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Flow Shop Scheduling Problem with Four Machines N-Job for Minimizing Makespan

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***Abstract*: In this paper we try to find algorithm for flow shop scheduling problem of 4 machines with n-jobs. Every job has to process on all machines which are arranged in series. After finishing of each job on first machine it will be transferred on second machine without delay with the help of transporting agent and so on. We also identify other factors which are responsible for scheduling like importance of jobs for arranging in sequence that is weightage assigned to each job, breakdown interval of machine, transportation time of jobs etc. By using Johnson's rule we try to explain the mathematical modelling, structure, suppositions, theorem with its statement and proof which is applicable to solve the numerical problem.**

Key words: Flow shop scheduling, Johnson's rule, setup time of machines, breakdown interval, transporting time

I INTRODUCTION

At present period, every manufacturer has to concentrate on innovative ways of production agenda to get success in today's competition of

rketing area. For this one should use available resources fruitfully and plan a proper schedule so that the purpose of optimization of production can be achieved.

The theory of scheduling is basically focused on Johnson's result of two machines. The analysis of all flow shop scheduling problems can be done by using this result.

Proper planning of scheduling in production and marketing is effective in decreasing price, superior quality of product to satisfy the demands of consumer in the competition of market. Ali Allahverdi [3] described the problem of three machine flow shop scheduling with setup and processing time treated separate from each other only start and end time is known. Hence he introduced the concept of dominance relation which helps to reduce the size of dominating schedule. Ali Allahverdi [2] summarized the data of scheduling literature on models with setup times for more than 300 papers. Bo Chen et al.[4] considered a case of flow shop scheduling problem of three machines and n jobs. He has used O(nlogn) time heuristic which depends on Johnson's algorithm to minimize the total completion time of the jobs. The completion time is 1.5 times the total time of the schedule. Ajaykumar Agarwal et.al[1]used heuristic approach for solving NP hard flow shop scheduling problem of m-machines n jobs by considering the criterion of reduction of total working hours by using RA's and CD's algorithm and Gantt chart. According to Kenneth Baker[10] flow shop in ordered way is one of the most important cases of scheduling problem. For this there are two conditions –

1) If the processing time of job i_1 on one of the machine is smaller than the processing time of another job i_2 on same machine then i_1 always takes less time than i_2 on other machines also.

2) If the processing time of job i_1 is least on first machine then it is same for other jobs also. First condition helps us to find the job size.

Scheduling problems with m-machines and n-jobs by using Johnson's rule and different heuristics are solved by many authors in their research [5, 6, 7, 8, 13].
Let us prove lemma which is helpful in the proof of theorem stated below.

Lemma:- If $min\{p_i + P_i + a_i\} \geq max\,\{q_i + Q_i + a_i\}$ then $CP_m + a_m \geq CQ_{m-1}$ where
p_i is the set up time of machine Pi & q_i, is the set up time of machine Qi. a_i, is the transportation time of item i from machine Pi to Qi,

By using the method of induction we can prove this lemma.

II THEOREM FOR GETTING AN OPTIMAL SOLUTION

When the articles i-1, i, i+1 are arranged in sequence, the optimal solution can be obtained so that

$$\text{Min}\ (\ p_i + P_i + a_i + q_i + Q_i + b_i + r_i + R_i + c_i\,,\ a_{i+1} + q_{i+1} + Q_{i+1} + b_{i+1} + r_{i+1} + R_{i+1} + c_{i+1} + s_{i+1} + S_{i+1}\,) < Min(p_{i+1} + P_{i+1} + a_{i+1} + q_{i+1} + Q_{i+1} + b_{i+1} + r_{i+1} + R_{i+1} + c_{i+1}\,,\ a_i + q_i + Q_i + b_i + r_i + R_i + c_i + s_i + S_i\,)$$

Let X and X' denotes sequences of items.

$$X = \{T_1, T_2, --\ T_{i-1}, T_i, T_{i+1}, T_{i+2}, ---T_n\}$$

$$X' = \{T'_1, T'_2, --\ T'_{i-1}, T'_i, T'_{i+1}, T'_{i+2}, ---T'_n\}$$

Let processing time of any item j on machine Y (ie. P, Q, R, S) for sequences X, X' is given by (Y_j, Y'_j) Completion time of any item j on machine Y (ie. P, Q, R, S) for sequences X, X' is given by (CY_j, CY'_j)

Let (a_j, a'_j) be the transportation time of item j from machine P to Q, (b_j, b'_j) be the transportation time of item j from machine Q to R, and (c_j, c'_j) be the transportation time of item j from machine R to S.

