Flow Distribution Network with Triangular Intuitionistic Fuzzy Number as EDGE Weight

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Abstract— **This paper deals with flow distribution of a network. The objective of flow distribution network is to maximize the flow subject to the edge capacities. In this paper an algorithm is proposed to find the path with maximum flow from source node to all other nodes of a network where edge weights are considered as triangular intuitionistic fuzzy number.**

***Keywords*— Maximal flow, Network, Label Setting Method, Intuitionistic Fuzzy Number.**

1. INTRODUCTION

Network is a graph consisting of a finite non-empty set of vertices (V), a set of edges (E) connecting the vertices (E ⊆ V x V) and a set of associated weights of the edges. Maximum flow problem for a given network with flow capacities on the edge weight is to determine the maximum possible flow from source node to destination node, with respect to the edge capacity. The first maximum flow algorithm, was proposed by Ford and Fulkerson [5] in 1956. The concept of flow problems was discussed by Andrew Goldberg &Tarjan [1]. Time varying cost flow problems was discussed by Cai et al. [2]. Applications of flow problem were given by Foulds [6]. The notion of fuzzy numbers was introduced by Dubois and Prade [3].In 2010, Deng Feng Li et al. [4] proposed a ranking method for triangular intuitionistic fuzzy number. In 2009, fuzzy quantities and relations in multi objective flow problem were analyzed by Mehdi Ghatee and Mehdi Hashemi [7].

In this paper an algorithm is proposed to find a path with maximum flow of a network with triangular intuitionisticfuzzy number as edge weight. The concept of label setting is used in this method. In Label Setting Method, there are two types of nodes, permanent labeled nodes and temporary labeled nodes. In this method, once a node gets permanent labeled, it continues to be permanent in further iterations.

2. PRELIMINARIES

2.1 Fuzzy Numbers

A fuzzy subset of the real line R with membership function is called a fuzzy number if

i. $\widetilde{A}$ is normal, (i.e.) there exists an element x_0 such that $\mu_{\widetilde{A}}(x_0) = 1$
ii. $\widetilde{A}$ is fuzzy convex, (i.e.) $\mu_{\widetilde{A}}[\lambda x_1 + (1-\lambda)x_2] \geq \mu_{\widetilde{A}}(x_1) \wedge \mu_{\widetilde{A}}(x_2), x_1, x_2 \epsilon\ R, \lambda \epsilon\ [0,1]$
iii. $\mu_{\widetilde{A}}$ is upper continuous and
iv. Supp$\widetilde{A}$ is bounded, where Supp $\widetilde{A} = \{x \epsilon\ R: \mu_{\widetilde{A}}(x) > 0\}$.

2.2 *Intuitionistic Fuzzy Number*

Let $\widetilde{A} = \{(x, \mu_{\widetilde{A}}(x), \gamma_{\widetilde{A}}(x)), x \epsilon X\}$ be an intuitionistic fuzzy set, then the pair $(\mu_{\widetilde{A}}(x), \gamma_{\widetilde{A}}(x)$ is refered here as an intuitionistic fuzzy number.

2.3. Triangular Intuitionistic Fuzzy Number

A triangular intuitionistic fuzzy number $\widetilde{A}$ in R, written as $(a_1, b_1, c_1 ; a_1', b_1, c_1')$ where $a_1' \leq a_1 \leq b_1 \leq c_1 \leq c_1'$ has the membership function

$$\mu_{\widetilde{A}}(x) = \begin{cases} \frac{x - a_1}{b_1 - a_1} & a_1 \leq x \leq b_1 \\ \frac{x - c_1}{b_1 - c_1} & b_1 \leq x \leq c_1 \\ 0 & \text{othererwise} \end{cases}$$

and non-membership function of $\widetilde{A}$ is given by

$$\gamma_{\widetilde{A}}(x) = \begin{cases} \frac{b_1 - x}{b_1 - a_1'} & a_1' \leq x \leq b_1 \\ \frac{x - b_1}{c_1' - b_1} & b_1 \leq x \leq c_1' \\ 1 & \text{othererwise.} \end{cases}$$

3. ALGORITHM

Following is the algorithm to find the path with maximum flow capacity from source node to all other nodes of a network where triangular intuitionistic fuzzy number is considered as weight of the edges.

Step 1:
Let N be the set of all nodes and $s \in N$be the source node.
Let TLN be the set of temporary labeled nodes and PLN be the set of permanently labeled nodes.
Let $PLN = \phi$ and $TLN = N - \{s\}$.
Let F_i^k represent the capacity of the path with maximum flow from source node to node i at k[th] iteration. Let $F_s^0 = (0,0,0\,;\,0,0,0)$. Let C_{ij} be the weight of the edge (i, j) and $C_{ij} = (0,0,0\,;\,0,0,0)$, if$(i,j) \notin E$.
Let $k = 0$.

