Runge-Kutta and differential quadrature.

It is the need to have a better understanding of the dynamics of the pipes conveying fluid. The various important factors that influence the dynamic behaviour of fluid conveying pipe are (i) Flow Velocity (ii) Support conditions and (iii) Interaction with supporting medium. Hence, the estimation of exact natural frequencies of pipes is presented with exact approach for finding the transverse vibration of elastically restrained pipes.

Nomenclature

EI bending stiffness of a pipe

m_p mass of pipe

U velocity of the fluid

w lateral deflection of pipe

γ mass ratio

λ non-dimensional parameter

m_f fluid mass

t time

x the axial coordinate

V non-dimensional Velocity

$\alpha \& \beta$ coefficient of trigonometric function

ξ natural boundary conditions.

II. MATHEMATICAL MODEL

A. FORMULATION OF PROBLEM

The governing differential equation of motion and boundary conditions corresponding to the transverse vibration of pipe has been derived by considering the equilibrium approach. Consider a straight uniform single span pipe conveying fluid of length L where K_1 and K_2 are translational and kt_1 and kt_2 are rotational stiffness parameters as shown in Figure1. The displacement of the pipe is assumed to be restrained in the z-x plane.

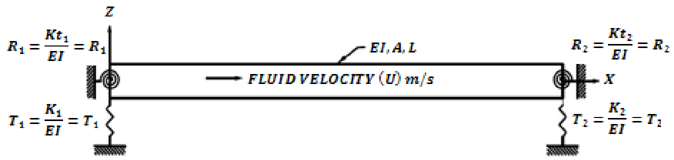

Fig.1. Elastically restrained pipe conveying fluid and with both ends supported

Where m_p is the mass per unit length of pipe, m_f is the mass of fluid per unit length, U is the flow velocity, E is the elastic modulus of the pipeline material, I is the moment of inertia of cross-section of the pipe and z is the lateral deflection of the pipeline; x and t are the axial co-ordinate and time, respectively. Considering only time dependent variables, and after simplification, we obtain the following transverse vibration equation of the pipeline conveying fluid.

From first principles and applying the Euler-Bernoulli beam theory and Hamilton's energy equations for the elastically restrained pipe conveying fluid, the differential equation of motion and boundary conditions are obtained as (3)

$$EI\frac{\partial^4 w}{\partial x^4} + (m_p + m_f)\frac{\partial^2 w}{\partial t^2} + m_f U^2\frac{\partial^2 w}{\partial x^2} + 2m_f U\frac{\partial^2 w}{\partial x\,\partial t} = 0 \quad (1)$$

EI = Bending stiffness of a pipe;

$m_p + m_f$ = mass of pipe and mass of fluid;

U= Velocity of the fluid; t = time;

$EI\frac{\partial^4 w}{\partial x^4}$ is stiffness term;

$(m_p + m_f)\frac{\partial^2 w}{\partial t^2}$ inertia force term;

$m_f U^2\frac{\partial^2 w}{\partial x^2}$ curvature term; $2m_f U\frac{\partial^2 w}{\partial x\,\partial t}$ coriolis force term.

The boundary conditions for the piping system are given below

$$\left[\left(-EI\frac{\partial^2 w}{\partial x^2}\right)\right]_{x=0} = 0 \quad (2)$$

$$\left[\left(EI\frac{\partial^3 w}{\partial x^3} - T^*.\frac{\partial w}{\partial x}\right)\right]_{x=0} = 0 \quad (3)$$

$$\left[\left(EI\frac{\partial^2 w}{\partial x^2}\right)\right]_{x=L} = 0 \quad (4)$$

$$\left[\left(-EI\frac{\partial^3 w}{\partial x^3} + T^*\frac{\partial w}{\partial x}\right)\right]_{x=L} = \quad (5)$$

Where $T^* = \alpha^2 - \beta^2$ and L=Length of the pipe supports or span.

The equation of motion Eq. (1) can be written in the following non-dimensional form:

$$\frac{\partial^4 w}{\partial \xi^4} + (V^2)\frac{\partial^2 w}{\partial \xi^2} + 2\gamma U\frac{\partial^2 w}{\partial \xi \partial t} - \lambda^4 w = 0 \quad (6)$$

Where,

γ = Mass ratio, ξ = Natural boundary condition,

ω =Natural frequency of pipe vibration,

V=Non-dimensional velocity and λ=Wavelength

$$V^2 = \left(\frac{m_f U^2 L^2}{EI}\right); \qquad \lambda^4 = \left(\frac{(m_p + m_f)\omega^2 L^4}{EI}\right)$$

$$\frac{\partial^4 w}{\partial \xi^4} + (V^2)\frac{\partial^2 w}{\partial \xi^2} - \lambda^4 w = 0 \quad (7)$$

$$2c^2 = (V^2) \ ; \qquad 2c = \sqrt{V^2}$$

When the natural frequency of the pipe approaches zero the critical flow velocity has been computed for all the end conditions. When the flow velocity is equal to the critical velocity, the pipe bows out and buckles, as the forces required to make the fluid deform to the pipe curvature are greater than the stiffness of the pipe. The term Coriolis force represents the damping of the system, and its effect on the frequency of vibration is negligible and so is omitted, as the present work aims to obtain upper bounds for the frequencies of vibration of the pipe conveying fluid. The damping term is omitted and Eq. (7) is a non-dimensional partial differential equation of higher order with boundary problem.