Let p_j, p'_j be the set up time of machine P, q_j, q'_j be the set up time of machine Q, r_j, r'_j be the set up time of machine R, s_j, s'_j set up time of item on machines S resp. for sequences X and X'.

The time required on machines Q, R & S for completion of j[th] item is given by

$CQ_j = \max(cP_j + a_j,\ cQ_{j-1}) + q_j + Q_j = cP_j + a_j + q_j + Q_j$

$CR_j = \max(cQ_J + b_j,\ cR_{j-1}) + r_J + R_J$

$= cQ_J + b_j + r_J + R_J$

$CS_j = \max(cR_j + c_j,\ cS_{j-1}) + s_j + S_j = \max(cP_j + a_j + q_j + Q_j + b_j + r_j + R_j + c_j,\ cS_{j-1}) + s_j + S_j$ ---- (1)

We will choose the sequence X such that $cS_n < c'S_n$ ---------- (2)

$\text{Max}\ (cP_m + a_m + q_m + Q_m + b_m + r_m + R_m + c_m,\ cS_{m-1}) + s_m + S_m$

$< \max(c'P_m + a'_m + q'_m + Q_m' + b'_m + r'_m + R'_m + c'_m,\ c'S_{m-1}) + s'_m + S'_m$

But $cP_m + a_m + q_m + Q_m + b_m + r_m + R_m + c_m = c'P_m + a'_m + q'_m + Q'_m + b'_m + r'_m + R'_m + c'_m$

Also $s_m = s'_m, S_m = S'_m$

Therefore equation 2 will be true if $cS'_{m-1} < c'S_{m-1}$

Proceeding in this way we get that inequality 2 is true if

$cS_j < c'S_j$ (j = i+1,i+2-----,m & i= 1,2,------m-1) ------------- (4)

We now calculate the values of cS_{i+1} & $c'S_{i+1}$

$cS_{i+1} = max(cR_{i+1} + c_{i+1}, cS_i) + s_{i+1} + S_{i+1}$

$= max(cP_{i+1} + a_{i+1} + q_{i+1} + Q_{i+1} + b_{i+1} + r_{i+1} + R_{i+1} + c_{i+1}, cs_i) + s_{i+1} + S_{i+1}$

$= max\{c P_{i+1} + a_{i+1} + q_{i+1} + Q_{i+1} + b_{i+1} + r_{i+1} + R_{i+1} + c_{i+1}, \max(cR_i + c_i, cS_{i-1}\} + s_i + S_i\} + s_{i+1} + S_{i+1}$

$= max\{c P_{i+1} + a_{i+1} + q_{i+1} + Q_{i+1} + b_{i+1} + r_{i+1} + R_{i+1} + c_{i+1}, cR_i + c_i + s_i + S_i, cS_{i-1} + s_i + S_i\} + s_{i+1} + S_{i+1}$

$= max\ \{c P_{i+1} + a_{i+1} + q_{i+1} + Q_{i+1} + b_{i+1} + r_{i+1} + R_{i+1} + c_{i+1}, \max(c P_i + a_i, cQ_{i-1}) + q_i + Q_i + b_i + r_i + R_i + c_i + s_i + S_i, cS_{i-1} + s_i + S_i\} + s_{i+1} + S_{i+1} = max\{c P_{i+1} + a_{i+1} + q_{i+1} + Q_{i+1} + b_{i+1} + r_{i+1} + R_{i+1} + c_{i+1}, c P_i + a_i + q_i + Q_i + b_i + r_i + R_i + c_i + s_i + S_i, cS_{i-1} + s_i + S_i\} + s_{i+1} + S_{i+1}$

