Flexural Fatigue Studies for SFRC under Variable Amplitude Loading For 1% Volume Fraction of Steel Fibres

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Abstract

*Cement concrete pavements are designed for flexural fatigue loading due to traffic. Due to its numerous advantages SFRC is finding its way in pavement construction. This paper investigates the fatigue strength of SFRC under variable amplitude loading considering 1% of steel fibres of weight of concrete. Fatigue study is conducted on the concrete to determine the fatigue life. Static flexural strength test is conducted to determine the static failure loads on prism specimen. SFRC prism specimens of size (75*100*500)mm containing 1% of steel fibres to weight of concrete were tested under two point flexural fatigue variable amplitude loading at a frequency of 4Hz at 0.65, 0.7, 0.75 stress ratios. The readings obtained from flexural fatigue test were used to plot graphs of P-N, ln(N)-ln(ln(1/Ln)) and also to perform statistical analysis using two parameter Weibull distribution. Recent literature shows that Weibull distribution is found to have more convincing physical features than the log normal distribution to describe the fatigue behaviour of concrete. It was observed that, addition of 1% volume fraction of steel fibres increased the static strength from 5.232 MPa to 5.404 MPa.*

Keywords

Variable amplitude loading, Forward and backward loading SFRC (Steel Fibre Reinforced Concrete), Probability of failure

1. Introduction

Transportation is a key infrastructure of a country. A country's economic status depends upon how well served the country is by its roads, railways, airports, ports, pipelines and shipping. The rate at which a country's economy grows is very closely linked to the rate at which the transport sector grows.

Traffic is the most researched load mechanism that is applied to a pavement. Traffic is usually the principle cause of reduced serviceability. Traffic induces stress in the pavement by applying a normal and shear force to the pavement surface.

Concrete pavements are by far the best long-term value because of their longer life expectations, durability and minimal maintenance requirements[8]. Concrete can be modified to perform in a more ductile form, by the addition of randomly distributed discrete fibres in the concrete matrix. The incorporation of steel fibers in concrete has been found to improve several of its properties, primarily cracking resistance, impact and wear resistance and ductility. For this reason fiber reinforced concrete (FRC) is now being used in increasing amounts in structures. Structures that are subjected to repeated loads are susceptible to failure due to fatigue. Fatigue is a process of progressive permanent internal changes in the materials that occur under the actions of cyclic loadings. These changes can cause progressive growth of cracks present in the concrete system and eventual failure of structures when high levels of cyclic loads applied for short times or low levels of loads are applied for long times.

The fibre reinforced concrete may be defined as a composite material made with portland cement, aggregate and incorporating discrete discontinuous fibres[1]. The plain concrete is a brittle material, with a low tensile strength and a low strain capacity. The role of randomly distributed discontinuous fibre is to bridge across the cracks and to provide some post cracking ductility to the structure. The fibre is a small piece of reinforcing material possessing certain characteristic properties. They can be either circular or flat. The fibre is often described by a convenient parameter called aspect ratio.

Some of the important applications of fibre reinforced concrete are as follows.

- Runway, Aircraft Parking, and Pavements.
- Tunnel Lining and Slope Stabilization.
- Blast Resistant Structures.
- The other applications include machine tool frames, lighting poles, water and oil tanks and concrete repairs etc.

2. Research significance

In the present investigation an attempt has been made to study the fatigue behaviour of SFRC for 1% volume fraction of fibres of weight of concrete, under variable amplitude fatigue loading. To investigate the flexural fatigue behaviour a series of prism specimens of size 75mm*100mm*500mm were tested under flexural fatigue loading.

In the present investigation, concrete containing 1% volume fraction of Shaktimaan steel fibres of round crimped type with diameter 0.55 mm and length 30 mm (aspect ratio = 54) manufactured by M/s. Stewols India (P) Limited, Nagpur were used. A total number of 120 SFRC prism specimens were tested under variable amplitude loading. Since most of the fatigue studies on concrete have mainly focused on simple S-

N relationships, little attention has been paid on probabilistic reliability analysis for concrete fatigue. In this study Weibull distribution is considered to impart probabilistic reliability against fatigue failure. An effort is made to develop relationships between stress level, fatigue life and probability of failure.