Now let $W(x) = c.e^{sx}$
(8)

$$\frac{\partial^4 w(x)}{\partial \xi^4} = c.s^4.e^{sx}; \ \frac{\partial^2 w}{\partial \xi^2} = c.s^2.e^{sx} \quad (9)$$

$$c.e^{s^4} + 2c.c^2.s^2 e^{sx} + \lambda^4.c.e^{sx} = 0$$

$s^4 + 2c^2.s^2 + \lambda^4 = 0$, where c and s are constants and substitution of eq. (8) in to (7) results.

$$(s^2)^2 + 2c^2.s^2 + (\lambda^2)^2 = 0$$
(10)

The roots of equation (10) is given by

$$s^2 = \frac{-2c^2 \pm \sqrt{(2c^2)^2 - 4\lambda^4}}{2}$$

$$s^2 = -c^2 \pm \sqrt{c^4 - \lambda^4}$$

$$s_1 = \alpha; \ s_2 = i\beta; \quad s_3 = -\alpha; \quad s_4 = -i\beta$$

Considering first two roots $s_1 = \alpha$ and $s_2 = i\beta$

Then $\alpha = \sqrt{-c^2 + \sqrt{c^4 + \lambda^4}}; \quad \beta = \sqrt{c^2 + \sqrt{c^4 + \lambda^4}}$
(11)

B. NATURAL FREQUENCY EVALUATION

Let the solution of the general equation (6) given as

$$W(x) = A\sinh\alpha x + B\cosh\alpha x + C\sin\beta x + D\cos\beta x$$
(12)

$$\frac{\partial W(x)}{\partial x} = A\alpha\cosh\alpha x + B\alpha\sinh\alpha x + C\beta\cos\beta x - D\beta\sin \qquad (13)$$

$$\frac{\partial^2 W(x)}{\partial x^2} = A\alpha^2\sinh\alpha x + B\alpha^2\cosh\alpha x - C\beta^2\sin\beta x - D\beta^2\cos\beta x$$

(14)

$$\frac{\partial^3 W(x)}{\partial x^3} = A\alpha^3\cosh\alpha x + B\alpha^3\sinh\alpha x - C\beta^3\cos\beta x + D\beta^3\sin\beta x$$
(15)

The above boundary conditions (2), (3), (4) and (5) are substituted in exact solution equations (12) to (15) to get the transcendental frequency equation can be written as follows

$$\begin{aligned}2\alpha\beta(\alpha^2\beta^2 + R_1T_1)(\alpha^2\beta^2 + R_2T_2) &+ \alpha\beta[(\alpha^4+\beta^4)(R_1+R_2)(T_1+T_2) \\ &+ 2\alpha^2\beta^2(R_1T_2+R_2T_1) - 2(\alpha^4\beta^4 \\ &+ R_1R_2T_1T_2)]\cosh(\alpha L)\cos(\beta L) \\ &- \alpha(\alpha^2+\beta^2)[(\alpha^4\beta^2 - T_1T_2)(R_1+R_2) \\ &+ \beta^2(\beta^2 - R_1R_2)(T_1 \\ &+ T_2)]\cosh(\alpha L)\sin(\beta L) \\ &- \beta(\alpha^2+\beta^2)[(\alpha^2\beta^4 + T_1T_2)(R_1+R_2) \\ &- \alpha^2(\alpha^2 + R_1R_2)(T_1 \\ &+ T_2)]\sinh(\alpha L)\cos(\beta L) \\ &- [\alpha^2\beta^2(\alpha^6-\beta^6) \\ &+ (\alpha^2+\beta^2)^2(\alpha^2\beta^2R_1R_2 - T_1T_2) \\ &- \alpha^2\beta^2(\alpha^2-\beta^2)(R_1T_1+R_2T_2) \\ &- (\alpha^2-\beta^2)R_1R_2T_1T_2]\sinh(\alpha L)\sin(\beta L) \\ &= 0 \qquad (16)\end{aligned}$$

Table I. Natural Frequencies (N=1) Ω_1, for Mass Ratios $\gamma_1 = \sqrt{0.1}$, $\gamma_2 = \sqrt{0.3}$ and $\gamma_3 = \sqrt{0.5}$ for Linearly and Rotationally Restrained End Conditions