$cS_{i+1} = \max\{c P_{i-1} + p_i + P_i + p_{i+1} + P_{i+1} + a_{i+1} + q_{i+1} + Q_{i+1} + b_{i+1} + r_{i+1} + R_{i+1} + c_{i+1} + s_{i+1} + S_{i+1}, c P_{i-1} + p_i + P_i + a_i + q_i + Q_i + b_i + r_i + R_i + c_i + s_i + S_i + s_{i+1} + S_{i+1}, cS_{i-1} + s_i + S_i + s_{i+1} + S_{i+1}\}$ ---- (5)

Similarly

$C'S_{i+1} = \max\left\{c'P_{i-1} + p'_i + P'_i + p'_{i+1} + P'_{i+1} + a'_{i+1} + q'_{i+1} + Q'_{i+1} + b'_{i+1} + r'_{i+1} + R'_{i+1} + c'_{i+1} + s'_{i+1} + S'_{i+1}, c'P_{i-1} + p'_i + P'_i + a'_i + q'_i + Q'_i + b'_i + r'_i + R'_i + c'_i + s'_i + S'_i + s'_{i+1} + S'_{i+1}, cS'_{i-1} + s'_i + S'_i + s'_{i+1} + S'_{i+1}\right\}$ -----------(6)

Comparing sequences S & S',

We get $c P_{i-1} = c' P_{i-1}$ & $c S_{i-1} = c' S_{i-1}$, $Y_i = Y'_{i+1}$, $Y_{i+1} = Y'_i$ where Y = P, Q, R, or S ---------(7)

Also $a_i = a'_{i+1}$, $a_{i+1} = a'_i$, $b_i = b'_{i+1}$, $b_{i+1} = b'_i$, $c_i = c'_{i+1}$, $c_{i+1} = c'_i$

Writing eq. 2 for j=i+1 & using eq. 7, we get

$\text{Max}\ \{c P_{i-1} + p_i + P_i + p_{i+1} + P_{i+1} + a_{i+1} + q_{i+1} + Q_{i+1} + b_{i+1} + r_{i+1} + R_{i+1} + c_{i+1} + s_{i+1} + S_{i+1}, c P_{i-1} + p_i + P_i + a_i + q_i + Q_i + b_i + r_i + R_i + c_i + s_i + S_i + s_{i+1} + S_{i+1}, cS_{i-1} + s_i + S_i + s_{i+1} + S_{i+1}\} < Max\{c P_{i-1} + p_{i+1} + P_{i+1} + p_i + P_i + a_i + q_i + Q_i + b_i + r_i + R_i + c_i + s_i + S_i, c P_{i-1} + p_{i+1} + P_{i+1} + a_{i+1} + q_{i+1} + Q_{i+1} + b_{i+1} + r_{i+1} + R_{i+1} + c_{i+1} + s_{i+1} + S_{i+1} + s_i + S_i, c S_{i-1} + s_{i+1} + S_{i+1} + s_i + S_i\}$ ---------(8)

Subtracting last term from both sides, we get

$\text{Max}\ \{c P_{i-1} + p_i + P_i + p_{i+1} + P_{i+1} + a_{i+1} + q_{i+1} + Q_{i+1} + b_{i+1} + r_{i+1} + R_{i+1} + c_{i+1} + s_{i+1} + S_{i+1}, c P_{i-1} + p_i + P_i + a_i + q_i + Q_i + b_i + r_i + R_i + c_i + s_i + S_i +$

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$s_{i+1} + S_{i+1} \} < \mathrm{Max}\{c\, P_{i-1} + p_{i+1} + P_{i+1} + p_i + P_i + a_i + q_i + Q_i + b_i + r_i + R_i + c_i + s_i + S_{i,}\ c\, P_{i-1} + p_{i+1} + P_{i+1} + a_{i+1} + q_{i+1} + Q_{i+1} + b_{i+1} + r_{i+1} + R_{i+1} + c_{i+1} + s_{i+1} + S_{i+1} + s_i + S_{i,} \}$