3. Objective

The objective of this experimental investigation is to study the performance of SFRC subjected to variable amplitude loading (fatigue test) for 1% volume fraction of fibres of weight of concrete compared with that of non fibrous specimen.

4. Laboratory tests

4.1 Materials and mix proportions

The ordinary Portland cement from single batch has been used in the present investigation. Aggregates were obtained from local source maximum size of coarse aggregate confined to 12mm. Shaktimaan steel fibres of round crimped type with diameter 0.55 mm and length 30 mm (aspect ratio = 54) manufactured by M/s. Stewols India (P) Limited, Nagpur were employed at constant volume fraction of 1% throughout in all mixes. Sulphonated naphthalene polymers based Conplast SP–430 Superplasticizer has been used as high range water redusing admixture (HWRA) to get desired workability. The reference concrete mix proportion used was 1: 0.998: 2.536 (cement: fine aggregate” coarse aggregate) with water/binder ratio of 0.41 with 2.5% reduction of cement content.

4.2 Test procedure and test results

4.2.1 Static testing

Cube specimens of size 150mm*150mm*150mm were used for determining compressive strength. For static flexural strength, specimens of similar size to that of fatigue specimens have been used. An effective span of 400mm has been used for both static flexural strength and fatigue strength determination. All the strength properties were determined after a curing period of 28 days. Static compressive strength values are shown in table.

Type of concrete	28 day compressive strength in MPa	28 day static flexural strength in MPa
Non-Fibrous	49.916	5.232
Fibrous	51.971	5.404

*Mean value of five specimens was taken

4.2.2 Variable amplitude fatigue testing

Fatigue test specimens were tested under one-third point loading using frequency of loading as 4Hz. Typical fatigue test set up and loading pattern used are shown in figure. All the fatigue specimens were cured for 28 days by ponding method. Minimum stress in fatigue loading was maintained at 1% of maximum stress. Minimum stress was used mainly to prevent any possible movement of specimens at support during testing and to stimulate residual stresses in the pavement to a certain degree[3]. For all the specimens tested under lower limit of stress level, was based on the criteria, when none of the specimens failed even after application of one lakh cycles of fatigue loading. Both NF(Non-Fibrous) and F(Fibrous) specimens were tested for stress ratio 0.65, 0.7, 0.75. The loading has been fed Under two pattern, **Forward loading and backward loading**, Under forward loading, the load according to stress ratio maintained at 1% of maximum stress has been fed in the variable amplitude loading set up in Increasing order, and accordingly vice versa ie, in Decreasing order under Backward loading, so as to facilitate the unevenness pattern of loading in Practice. Fatigue life values have been arranged in the increasing order so as to facilitate probability analysis[5].

In Variable Amplitude loading, Different magnitude of loading are applied at un-even time lag. Under Variable amplitude loading, the definition of one cycle is not clear and hence reversals of stress are often considered.

In the literature[4,7] variable amplitude fatigue studies have been carried out on plain concrete to verify the validity of Miner’s hypothesis. Miner’s hypothesis assumes that damage accumulates linearly with the number of cycles applied at a particular stress level. As per Miner’s hypothesis the failure criterion is written as:

$$\sum_{i=1}^{k} = \left(\frac{ni}{Ni}\right) = 1.0 \qquad \ldots(1)$$

Where, ni= number of cycles applied at stress level i

Ni= number of cycles to failure at stress level i

K= number of stress levels used

Studies carried out by Siemens [4] on plain concrete proved the validity of Minor’s rule. But the studies carried out by Holmen[7] found variable amplitude loading to be more damaging than that by Minor’s hypothesis.