V	$\Omega_1@\gamma_1$	$\Omega_1@\gamma_2$	$\Omega_1@\gamma_3$	V	$\Omega_1@\gamma_1$	$\Omega_1@\gamma_2$	$\Omega_1@\gamma_3$
0.1	2.7729	3.1203	3.3381	6.0	2.4613	2.8477	3.0852
0.2	2.7727	3.1201	3.3379	6.1	2.4502	2.8381	3.0763
0.3	2.7723	3.1197	3.3375	6.2	2.4388	2.8282	3.0673
0.4	2.7717	3.1192	3.3371	6.3	2.4271	2.8182	3.0581
0.5	2.7710	3.1186	3.3365	6.4	2.4152	2.8080	3.0487
0.6	2.7701	3.1178	3.3357	6.5	2.4030	2.7976	3.0391
0.7	2.7690	3.1168	3.3348	6.6	2.3906	2.7870	3.0294
0.8	2.7678	3.1157	3.3338	6.7	2.3780	2.7761	3.0194
0.9	2.7664	3.1145	3.3327	6.8	2.3651	2.7651	3.0093
1.0	2.7648	3.1131	3.3314	6.9	2.3519	2.7539	2.9990
1.1	2.7631	3.1116	3.3300	7.0	2.3384	2.7424	2.9885
1.2	2.7612	3.1099	3.3284	7.1	2.3247	2.7307	2.9778
1.3	2.7592	3.1081	3.3267	7.2	2.3107	2.7189	2.9669
1.4	2.7570	3.1062	3.3249	7.3	2.2964	2.7068	2.9559
1.5	2.7546	3.1040	3.3229	7.4	2.2818	2.6944	2.9446
1.6	2.7520	3.1018	3.3208	7.5	2.2670	2.6819	2.9331
1.7	2.7493	3.0994	3.3186	7.6	2.2518	2.6691	2.9215
1.8	2.7464	3.0968	3.3162	7.7	2.2363	2.6561	2.9096
1.9	2.7434	3.0941	3.3137	7.8	2.2205	2.6428	2.8975
2.0	2.7402	3.0913	3.3110	7.9	2.2044	2.6293	2.8829
2.1	2.7368	3.0883	3.3082	8.0	2.1879	2.6156	2.8728
2.2	2.7332	3.0851	3.3053	8.1	2.1712	2.6016	2.8600
2.3	2.7295	3.0818	3.3022	8.2	2.1540	2.5874	2.8471
2.4	2.7256	3.0784	3.2990	8.3	2.1366	2.5729	2.8340
2.5	2.7215	3.0748	3.2957	8.4	2.1187	2.5581	2.8206
2.6	2.7173	3.0711	3.2922	8.5	2.1005	2.5431	2.8070
2.7	2.7128	3.0672	3.2885	8.6	2.0819	2.5278	2.7932
2.8	2.7082	3.0631	3.2848	8.7	2.0630	2.5122	2.7791
2.9	2.7035	3.0589	3.2809	8.8	2.0436	2.4964	2.7649
3.0	2.6985	3.0545	3.2768	8.9	2.0238	2.4803	2.7503
3.1	2.6934	3.0500	3.2726	9.0	2.0036	2.4638	2.7355
3.2	2.6881	3.0453	3.2682	9.1	1.9830	2.4471	2.7205
3.3	2.6826	3.0405	3.2638	9.2	1.9619	2.4301	2.7052
3.4	2.6770	3.0355	3.2591	9.3	1.9403	2.4128	2.6897
3.5	2.6711	3.0304	3.2543	9.4	1.9182	2.3951	2.6739
3.6	2.6651	3.0251	3.2494	9.5	1.8957	2.3771	2.6579
3.7	2.6589	3.0196	3.2443	9.6	1.8726	2.3588	2.6415
3.8	2.6525	3.0140	3.2391	9.7	1.8490	2.3404	2.6249
3.9	2.6459	3.0082	3.2338	9.8	1.8249	2.3212	2.6080
4.0	2.6391	3.0023	3.2282	9.9	1.8002	2.3019	2.5909
4.1	2.6322	2.9962	3.2226	10.0	1.7748	2.2822	2.5734
4.2	2.6250	2.9899	3.2168	10.1	1.7488	2.2621	2.5556
4.3	2.6177	2.9835	3.2108	10.2	1.7222	2.2416	2.5376
4.4	2.6101	2.9769	3.2047	10.3	1.6949	2.2207	2.5192
4.5	2.6024	2.9701	3.1984	10.4	1.6669	2.1995	2.5005
4.6	2.5944	2.9632	3.1920	10.5	1.6381	2.1778	2.4815
4.7	2.5863	2.9561	3.1854	10.6	1.6085	2.1556	2.4621
4.8	2.5780	2.9488	3.1786	10.7	1.5780	2.1331	2.4424
4.9	2.5694	2.9414	3.1718	10.8	1.5467	2.1100	2.4223
5.0	2.5607	2.9338	3.1647	.	.	.	.
5.1	2.5517	2.9260	3.1575	.	.	.	.
5.2	2.5425	2.9180	3.1501	12.9	-	0.11260	0.9678
5.3	2.5332	2.9098	3.1426	.	.	0.10649	0.8940
5.4	2.5236	2.9015	3.1349	.	.	.	.
5.5	2.5138	2.8930	3.1270	14.6	.	-	0.7223
5.6	2.5037	2.8843	3.1190	.	.	.	0.6177
5.7	2.4935	2.8754	3.1108	.	.	.	.
5.8	2.4830	2.8664	3.1024	.	.	.	.
5.9	2.4723	2.8571	3.0939	15.6	.	.	-