Subtracting $c\, P_{i-1} + p_i + P_i + p_{i+1} + P_{i+1} + a_i + a_{i+1} + b_i + b_{i+1} + c_i + c_{i+1} + q_i + Q_i + q_{i+1} + Q_{i+1} + r_i + R_i + r_{i+1} + R_{i+1} + s_i + S_i + s_{i+1} + S_{i+1},$ From both sides, we get

$$\mathrm{Min}\ (p_i + P_i + a_i + q_i + Q_i + b_i + r_i + R_i + c_i\, ,\ a_{i+1} + q_{i+1} + Q_{i+1} + b_{i+1} + r_{i+1} + R_{i+1} + c_{i+1} + s_{i+1} + S_{i+1})$$

$$< Min(p_{i+1} + P_{i+1} + a_{i+1} + q_{i+1} + Q_{i+1} + b_{i+1} + r_{i+1} + R_{i+1} + c_{i+1}\, ,\ a_i + q_i + Q_i + b_i + r_i + R_i + c_i + s_i + S_i)$$

Hence proved.

III FOUR MACHINES ALGORITHM

This theorem can be used in solving four machine problem of optimization for getting total elapsed time that is numerical problem solution can be obtained by applying above proved theorem. In this problem four machines with their setup time, processing time, transporting time, as well as waiting time are considered.

The problem can be given in tabular form as follows:

A_i	p_i	P_i	a_i	q_i	Q_i	b_i	r_i	R_i	c_i	s_i	S_i	wt_i
A_1	p_1	P_1	a_1	q_1	Q_1	b_1	r_1	R_1	c_1	S 1	S_1	wt_1
A_2	p_2	P_2	a_2	q_2	Q_2	b_2	r_2	R_2	c_2	S_2	S_2	wt_2
A_3	p_3	P_3	a_3	q_3	Q_3	b_3	r_3	R_3	c_3	S_3	S_3	wt_3
A_4	p_4	P_4	a_4	q_4	Q_4	b_4	r_4	R_4	c_4	S_4	S_4	wt_4
A_5	p_5	P_5	a_5	q_5	Q_5	b_5	r_5	R_5	c_5	S_5	S_5	wt_5

In the given problem four machines namely P, Q, R, S are arranged in series such that the articles A_1, A_2, A_3, ------ A_n are transported by transporting agent from machine P to machine Q, machine Q to machine R, machine R to machine S in such a way that after delivering the articles to machine S without delay come back to machine P for transferring the next item. The problem is solved for finding the solution which is optimum for the given production system in minimum time period.

Let $\mathbf{a_i}$ denotes transporting time that is the time required for transportation of the articles from P to Q, $\mathbf{b_i}$ denotes transporting time required for transportation of the articles from Q to R, $\mathbf{c_i}$ denotes transporting time required for transportation of the articles from R to S.

For producing the articles machines required the set up time which is denoted by $\mathbf{p_i}$:- set up time of machine P, $\mathbf{q_i}$, :- set up time of machine Q, $\mathbf{r_i}$, :- set up time of machine R, $\mathbf{s_i}$:- set up time of machine S

Let R_i is the time required for the transporting agent from S and P which is called as the returning time. When the machine P has completed the production of A_{i-1} article and the transporting agent delivering this article to machine S come back to machine P but if the processing of last article by machine S is completed as it starts this processing instantly after giving out of previous article then machine P has to wait for the transporting agent if it didn't come back by that time.

