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Flexural-Distortional Performance of Thin- Walled Mono Symmetric Box Girder Structures

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Abstract

Thin-walled mono symmetric box girder structures are commonly found in the form of trapezoidal cross sections of either concrete or steel. Such structures resist eccentric vertical loads in bending action and torsion. The torsional component of eccentric loads on such structures give rise to pure torsion (Saint Venant torsion), distortion and flexure about the non symmetric axis of the box girder section. In order to provide an improved understanding of the complex interactions between these strain fields, this paper examined the interaction between the distortional strain mode and flexural strain mode and derived a general differential equations of equilibrium for flexural-distortional analysis of mono symmetric box girder structures. In addition the derived equations were used to analyze a double cell mono symmetric box girder section to obtain flexural and distortional deformations.

1. Introduction

Thin walled structures are structures in which the ratio of the thickness t, to the two other linear dimensions (length l, and width w) ranges within the limits t/l or t/w = 1/50 to 1/10, Rekach [1]. Thus a thin walled structure has two dimensions of the structural element much larger than the third one, i.e., the thickness. When two or more plates are joined together to form an open or closed structure strength and rigidity are increased. For example, tanks, boilers, etc, are cylindrical shell structures with increased strength and rigidity. Conical shell structures are also common features in construction, mechanical engineering and aeronautical design. Thin-walled structures are used extensively in steel and concrete bridges, ships, air crafts, mining head frames and gantry frames. These are seen in the form of box girders, plate girders, box columns and purlins (z and channel sections). Because of their thin wall thicknesses, the shearing resistances are constant across the thickness of the plate. On the other hand thin walled box structures may be subjected to bending, torsional and distortional stresses. Distortion alters the geometry of the cross section and generates some additional stresses thereby reducing the bearing capability of the box structural component.

Research [2], has shown that a mono symmetric thin walled box girder has three strain modes interactions: torsion interacts with distortion and each of these interacts with flexure about the non axis of symmetry. Thus we have torsional-distortional interaction, flexural-torsional interaction, and flexural-distortional interaction. In this work, the interaction of flexural strain mode about the non axis of symmetry with distortional strain mode of a mono symmetric box girder structure is examined.

2. Literature review

Recent literatures, Hsu et al [3], Fan and Helwig [4], Sennah and Kennedy [5], on straight and curved box girder bridges deal with analytical formulations to better understand the behaviour of these complex structural systems. Few authors, Okil and El-tawil [6], Sennah and Kennedy [5], have undertaken experimental studies to investigate the accuracy of existing methods. Before the advent of Vlasov's 'theory of thin-walled beams', [7], the conventional method of predicting warping and distortional stresses is by beam on elastic foundation (BEF) analogy. This analogy ignores the effect of shear deformations and takes no account of the cross sectional deformations which are likely to occur in a thin walled box girder structure

Several investigators; Bazant and El-Nimeiri [8], Zhang and Lyons [9], Boswell and Zhang [10], Usuki [11], Waldron [12], Paavola [13], Razaqpur and Lui [14], Fu and Hsu [15], Tesar [16], have combined thin-walled beam theory of Vlasov and the finite element technique to develop a thin walled box element for elastic analysis of straight and curved cellular bridges. Osadebe and Chidolue [17], [18], obtained fourth order differential equations of torsional-distortional equilibrium and flexural-torsional equilibrium for the analysis of mono symmetric box girder structures using Vlasov's theory with modifications by Varbanov [19].

Various theories were therefore postulated by different authors examining methods of analysis, both classical and numerical. A few others however carried out tests on prototype models to verify the authenticity of the theories. The authors are of the view that Vlasov's theory captures all peculiarities of cross sectional deformation such as warping, torsion, distortion etc, and is therefore adopted in this work.