Fig. 1: Flexural fatigue test setup

Fibrous (0.65, 0.7, 0.75) forward loading		
Specimen No.	Wt(Kg)	No. of cycles
1	9.03	174
2	9.63	9900
3	9.03	1785
4	9.28	3494
5	9.38	14514
6	10.34	19500

Fibrous (0.65, 0.7, 0.75) backward loading		
Specimen No.	Wt(Kg)	No. of cycles
1	9.4	279
2	9.71	53615
3	8.89	1123
4	9.01	2063
5	8.79	76
6	9.93	23311

Non-fibrous(0.65, 0.7, 0.75) forward loading		
Specimen no.	Wt(Kg)	No. of cycles
1	9.7	747
2	9.57	5377
3	9.4	10596
4	10.31	40101
5	9.24	39
6	9.65	63

Non-fibrous (0.65, 0.7, 0.75) backward loading		
Specimen no.	Wt(Kg)	No. of cycles
1	9.13	16
2	9.37	11
3	9.49	1795
4	9.24	157
5	9.04	166
6	9.55	2679

5. Probability of failure

Probability is defined as measure of what is expected to happen on the average, if a given observation or measurement is repeated a large number of times under identical conditions. Probability of failure Pf is calculated by dividing the rank of each specimen 'm' by (n + 1), where 'n' equals the total number of specimens tested at a particular stress level. The graph is plotted between Pf v/s Log N[6]. Regression equation are developed for probability of failure (Pf) by plotting a graph between varying high stress ratios(S) and the obtained value of Log N from the linear regression equation[6].

Fibrous (0.65, 0.70, 0.75) forward loading

No. of cycles	Pf
174	0.143
1785	0.286
3494	0.428
9900	0.571
14514	0.714
19500	0.857

Fibrous (0.65, 0.70, 0.75) backward loading

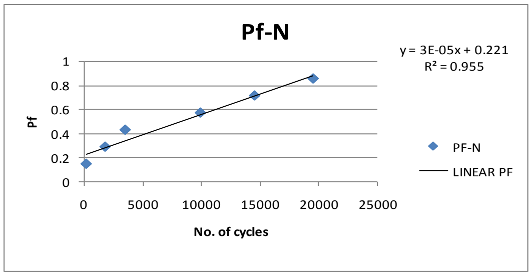
No. of cycles	Pf
76	0.143
279	0.286
1123	0.428
2063	0.571
23311	0.714
53615	0.857

Non-fibrous (0.65, 0.7, 0.75) forward loading

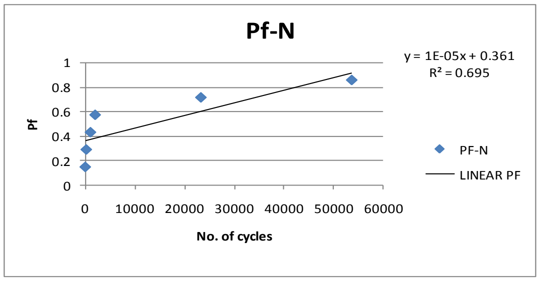
No. of cycles	Pf
39	0.143
63	0.286
747	0.428
5377	0.571
10596	0.714
40101	0.857

Non-fibrous (0.65, 0.7, 0.75) backward loading

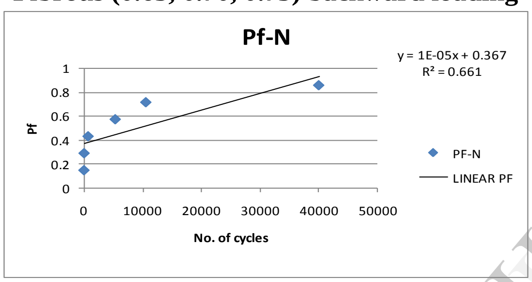
No. of cycles	Pf
11	0.143
16	0.286
157	0.428
166	0.571
1795	0.714
2679	0.857



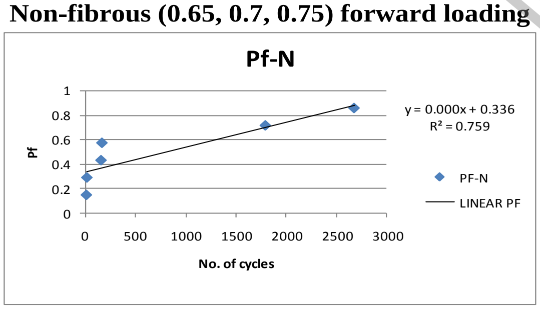
Fibrous (0.65, 0.70, 0.75) forward loading