Let P_i :- Processing time of machine P, Q_i :- Processing time of machine Q, R_i :- Processing time of machine R, and S_i:- Processing time of machine S,

Step I: Four machine problem reduced into three machine problem by introducing three assumed machines E, F and G with the service time E_i, F_i and G_i resp. Where,

$E_i = p_i + P_i + a_i + q_i + Q_i + b_i$,

$F_i = a_i + q_i + Q_i + bi + r_i + R_i$

$G_i = bi + r_i + Q_i + c_i + s_i + S_i$

Article	E_i	F_i	G_i	wt_i
A_1	E_1	F_1	G_1	wt_1
A_2	E_2	F_2	G_2	wt_2
A_3	E_3	F_3	G_3	wt_3
A_4	E_4	F_4	G_4	wt_4
A_5	E_5	F_5	G_5	wt_5

1) Min $(p_i + P_i + a_i) \geq$ Max $(a_i + q_i + Q_i)$ 2) Min $(bi + r_i + R_i) \geq$ Max $(a_i + q_i + Q_i)$ 3) Min $(c_i + s_i + S_i) \geq$ Max $(c_i + r_i + R_i)$

Step II:

Now this problem will be reduced into two machine problem by considering two fictitious machines H and K where $Hi = E_i + F_i$ and $Ki = F_i + G_i$.

1) If min (H,K) = H_i then $H'_i = H_i - wt_i$ and $K'_i = K_i$

2) If min (H,K) = K_i then $H'_i = H_i$ and $K'_I = K_i + wt_i$

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Step III:

For getting the proper sequence the new problem is defined as follows and represented in the table form: $H'i = H'i/ wt_i$ and $K'i = K'i/ wt_i$

Article	H'_i / wt_i	K'_i / wt_i	wt_i
A_1	H'_1 / wt_1	K'_1 / wt_1	wt_1
A_2	H'_2/ wt_2	K'_2/ wt_2	wt_2
A_3	H'_3 / wt_3	K'_3 / wt_3	wt_3
A_4	H'_4 / wt_4	K'_4 / wt_4	wt_4
A_5	H'_5 / wt_5	K'_5 / wt_5	Wt_5

Step IV:

Considering the breakdown interval which is already decided or known and the effect of this breakdown interval should be observed on all jobs. With the effect of this breakdown interval, the problem will be redefined as follows:

If the job is affected by the breakdown interval (u_1-u_2) then $P'_i = P_i + (u_1-u_2)$, $Q'_i = Q_i + (u_1-u_2)$, $R'_i = R_i + (u_1-u_2)$, $S'_i = S_i + (u_1-u_2)$

If the job is not affected by the breakdown interval then $P'_i = P_i$, $Q'_i = Q_i$, $R'_i = R_i$ and $S'_i = S_i$

Step V: Applying steps I, II, III, IV the problem has been solved for getting the optimal sequence.
Every job has not equal importance hence all the jobs are assigned with weight according to their importance in the sequence of production and it is considered as one of the measure in the computation of total make-span. So the entirety weighted flow time is summation of product value of weighted value of the job and its flow time. Also the mean weighted flow time is the ratio of weighted flow time and sum of the values of weight of all the jobs.

Hence, Mean weighted flow time = weighted flow time/ sum of weights

The scheduling of all the course of action is in such a way that the minimum time should be required for getting the optimum solution or whole production For finding the algorithm the above information can be symbolized and represented in table for finding the sequence by using Johnson's rule of sequencing.

IV PROBLEM DESCRIPTION:

Let us consider the problem of four machines with set up time, processing time, a transporting time, weight of the jobs, arranged in series:

Ai	p_i	P_i	a_i	q_i	Q_i	b_i	r_i	R_i	c_i	s_i	S_i	wt_i
A_1	2	6	4	3	5	3	4	6	2	5	6	2
A_2	3	5	6	3	2	5	2	5	3	6	5	4
A_3	3	4	5	2	4	2	4	6	3	4	6	3
A_4	4	5	3	2	4	4	2	7	2	3	8	5
A_5	2	8	3	1	3	6	1	8	2	4	8	2

Step I: Min $(p_i + P_i + a_i)$= 12 and Max $(a_i + q_i + Q_i)$= 12

As discussed earlier about three fictitious machines E, F and G and their service times E_i, F_i, and G_i. The problem is reduced and it is given in the following table.