The objective of this study is to derive a set of differential equations governing the flexural-distortional behaviour of thin- walled mono symmetric box girder structures on the basis of Vlasov's theory and to apply the obtained equations in the analysis of double cell mono symmetric box girder structure to obtain flexural and distortional deformations.

3. Vlasov's stress – strain relations

The longitudinal warping and transverse (distortional) displacements given by Vlasov [7], are

$$u(x,s)=U(x)\varphi(s)$$
$$v(x,s)=V(x)\psi(s) \qquad (1)$$

The displacements may be represented in series form as;

$$u(x,s)=\sum_{i=1}^{m}U_i(x)\varphi_i(s)$$
$$v(x,s)=\sum_{k=1}^{n}V_k(x)\psi_k(s) \qquad (2)$$

where, $U_i(x)$ and $V_k(x)$ are unknown functions which express the laws governing the variation of the displacements along the length of the box girder frame. $\varphi_i(s)$ and $\psi_k(s)$ are elementary displacements of the strip frame, respectively out of the plane (m displacements) and in the plane (n displacements). These displacements are chosen among all displacements possible, and are called the generalized strain coordinates of a strip frame.

From the theory of elasticity the strains in the longitudinal and transverse directions are given by;

$$\frac{\partial u(x,s)}{\partial x}=\sum_{i=1}^{m}U_i'(x)\varphi_i(s)$$
$$\frac{\partial v(x,s)}{\partial x}=\sum_{k=1}^{n}V_k'(x)\psi_k(s) \qquad (3)$$

The expression for shear strain is $\gamma(x,s)=\dfrac{\partial u}{\partial s}+\dfrac{\partial v}{\partial x}$

$$\Rightarrow \gamma(x,s)=\sum_{i=1}^{m}\varphi_i'(s)U_i(x)+\sum_{k=1}^{n}\psi_k(s)V_k'(x) \qquad (4)$$

Using the above displacement fields φ_i *and* ψ_i, and basic stress-strain relationships of the theory of elasticity, the expressions for normal and shear stresses become:

$$\sigma(x,s)=E\frac{\partial u(x,s)}{\partial x}=E\sum_{i=1}^{m}\varphi_i(s)U_i'(x) \qquad (5)$$

$$\tau(x,s)=G\gamma(x,s)=G\sum_{i=1}^{m}\varphi_i'(s)U_i(x)+$$
$$+\quad G\sum_{k=1}^{n}\psi_k(s)V_k'(x) \qquad (6)$$

Transverse bending moment generated in the box structure due to distortion is given by;

$$M(x,s)=\sum_{k=1}^{n}M_k(s)V_k(x) \qquad (7)$$

where $M_k(s)$ = bending moment generated in the cross sectional frame of unit with due to a unit distortion $V(x)$ = 1.

4. Energy formulation of the equilibrium equations

The potential energy of a box structure under the action of a distortional load of intensity q is given by:

$$\Pi=U+W_E \qquad (8)$$

where,

Π = the total potential energy of the box structure,

U = Strain energy,

V_E = External potential or work done by the external loads.

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From strength of materials, the strain energy U, of a structure is given by:

$$U = \frac{1}{2}\int_L \int_S \left[\left(\frac{\sigma^2(x,s)}{E} + \frac{\tau^2(x,s)}{G} \right) t(s) + \frac{M^2(x,s)}{EI_{(s)}} \right] dxds \quad (9)$$

Work done by external load is given by:

$$W_E = qv(x,s)dxds$$
$$= \int_s \int_x q \sum V_h(x)\varphi_h(s)dsdx$$
$$= \int_x \sum q_h V_h dx \quad (10)$$