Fibrous (0.65, 0.70, 0.75) backward loading



Non-fibrous (0.65, 0.7, 0.75) forward loading



Non-fibrous (0.65, 0.7, 0.75) backward loading

6. Determination of fatigue-life distribution

A number of mathematical probability models have been employed for the statistical description of fatigue data. The distributions of fatigue life can be determined on the basis of experimental data. Since the fatigue-life distribution of concrete was found previously to follow the Weibull distribution, the verification of this distribution will be attempted first and the parameter of the distribution will then be determined from experimental data for each different fatigue stress level. The graphical method is employed to estimate the distribution parameters[2].

The two parameter weibull probability density function f_N (n) and its cumulative distribution function F_N (n) for fatigue life may be written as follows. The two parameter weibull probability density function $F_N(n)$ for fatigue life may be written as follows.

$\mathbf{f_N(n) = \alpha/u.\ (n/u)^{\alpha-1} \exp(-(n/u)^{\alpha})}$ $\quad n \geq 0$ ------ (1)

$\mathbf{F_N(n)=1- \exp(-(n/u)^{\alpha})}$ $\quad n \geq 0$ ------ (2)

In which α= shape parameter or weibull slope at stress level S.

u= Characteristics extreme value (Scale parameter at stress level S)

n= location parameter or minimum value (local parameter at stress level S)

N= specific value of the random variable.

The reliability function $L_N(n)$ may be written as from eq (2)

$\mathbf{L_N(n)=1- F_N(n)= \exp(-(n/u)^{\alpha})}$ ---------(3)

Taking the logarithm twice for both side of eq(3) gives

$\mathbf{\ln (l(1/ L_N)) = \ln (\ln (1/(1- F_N))}$

$\mathbf{\ln (l(1/ L_N)) =\alpha \ln (n)- \alpha \ln u}$ ---------(4)

This equation (4) can be used to verify whether the statistical distribution of equivalent fatigue- life of SFRC, at given stress level S, follows the two parameter weibull distribution

Taking $\quad Y =\alpha x - \beta$ ----(5)

In which $\quad Y= \ln (l (1 / L_N))$

$\quad X = \ln (n)$

And $\quad \beta = \alpha \ln u$

The parameter u may be obtained from eq(3) by putting n=u

$L_N (n)= L_N (u) = 1/e =0.368$ -------(6)

Eq (5) represents a linear relationship between X and Y. Therefore a linear trend is obtained for the fatigue data, the parameter of the distribution may be obtained from the straight line. The fatigue data at a given stress level must be first arranged in ascending order to obtained a graph from eq (5). The empirical survivorship function L_N is then calculated as follows.

$L_N =1-i/k+1$

In which i= the failure order number

K= the number of fatigue data or sample size

A graph is plotted between ln (ln(1/ L_N)) and ln (n), and if a linear trend is observed for the equivalent fatigue-life data at a given stress level S, it can be assumed that the two parameter weibull distribution is a reasonable assumption for the statistical description of equivalent fatigue-life data at that stress level. The parameter α and u can be directly obtained from the graph.

Fibrous (0.65, 0.7, 0.75) forward loading

ln(N)	ln(ln(1/Ln))
5.16	-1.867
7.49	-1.089
8.16	-0.579
9.2	-0.164
9.58	0.224
9.89	0.66

Fibrous (0.65, 0.7, 0.75) backward loading

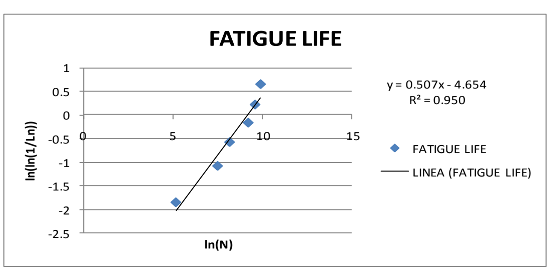
ln(N)	ln(ln(1/Ln))
4.33	-1.867
5.63	-1.089
7.02	-0.579
7.63	-0.164
10	0.224
10.89	0.66