Equation (16) is the general frequency equation of elastically restrained pipe conveying fluid.

Assuming C = 0 then $\alpha = \beta = \lambda$

a) Applying the B.C, $T_1 = T_2 = T$, $R_1 = R_2 = R$, *in the general frequency equation (11), for*

Linearly & Rotationally Restrained End Condition (Reference Fig.1) will result as

$$\begin{aligned}&2\alpha\beta(\alpha^2\beta^2 + RT)^2 + \alpha\beta[(\alpha^4+\beta^4)(4RT) + 4\alpha^2\beta^2RT - \\ &2(\alpha^4\beta^4 + R^2T^2)]\text{Cosh}\alpha x.\text{Cos}\beta x - \alpha(\alpha^2+\beta^2)[(\alpha^4\beta^2 - \\ &T^2)(2R) + \beta^2(\beta^2 - R^2)2T]\text{Cosh}\alpha x.\text{Sin}\beta x - \beta(\alpha^2 + \\ &\beta^2)[(\alpha^2\beta^2 + T^2)2R - \alpha^2(\beta^2 + R^2)2T]\text{Sinh}\alpha x.\text{Cos}\beta x - \\ &[\alpha^2\beta^2(\alpha^6-\beta^6)(\alpha^2\beta^2R^2 - T^2) - \alpha^2\beta^2(\alpha^2-\beta^2)2RT - \\ &(\alpha^2-\beta^2)R^2T^2]\text{Sinh}\alpha x.\text{Sin}\beta x = 0 \qquad (17)\end{aligned}$$

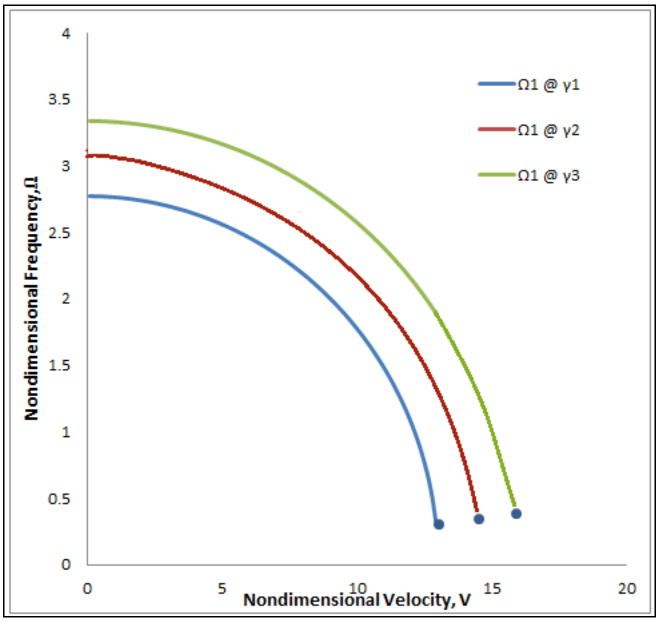

Fig.2. Linearly and Rotationally Restrained End Conditions

b) Applying the B.C, $T_1 = T_1$, $T_2 = T_2$, $R_1 = R_1$, $R_2 = 0$, in the general frequency equation [16],

for Rotationally Restrained-Linearly Restrained and Linearly Restrained Condition will result as

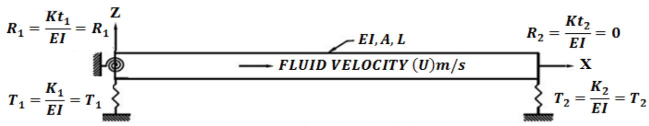

Fig.3.Rotationally Restrained –Linearly Restrained and Linearly Restrained End Condition

$$2\alpha^3\beta^3R_1 + \alpha\beta[(\alpha^4 + \beta^4)R_1]Cosh\alpha x.\,Cos\beta x - \alpha(\alpha^2 + \beta^2)[\beta^4 - R_1T_2]Cosh\alpha x.\,Sin\beta x + \beta(\alpha^2 + \beta^2)[\alpha^4 - R_1T_2]Sinh\alpha x.\,Cos\beta x + [(\alpha^2+\beta^2)^2T_2 + \alpha^2\beta^2(\alpha^2 - \beta^2)R_1]Sinh\alpha x.\,Sin\beta x = 0 \quad (18)$$