Step 2:
For all$i \in TLN, if\,(s, i) \in E, then\ F_i^0 = C_{si}$

Step 3:
The value of the membership function of the triangular intuitionistic fuzzy number $F_i^k = (a_1, b_1, c_1\,; a_1{'}, b_1, c_1{'})$ and $F_j^k = (a_2, b_2, c_2\,; a_2{'}, b_2, c_2{'})$ is calculated as follows:
If $v_\mu\,(F_i^k) > v_\mu\,(F_j^k)$ then $F_i^k > F_j^k$
If $v_\mu\,(F_i^k) < v_\mu\,(F_j^k)$ then $F_i^k < F_j^k$
If $v_\mu\,(F_i^k) = v_\mu\,(F_j^k)$ then $F_i^k = F_j^k$, where
$v_\mu\,(F_i^k) = v_\mu(a_1, b_1, c_1\,; a_1{'}, b_1, c_1{'}) = (a_1 + 4b_1 + c_1)/6$.

Step 4:
Using step 3, select a node y fromTLN, such that $F_y^K$is maximum. Tie can be broken arbitrarily
Let $N^{k+1} = y$
Let $TLN = TLN - \{y\}$ and $PLN = PLN \cup \{y\}$
If $TLN = \{\,\}$, then
{For all $i \in PLN, F_i^{k+1} = F_i^k$
Let $k = k + 1$and go to step 6},
otherwise go to step 5.

Step 5:
For all $j \in TLN$, if $(y, j) \in E$ compute $F_j^{k+1} =$ Max[F_j^k, $\min(F_y^k, C_{yj})$]
Else, if $(y, j) \notin E$ then $F_j^{k+1} = F_j^k$.
For all$i \in PLN, Let\ F_i^{k+1} = F_i^k$
Let $k = k + 1$
Go to step 4

Step 6*: (Determination of path from source node s to node i)*
Select a node x arbitrarily from PLN.
Let $PLN = PLN - \{x\}$
Let$j = 0$ and Path= < x >

Step 7:
If $F_x^{k-j} = F_x^{k-(j+1)}$ then go to step 9
Otherwise go to step 8

Step 8:
Path = $< N^{k-j} > \oplus$ Path
Let $x = N^{k-j}$
Go to step 7.

Step 9:
Let $j = j + 1$
If $k - j = 0$, then
{Path = < s >⊕ Path
If $TLN \neq \phi$, go to step 6
Else Terminate}
Else go to step 7.

4. NUMERICAL ILLUSTRATION

Consider a simple directed network given in figure 4.1 with seven vertices and twelve edges. Here triangular intuitionistic fuzzy numbers are considered as weight of the edges and is given in table 4.1. The problem is to find a path with maximum flow capacity from node 1 to all other nodes.

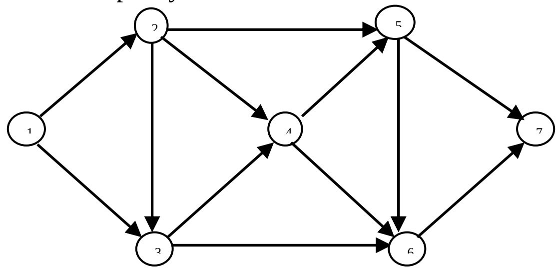

Figure 4.1 Network

Table 4.1 Edge weights of the Network for figure 4.1

Edge	*weight of the edge*
(1, 2)	(75,80,86; 70,80,90)
(1, 3)	(70,76,80; 60,76,85)
(2, 3)	(30,32,36; 28,32,38)
(2, 4)	(48,52,56; 40,52,58)
(2, 5)	(30,36,42; 26,36,46)
(3, 4)	(64,70,74; 50,70,80)
(3, 6)	(20,26,30; 17,26,33)
(4, 5)	(60,64,68; 50,64,70)
(4, 6)	(48,50,54; 40,50,56)
(5, 6)	(22,25,28; 20,25,30)
(5, 7)	(28,33,35; 26,33,38)
(6, 7)	(75,80,85; 70,80,88)

Applying the algorithm, one of the paths with maximum flow capacity from node 1 to all other nodes is computed and is given in table 4.2.

Node i	*Path with maximum flow from node 1 to node i*	*Maximum flow of the path*
2	1-2	(75,80,86; 70,80,90)
3	1-3	(70,76,80; 60,76,85)
4	1-3-4	(64,70,74; 50,70,80)
5	1-3-4-5	(60,64,68; 50,64,70)
6	1-3-4-6	(48,50,54; 40,50,56)
7	1-3-4-6-7	(48,50,54; 40,50,56)

Table 4.2 Path and its maximum flow

5. CONCLUSION

In this paper a new method is proposed to find a path with maximum flow from source node to all other nodes with triangular intuitionistic fuzzy numbers as edge weights. An example is provided to illustrate the method.

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