Non-fibrous (0.65, 0.7, 0.75) forward loading

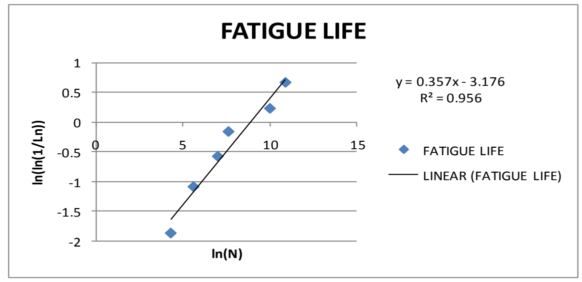
ln(N)	ln(ln(1/Ln))
3.66	-1.867
4.14	-1.089
6.62	-0.579
8.59	-0.164
9.27	0.224
10.6	0.66

Non-fibrous (0.65, 0.7, 0.75) backward loading

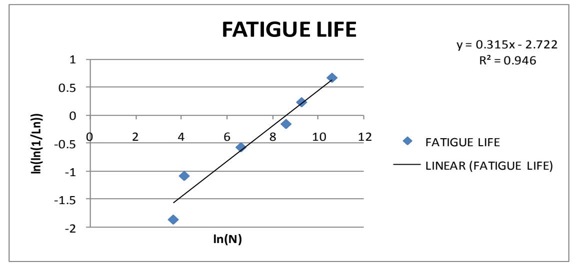
ln(N)	ln(ln(1/Ln))
2.4	-1.867
2.77	-1.089
5.06	-0.579
5.11	-0.164
7.49	0.224
7.89	0.66



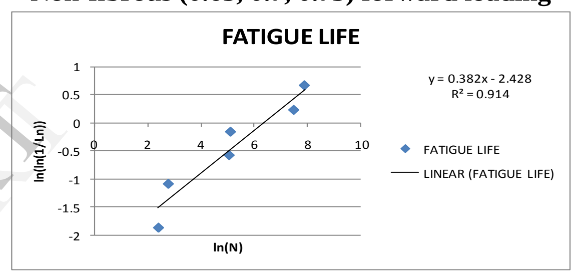
Fibrous (0.65, 0.7, 0.75) forward loading



Fibrous (0.65, 0.7, 0.75) backward loading



Non-fibrous (0.65, 0.7, 0.75) forward loading



Non-fibrous (0.65, 0.7, 0.75) backward loading

7. Weibull distribution verification

According to the probability theory of Weibull distribution, the failure probability Pf(N) corresponding to the failure life N can be expressed by:

$$PN = \frac{1}{(k+1)}$$

Where, k is the total number of the fatigue test data at certain stress level; i is the sequence number of failure specimens at certain stress level, i=1,2,...... k. The linear regression is carried out for the fatigue test data according to Eq. (5) for all stress levels. The verification of the distribution is assayed first and it has been found out that the fatigue life distributions of the test results in this investigation followed the Weibull distribution.

Stress ratio (0.65, 0.7, 0.75)

Loading pattern	% of fibres	Regression equation	R^2	α	β	U
Fibrous (Forward)	1%	y=0.507x-4.654	0.95	0.507	4.654	9701.1
Fibrous (Backward)	1%	y=0.357x-3.176	0.956	0.357	3.176	7259
Non-fibrous (Forward)	1%	y=0.315x-2.722	0.946	0.315	2.722	5653.3
Non-fibrous (Backward)	1%	Y=0.382x-2.428	0.915	0.382	2.428	578.2

8. Fatigue life and survivorship function

Fatigue life and survivorship function for fibrous (0.65, 0.7, 0.75) forward loading