Table II. Natural Frequencies (N=1) Ω_1, for Mass Ratios $\gamma_1 = \sqrt{0.1}$, $\gamma_2 = \sqrt{0.3}$ and $\gamma_3 = \sqrt{0.5}$ for Rotationally Restrained –Linearly Restrained and linearly Restrained End Condition

V	Ω_1@ γ_1	Ω_1@ γ_2	Ω_1@ γ_3	V	Ω_1@ γ_1	Ω_1@ γ_2	Ω_1@ γ_3
0.1	3.1754	3.2105	3.2340	1.5	2.9333	2.9776	3.0069
0.2	3.1599	3.1955	3.2193	1.6	2.9136	2.9587	2.9886
0.3	3.1441	3.1802	3.2044	1.7	2.8934	2.9395	2.9700
0.4	3.1281	3.1648	3.1893	1.8	2.8728	2.9199	2.9510
0.5	3.1119	3.1491	3.1740	1.9	2.8518	2.8998	2.9316
0.6	3.0954	3.1332	3.1585	2.0	2.8303	2.8794	2.9118
0.7	3.0786	3.1170	3.1427	2.1	2.8083	2.8585	2.8916
0.8	3.0615	3.1006	3.1267	2.2	2.7857	2.8371	2.8709
0.9	3.0442	3.0839	3.1104	2.3	2.7626	2.8153	2.8499
1.0	3.0266	3.0669	3.0939	2.4	2.7389	2.7929	2.8283
1.1	3.0086	3.0497	3.0771	2.5	2.7146	2.7699	2.8062
1.2	2.9903	3.0321	3.0600	2.6	2.6895	2.7464	2.7836
1.3	2.9717	3.0143	3.0426	2.7	2.6638	2.7223	2.7605
1.4	2.9527	2.9961	3.0249	2.8	2.6373	2.6975	2.7367
1.5	2.9333	2.9776	3.0069	2.9	2.6373	2.6720	2.7123
1.6	2.9136	2.9587	2.9886	3.0	2.5818	2.6457	2.6872
1.7	2.8934	2.9395	2.9700	3.1	2.5526	2.6187	2.6614
1.8	2.8728	2.9199	2.9510	3.2	2.5224	2.5908	2.6349
1.9	2.8518	2.8998	2.9316	3.3	2.4911	2.5619	2.6075
2.0	2.8303	2.8794	2.9118	3.4	2.4585	2.5320	2.5792
2.1	2.8083	2.8585	2.8916	3.5	2.4246	2.5011	2.5499
2.2	2.7857	2.8371	2.8709	3.6	2.3893	2.4689	2.5196
2.3	2.7626	2.8153	2.8499	3.7	2.3522	2.4355	2.4882
2.4	2.7389	2.7929	2.8283	3.8	2.3134	2.4006	2.45550
2.5	2.7146	2.7699	2.8062	3.9	2.2724	2.3641	2.4215
2.6	2.6895	2.7464	2.7836	4.0	2.2292	2.3258	2.3859
2.7	2.6638	2.7223	2.7605	4.1	2.1832	2.2856	2.3488
2.8	2.6373	2.6975	2.7367	4.2	2.1342	2.2431	2.3097
2.9	2.6373	2.6720	2.7123	4.3	2.0815	2.1980	2.2686
3.0	2.5818	2.6457	2.6872	4.4	2.0245	2.1500	2.2251
0.1	3.1754	3.2105	3.2340	4.5	1.9622	2.0985	2.1789
0.2	3.1599	3.1955	3.2193	4.6	1.8933	2.0429	2.1295
0.3	3.1441	3.1802	3.2044	4.7	1.8160	1.9824	2.0765
0.4	3.1281	3.1648	3.1893	4.8	1.7273	1.9158	2.0190
0.5	3.1119	3.1491	3.1740	4.9	1.6224	1.8414	1.9562
0.6	3.0954	3.1332	3.1585	5.0	1.4919	1.7567	1.8866
0.7	3.0786	3.1170	3.1427	5.1	1.3140	1.6576	1.8084
0.8	3.0615	3.1006	3.1267	5.2	1.0018	1.5367	1.7185
0.9	3.0442	3.0839	3.1104	5.3	-	1.3777	1.6117
1.0	3.0266	3.0669	3.0939	5.4		1.1297	1.4782
1.1	3.0086	3.0497	3.0771	5.5		-	1.2937
1.2	2.9903	3.0321	3.0600	5.6			0.9536
1.3	2.9717	3.0143	3.0426	5.7			-
1.4	2.9527	2.9961	3.0249				