Substituting eqns (9) and (10) into eqn. (8) we obtain that:

$$\Pi = \int_L \int_S \left[\frac{\sigma^2(x,s)}{2E} + \frac{\tau^2(x,s)}{2G} \right] t(s)dxds + \frac{1}{2}\int_L \int_S \left[\frac{M^2(x,s)}{EI(s)} - qv(x,s) \right] dxds \quad (11)$$

where,

$\sigma(x,s)$ = normal stress

$\tau(x,s)$ = shear stress

$M(x,s)$ = transverse distortional bending moment

q = line load per unit area applied in the plane of the plate

$I_{(s)} = \frac{t^3(s)}{12(1-\nu^2)}$ = moment of inertia

E = modulus of elasticity
G = shear modulus
ν = poisson ratio
t = thickness of plate

Substituting eqns (1), (5), (6), and (7) into eqn.(11) and simplifying, noting that $t(s)ds = dA$ we obtain the potential energy of the box structure as follows.

$$\Pi = \frac{E}{2}\sum a_{ij}U_i{'}(x)U_j{'}(x)dx + \frac{G}{2}\left[\sum b_{ij}U_i(x)U_j(x) + \sum c_{kj}U_k(x)V_j{'}(x)\right]dx$$
$$+ \frac{G}{2}\left[\sum c_{ih}U_i(x)V_h{'}(x) + \sum r_{kh}V_k{'}(x)V_h{'}(x)\right]dx + \frac{E}{2}\sum s_{hk}V_k(x)V_h(x)dx - \sum q_h V_h dx \quad (12)$$

where the (Vlasov's) coefficients are defined as follows.

$$\begin{aligned}
&a_{ij} = a_{ji} = \int \varphi_i(s)\varphi_j(s)dA && \text{(a)} \\
&b_{ij} = b_{ji} = \int \varphi_i^{'}(s)\varphi_j^{'}(s)dA && \text{(b)} \\
&c_{kj} = c_{jk} = \int \varphi_k^{'}(s)\psi_j(s)dA && \text{(c)} \\
&c_{ih} = c_{hi} = \int \varphi_i^{'}(s)\psi_k(s)dA && \text{(d)} \quad (13) \\
&r_{kh} = r_{hk} = \int \psi_k(s)\psi_h(s)dA; && \text{(e)} \\
&s_{kh} = s_{hk} = \frac{1}{E}\int \frac{M_k(s)M_h(s)}{EI_{(s)}}ds && \text{(f)} \\
&q_h = \int q\psi_h ds && \text{(g)}
\end{aligned}$$

The governing equations of flexural-distortional equilibrium are obtained by minimizing the energy functional eqn. (12), with respect to its functional variables u(x) and v(x) using Euler Lagrange technique [20]. Minimizing with respect to u(x) we obtain;

$$k\sum_{i=1}^{m} a_{ij}U_i{''}(x) - \sum_{i=1}^{m} b_{ij}U_i(x) - \sum_{k=1}^{n} c_{kj}V_k{'}(x) = 0 \quad (14)$$

Minimizing with respect to v(x) we have;

$$\sum c_{ih}U_i{'}(x) - \kappa\sum s_{hk}V_k(x) + \sum r_{kh}V_k{''}(x) + \frac{1}{G}\sum q_h = 0 \quad (15)$$

where $\kappa = \frac{E}{G} = 2(1+\nu)$

Equations (14) and (15) are Vlasov's generalized differential equations of distortional equilibrium for a box girder structure.

5. Generation of Strain Modes Diagrams

Consider a simply supported girder loaded as shown in Fig 1(a). If we assume the normal beam theory, i.e., neutral axis remaining neutral before and after bending, then the distortion of the cross section will be as shown in Fig. 1(b) where, θ is the distortion angle (rotation of the vertical axis). The displacement φ_1 at any distance R, from the centroid is given by $\varphi_1 = R\theta$. If we assume a unit rotation of the vertical (z) axis then $\varphi_1 = R$, at any point on the cross section. Note that φ_1 can be positive or negative depending on the value of R, in the tension or compression zone of the girder. Thus, φ_1 is a property of the cross section obtained by plotting the displacement of the members of the cross section when the vertical (z-z) axis is rotated through a unit radian.