Nj	Ln=1-(j/k+1)	Fn= 1-Ln	X= ln(Nj)	Y=ln (ln(1/Ln))	Fn%
174	0.857	0.143	5.16	-1.867	14.30%
1785	0.714	0.286	7.49	-1.089	28.60%
3494	0.571	0.429	8.16	-0.579	42.90%
9900	0.428	0.572	9.2	-0.164	57.20%
14514	0.286	0.714	9.58	0.224	71.40%
19500	0.143	0.857	9.89	0.66	85.70%

Fatigue life and survivorship function for fibrous (0.65, 0.7, 0.75) backward loading

Nj	Ln=1-(j/k+1)	Fn= 1-Ln	X= ln(Nj)	Y=ln (ln(1/Ln))	Fn%
76	0.857	0.143	4.33	-1.867	14.30%
279	0.714	0.286	5.63	-1.089	28.60%
1123	0.571	0.429	7.02	-0.579	42.90%
2063	0.428	0.572	7.63	-0.164	57.20%
23311	0.286	0.714	10	0.224	71.40%
53615	0.143	0.857	10.89	0.66	85.70%

Fatigue life and survivorship function for non-fibrous(0.65, 0.7, 0.75) forward loading

Nj	Ln=1-(j/k+1)	Fn= 1-Ln	X=ln (Nj)	Y=ln (ln(1/Ln))	Fn%
39	0.857	0.143	3.66	-1.867	14.30%
63	0.714	0.286	4.14	-1.089	28.60%
747	0.571	0.429	6.62	-0.579	42.90%
5377	0.428	0.572	8.59	-0.164	57.20%
10596	0.286	0.714	9.27	0.224	71.40%
40101	0.143	0.857	10.6	0.66	85.70%

Fatigue life and survivorship function for non-fibrous(0.65, 0.7, 0.75) backward loading

Nj	Ln=1-(j/k+1)	Fn= 1-Ln	X=l n(Nj)	Y=ln (ln(1/Ln))	Fn%
11	0.857	0.143	2.4	-1.867	14.30%
16	0.714	0.286	2.77	-1.089	28.60%
157	0.571	0.429	5.06	-0.579	42.90%
166	0.428	0.572	5.11	-0.164	57.20%
1795	0.286	0.714	7.49	0.224	71.40%
2679	0.143	0.857	7.89	0.66	85.70%

9. Observations

1. It was observed that, the nature of failure of specimens under static flexural test was ductile in nature for fibrous concrete and brittle for non-fibrous concrete.
2. Under variable amplitude fatigue loading test, it was observed that, the nature of failure was ductile for fibrous specimens and brittle for all non-fibrous specimens.
3. When compared for forward and backward loading, for both fibrous and non-fibrous specimens, it was observed that, number of cycles for forward loading were more than that of backward loading.
4. The Mean and Standard Deviation doesn't show much larger scatter with mean values of the number of cycles obtained for both fibrous and non-fibrous specimens under forward and backward loading.
5. The shape parameter(α) in Weibull Distribution ranges from 0.357 to 0.507 for fibrous specimens considering forward and backward loading, compared to that of 0.315 to 0.382 for non-fibrous, which shows larger difference and more spread out distribution.
6. The regression equation co-efficient (R^2) obtained for fibrous specimens showed Good values all reaching unity, compared with that of non-fibrous specimens.

10. Conclusions

1. The maximum compressive strength of M-40 grade concrete for 28 days of curing attained was 51.97MPa for 1% Volume fraction of fibres.
2. The flexural strength of M-40 grade concrete was found to be 5.404MPa for fibrous specimen's with 1% volume fraction of Fibres, compared to that of 5.232MPa obtained for Non-Fibrous specimens.

3. The characteristic extreme value(u) in Weibull distribution, which represents scale parameter, shows much larger distribution for non-fibrous(578.2-5653.3), when compared with fibrous specimens (7259-9701.1).

4. The number of cycles of fatigue life(N), obtained for fibrous specimen's were comparatively more than, that obtained for non-fibrous specimens.

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