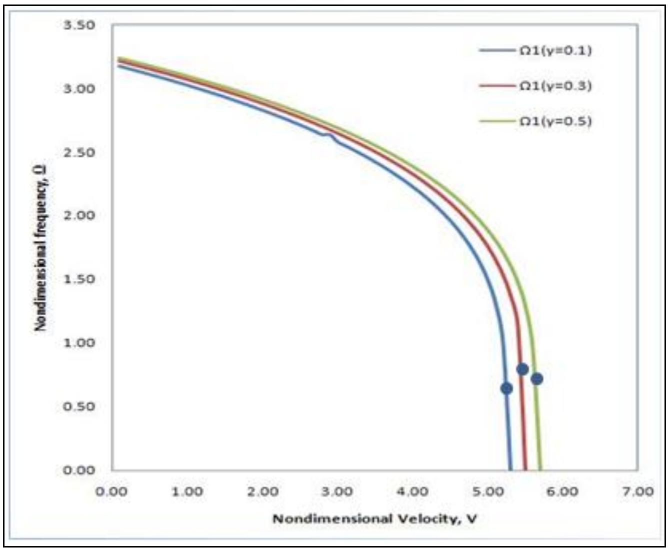

Fig.4. Rotationally Restrained-Linearly Restrained and Linearly Restrained

c) Applying the B.C, $R_1 = R_1$, $R_2 = \infty$, $T_1 = T_1$, $T_2 = 0$, in the general frequency equation [16],

for Rotationally Restrained-Linearly Restrained and Fixed End Condition will result as

$$\alpha\beta[(\alpha^4 + \beta^4)T_1 + 2\alpha^2\beta^2T_1]Cosh\alpha x.Cos\beta x - \alpha(\alpha^2 + \beta^2)[\alpha^4\beta^2 - \beta^2R_1T_1]Cosh\alpha x.Sin\beta x - \beta(\alpha^2 + \beta^2)[\alpha^2\beta^4 - \alpha^2R_1T_1]Sinh\alpha x.Cos\beta x - [(\alpha^2 + \beta^2)^2\alpha^2\beta^2R_1]Sinh\alpha x.Sin\beta x = 0 \quad (19)$$

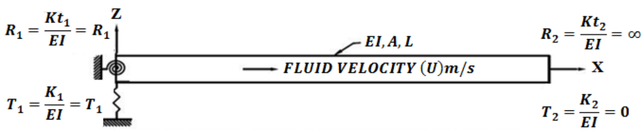

Fig.5. Rotationally Restrained –Linearly Restrained and Fixed End Condition

Table III. Natural Frequencies (N=1) Ω_1 for Mass Ratios $\gamma_1=\sqrt{0.1}$, $\gamma_2=\sqrt{0.3}$ and $\gamma_3=\sqrt{0.5}$ for Rotationally Restrained and Linearly Restrained and Fixed End Condition

V	Ω_1@ γ_1	Ω_1@ γ_2	Ω_1@ γ_3	V	Ω_1@ γ_1	Ω_1@ γ_2	Ω_1@ γ_3
0.1	3.1754	3.2105	3.2340	2.7	2.6638	2.7223	2.7605
0.2	3.1599	3.1955	3.2193	2.8	2.6373	2.6975	2.7367
0.3	3.1441	3.1802	3.2044	2.9	2.610	2.6720	2.7123
0.4	3.1281	3.1648	3.1893	3.0	2.5818	2.6457	2.6872
0.5	3.1119	3.1491	3.1740	3.1	2.5526	2.6187	2.6614
0.6	3.0954	3.1332	3.1650	3.2	2.5224	2.5908	2.6349
0.7	3.0786	3.1170	3.1427	3.3	2.4911	2.5619	2.6075
0.8	3.0615	3.1006	3.1267	3.4	2.4585	2.5320	2.5792
0.9	3.0442	3.0839	3.1104	3.5	2.4246	2.5011	2.5499
1.0	3.0266	3.0669	3.0939	3.6	2.3893	2.4689	2.5196
1.1	3.0086	3.0497	3.0771	3.7	2.3522	2.4355	2.4882
1.2	2.9903	3.0321	3.0600	3.8	2.3134	2.4006	2.4555
1.3	2.9717	3.0143	3.0426	3.9	2.2724	2.3641	2.4215
1.4	2.9527	2.9961	3.0249	4.0	2.2292	2.3258	2.3859
1.5	2.9333	2.9776	3.0069	4.1	2.1832	2.2856	2.3488
1.6	2.9136	2.9587	2.9886	4.2	2.1342	2.2856	2.3097
1.7	2.8934	2.9395	2.9700	4.3	2.0815	2.1980	2.2686
1.8	2.8728	2.9199	2.9510	4.4	2.0245	2.1500	2.2251
1.9	2.8518	2.8998	2.9316	4.5	1.9622	2.0985	2.1789
2.0	2.8303	2.8794	2.9118	.	.	.	.
2.1	2.8083	2.8585	2.8916	5.1	-	1.6042	1.7679
2.2	2.7857	2.8371	2.8709	5.2		1.5367	1.7185
2.3	2.7626	2.8153	2.8499	.	.	.	.
2.4	2.7389	2.7929	2.8283	5.3		-	1.5093
2.5	2.7146	2.7699	2.8062	5.4	.	.	1.4782
2.6	2.6895	2.7464	2.7836	5.5			-