Similarly, if the load is acting in horizontal (y- y) direction, normal to the x-z plane in Fig.1(a), then the bending is in x-z plane and y axis is rotated through angle θ_2 giving rise to φ_2 displacement out of plane. The values of φ_2 are obtained for the members of the cross section by plotting the displacement of the cross section when y-axis is rotated through a unit radian.

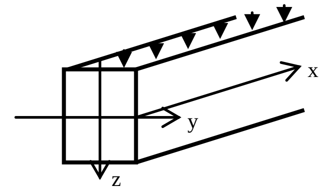
The warping function φ_3 of the beam cross section is obtained as detailed in [1] and [2]. It has been explained that the warping function is the out of plane displacement of the cross section when the beam is twisted about its axis through the pole, one radian per unit length without bending in either x or y direction and without longitudinal extension.

ψ_1 and ψ_2 are in-plane displacements of the cross section in x-z and x-y planes respectively while ψ_3 is the distortion of the cross section.

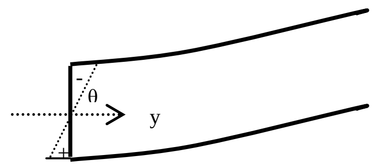
The authors have shown that these in-plane displacement quantities ψ_1, ψ_2 and ψ_3 are the same as the derivatives of their corresponding out of plane displacements. Consequently, ψ_1, ψ_2 and ψ_3 are obtained by numerical differentiation of φ_1, φ_2 and φ_3 diagrams respectively.

ψ_4 is the displacement diagram of the beam cross section when the section is rotated one radian in say, a clockwise direction, about its centroidal axis. Thus, ψ_4 is directly proportional to the perpendicular distance (radius of rotation) from the centroidal axis to the members of the cross section. It is assumed to be positive if the member moves in the positive directions of the coordinate axis and negative otherwise.

The generalized strain modes for the double cell mono-symmetric frame are shown in Fig 2

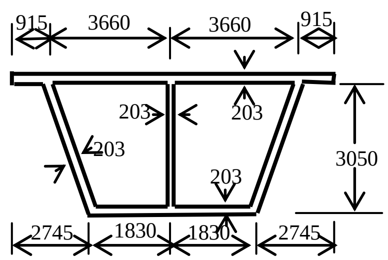


(a) Simply supported girder section

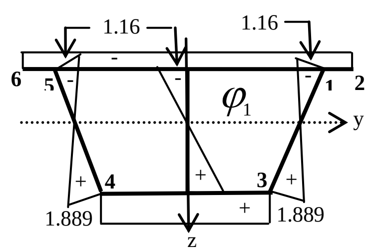


(b) Cross section distortion

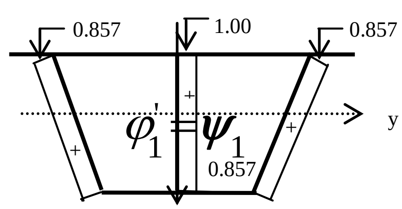
Fig. 1 Simply Supported Girder and Cross Section Distortion



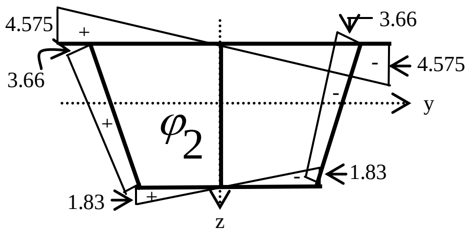
(a) Double cell box girder section



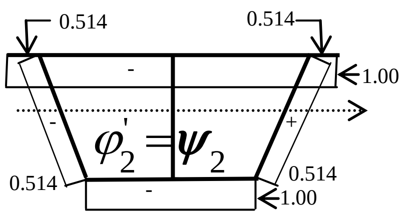
(b) Longitudinal strain mode diagram (Bending about y-y Axis)