Ai	E_i	F_i	G_i	wt_i
A_1	23	25	26	2
A_2	24	23	26	4
A_3	20	23	25	3
A_4	22	22	26	5
A_5	23	22	29	2

Step II:Now this problem will be reduced into two machine problem by considering two fictitious machines H and K where $Hi = E_i + F_i$ and $Ki = F_i + G_i$

Ai	Hi	Ki	wt_i
A_1	48	51	2
A_2	47	49	4
A_3	43	48	3
A_4	44	48	5
A_5	45	51	2

If min (H,K) = H_i then $H'_i = H_i - wt_i$ and $K'_i = K_i$ & if min (H,K) = K_i then $H'_i = H_i$ and $K'_I = K_i + wt_i$

Ai	H'_i	K'_i	wt_i
A_1	46	51	2
A_2	43	49	4
A_3	40	48	3
A_4	39	48	5
A_5	43	51	2

Step III: The ratio of weights with Hi and Ki is taken so that we can decide the sequence of jobs according to Johnson's rule.

Article	H'_i / w_i	K'_i / w_i	wt_i
A_1	23	25.5	2
A_2	10.75	12.25	4
A_3	13.33	16	3
A_4	7.8	9.6	5
A_5	21.5	25.5	2

By Johnson's rule the optimal sequence obtained for above reduced problem is 4, 2, 3, 5, 1. Then the time required for total processing of articles by using above scheduling sequence i.e minimum time for entire production can be calculated by considering the time required by the transporting agent when it returns back to machine M_1 to load the next article and also the time when it reaches to machine M_2 for unloading of an article.

Step IV:

A_i	p_i	P_i		a_i	q_i	Q_i		b_i	r_i	R_i		c_i	s_i	S_i		w_i
		I	O			I	O			I	O			I	O	
4	4	4	9	3	2	14	18	4	2	24	31	2	3	**36**	**50**	5
2	3	12	17	6	3	26	28	5	2	**35**	**48**	3	6	57	62	4
3	3	20	24	5	2	31	35	2	4	52	58	3	4	66	72	3
5	2	26	34	3	1	**38**	**49**	6	1	59	67	2	4	76	84	2
1	2	**36**	**50**	4	3	57	62	3	4	71	77	2	5	89	95	2

Effect of breakdown interval (36, 44):

The effect of breakdown interval is on jobs P_i1 and on Q_i5, R_i2, S_i4 hence the original problem is converted into new problem. As mentioned above in step IV we redefine the problem by considering the effect of breakdown interval

For finding the sequence and getting the optimal solution repeating the same procedure of step 1, 2, 3

A_i	p_i	P_i		a_i	q_i	Q_i		b_i	r_i	R_i		c_i	s_i	S_i		w_i
		I	O			I	O			I	O			I	O	
4	4	4	9	3	2	14	18	4	2	24	31	2	3	**36**	**50**	5
2	3	12	17	6	3	26	28	5	2	**35**	**48**	3	6	57	62	4
3	3	20	24	5	2	31	35	2	4	52	58	3	4	66	72	3
5	2	26	34	3	1	**38**	**49**	6	1	59	67	2	4	76	84	2
1	2	**36**	**50**	4	3	57	62	3	4	71	77	2	5	89	95	2

(Note: two digits in one box of every column are two digit numbers ex.1 &4 means 14 and so on).

Repeating the same procedure of step 1, 2, 3 for finding the sequence and getting the optimal solution.By Johnson's rule the optimal sequence obtained for above reduced problem is 4, 2, 3, 5, 1.

(Note: two digits in one box of every column are two digit numbers ex.1 &4 means 14 and so on).

Minimum weighted flow time (MWFT)

= (50*5) + (62- 9) *4 + (72-17)*3 +(84-24)*2 + (95-34)*2122 / 5+4+3+2+2

= 54 hours

V CONCLUSION:

From above table it is shown that the time gets reduced for total production by using the sequence obtained with the help of Johnson's rule. The total elapsed time for the complete process is 95 hrs and minimum weighted flow time is 54 hrs.

In this paper setup time and processing time both are treated separately. Also effect of break down interval is calculated which helps to get proper schedule and reduce the total time span also. One can prove the theorem for less than four machines also and solve the problem by using Johnson's rule. If the number of machines are less in number then the makespan gets decreased as well as minimum weighted flow time also gets reduced.

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