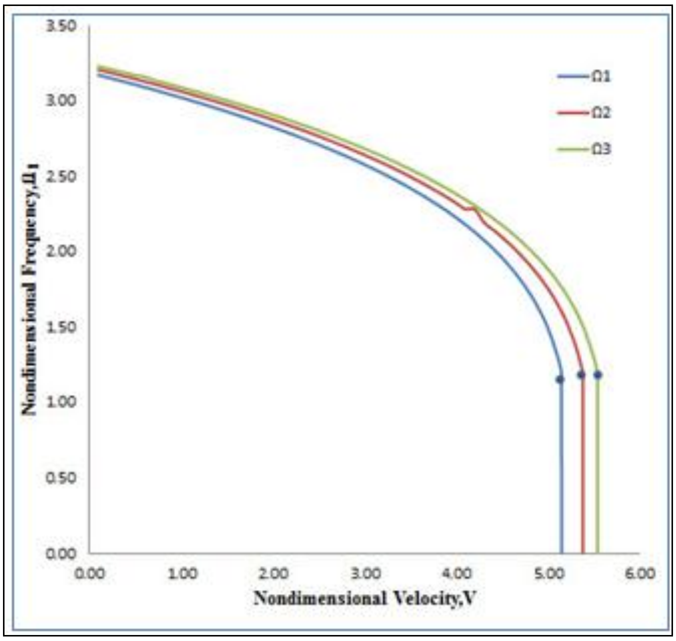

Fig.6. Rotationally Restrained and Linearly Restrained and Fixed End Condition

d) Applying the B.C, $R_1 = R_1$, $R_2 = \infty$, $T_1 = 0$, $T_2 = \infty$, in the general frequency equation [16], for Rotationally Restrained and Guided End Condition will result as

$$2\alpha^2\beta^2 + (\alpha^4+\beta^4)Cosh\alpha x.Cos\beta x + \beta(\alpha^2 + \beta^2)R_1 Cosh\alpha x.Sin\beta x + \alpha(\alpha^2+\beta^2)R_1 Sinh\alpha x.Cos\beta x - \alpha\beta(\alpha^2-\beta^2)Sinh\alpha x.Sin\beta x = 0 \quad (20)$$

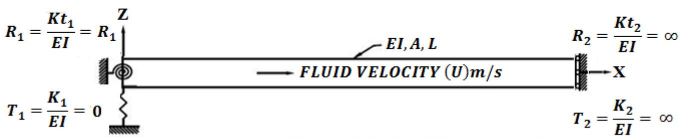

Fig.7. Rotationally Restrained and Guided End Condition

Table IV. Natural Frequencies (N=1) Ω_1 for Mass Ratios $\gamma_1=\sqrt{0.1}$, $\gamma_2=\sqrt{0.3}$ and $\gamma_3=\sqrt{0.5}$ for Rotationally Restrained and Guided End Condition

V	Ω_1@γ_1	Ω_1@γ_2	Ω_1@γ_3
0.1	1.8577	1.8336	1.8156
0.2	1.8665	1.8445	1.8269
0.3	1.8740	1.8547	1.8381
0.4	1.8800	1.8639	1.8488
0.5	1.8843	1.8718	1.8586
0.6	1.8869	1.8782	1.8672
0.7	1.8875	1.8831	1.8746
0.8	1.8862	1.8863	1.8804
0.9	1.8827	1.8875	1.8846
1.0	1.8768	1.8868	1.8870
1.1	1.8685	1.8840	1.8875
1.2	1.8573	1.8789	1.8860
1.3	1.8429	1.8714	1.8823
1.4	1.8250	1.8611	1.8762
1.5	1.8026	1.8478	1.8676
1.6	-	1.8311	1.8561
1.7		1.8102	1.8415
1.8		-	1.8231
1.9			1.8003
2.0			-

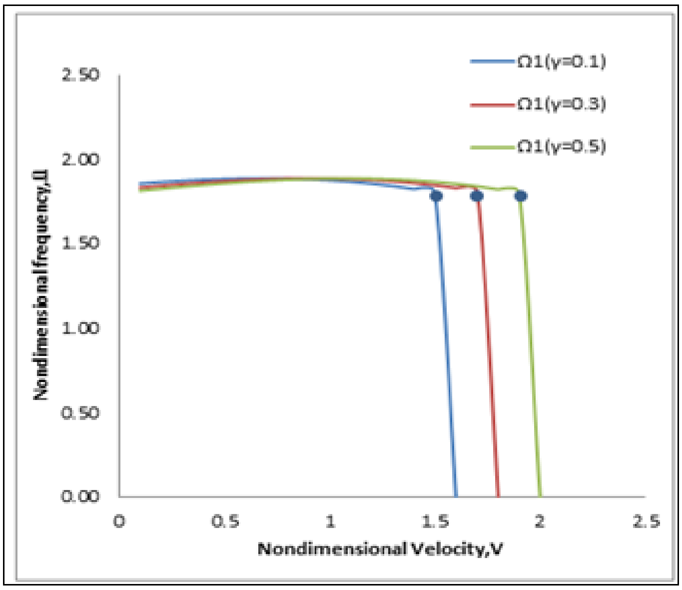

Fig.8. Rotationally Restrained and Guided End Condition

III. RESULTS AND DISCUSSIONS

Table 1, shows the graph of non-dimensional natural frequencies with non-dimensional velocities. In the case of a) Linearly and Rotationally Restrained end conditions the instability region lies in the range of 12.99435 to 15.66188. However, in other three cases like b) Rotationally Restrained –Linearly Restrained and Linearly Restrained end conditions c) Rotationally Restrained and Linearly Restrained and Fixed end conditions d) Rotationally Restrained and Guided end conditions are shown in Tables 2, 3 & 4 the pipe flutters at a much lower velocity, in the flow region of b) V=5.3 to V=5.7, c) V=5.147 to V=5.338 and d) V=1.6 to V=2.0. Figures 2, 4, 6 and 8 shows the points of flutter for three mass ratios. The percentage reduction in frequency as velocity increases from (reference fig.2) V=0.1 to V=2.77298 is 72 %, For Tables 2 and 3 (reference fig.4 & 6) from V=0.1 to V=3.175468 shows 68.24%, reduction in frequency. Table 4 shows the frequency reduction (reference fig.8) from V=0.1 to V=1.85777 is 81.42%. It is found that the natural frequencies remain same for all the three mass ratios, which means that the instability condition is close with higher mass ratio and fluid velocity.

IV. CONCLUSIONS

- Exact method is developed for pipes conveying fluid for Linearly Restrained and Rotationally restrained end conditions, Rotationally Restrained –Linearly Restrained and Linearly Restrained end conditions, Rotationally Restrained –Linearly Restrained and Fixed end conditions and Rotationally Restrained and Guided end conditions
- The frequencies of the first mode of vibration are computed by varying the fluid velocity
- Critical velocity for different mass ratios are found
- A FORTRAN program is developed for computation of natural frequencies by using Mueller’s Iteration method for non-linear equations (Bi-section) and the iterated value of x (non-dimensional) natural frequency is found by the Inverse Parabolic Interpolation method
- The natural frequencies are obtained by varying the fluid velocities.

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REFERENCES

[1] Ashley H, Haviland G, “Bending Vibration of a Pipeline containing flowing fluid”, Transactions of ASME, Journal of Applied Mechanics, Sept 1950, pp 229-232

[2] G.W. Housner, “Bending vibration of a pipe line containing flowing fluid”, J. Appl. Mech., 19, 6(1952), pp205-208

[3] Rao. SS., "Vibration of Continuous Systems" John-Wiley &Sons Inc. Publishing company USA 2007, pp. 317-328.

[4] S. Naguleswaran & C. J. H. Williams.," Lateral Vibration of a Pipe Conveying a Fluid" published in Journal of Mechanical Engineering Science June 1968 vol. 10 no. 3, pp 228-238

[5] Chen S-S. ,"Free Vibration of a coupled fluid/structural system "Journal of Sound and Vibration (1972) 21(4) , pp387-398

[6] M.P Paidoussis., "Dynamic Stability of pipes conveying fluid" Journal of Sound and Vibration (1974)33(3), pp 267- 294

[7] Huang Yi-min, Liu Yong-shou, Li Bao-hui, Li Yan-jiang, Yue Zhu-feng, “Natural frequency analysis of fluid conveying pipeline with different boundary conditions”, Nuclear Engineering and design 240(2010), pp 461-467

[8] Stein R. A., Tobriner M.W., “Vibration of pipes containing flowing fluid”, Transactions of the ASME, Dec 1970, pp 906-916

[9] Wang Shizhong, Liu Yulan, Huang Wenhu, “Research on Solid-Liquid coupling Dynamics of pipe conveying fluid”, Journal of Applied Mathematics and Mechanics, Vol.19, No. 11, Nov 1998, pp 1065-1071

[10] Weaver D.S and Unny T.E., “On the Dynamic Stability of Fluid conveying Pipes”, Transactions of ASME, March 1973, 48-52

[11] Chaos, Solitons& Fractals, “Class of analytical closed-form polynomial solutions for clamped–guided inhomogeneous beams”, Volume 12, Issue 9, July 2001, Pages 1657-1678