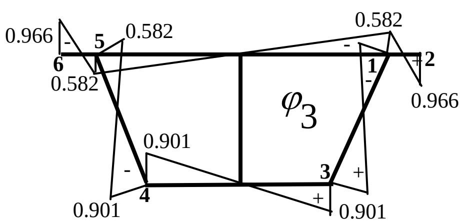
(c) Transverse stain mode in y-y direction



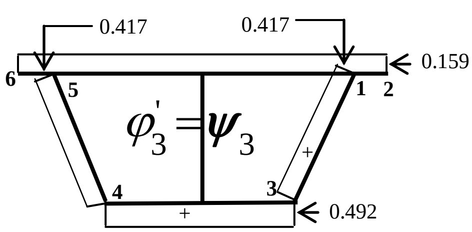
(d) Longitudinal strain mode (Bending about z-z axis)



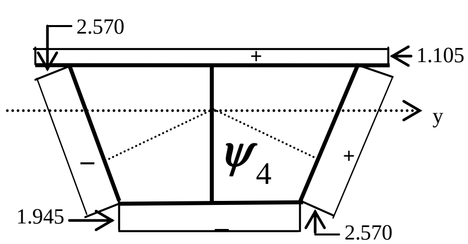
(e) Transverse strain mode in z-direction



(f) Warping function diagram



(g) Distortion diagram



(h) Pure rotation diagram

Fig. 2 Generalized strain modes for double cell mono-symmetric box girder frame

Computation of Vlasovs coefficients

The coefficients $a_{ij}, b_{ij}, c_{kj}, c_{ih}$ and r_{kh}, of the differential equations of equilibrium are computed with the aid of Morh's integral chart. Thus:

$$a_{ij} = a_{ji} = \int \varphi_i(s)\varphi_j(s)dA$$

$$a_{22} = \int_s \varphi_2(s).\varphi_2(s)dA = 25.073$$

$$a_{23} = a_{32} = \int \varphi_2(s)\varphi_3(s)dA = 0.425$$

$$a_{33} = \int_s \varphi_3(s).\varphi_3(s)dA = 0.750$$

$$b_{ij} = b_{ji} = \int_s \varphi_i'(s).\varphi_j'(s)dA$$

$$b_{22} = \int_s \varphi_2'(s).\varphi_2'(s)dA = 2.982$$

$$b_{23} = b_{32} = \int_s \varphi_2'(s).\varphi_3'(s)dA = -0.449$$

$$b_{33} = \int_s \varphi_3'(s).\varphi_3'(s)dA = 1.533$$

$$c_{kj} = c_{jk} = \int_s \psi_k(s).\varphi_j'(s)dA$$

$$c_{22} = \int_s \psi_2(s).\varphi_2'(s)dA = 2.982$$

$$c_{23} = c_{32} = \int_s \psi_2(s).\varphi_3'(s)dA = -0.449$$

$$c_{33} = \int_s \psi_3(s).\varphi_3'(s)dA = 1.533$$

$$r_{kh} = r_{hk} = \int_s \psi_k(s).\psi_h(s)dA$$

$$r_{22} = \int_s \psi_2(s).\psi_2(s)dA = 2.982$$

$$r_{23} = r_{32} = \int_s \psi_2(s).\psi_3(s)dA = -0.449$$

$$r_{33} = \int_s \psi_3(s).\psi_3(s)dA = 1.533$$

6.1 Evaluation of distortional bending moment coefficients, s_{hk}

The distortional bending moment coefficients s_{hk}, given by eqn. (13f) depend on the bending deformation of the strip frame characterized by the distortional bending moment, M_k (for k = 1, 2, 3, 4). To compute the coefficients we need to construct the diagram of the bending moments due to strain modes ψ_1, ψ_2, ψ_3 and ψ_4. Incidentally, ψ_1, ψ_2 and ψ_4 strain modes do not generate distortional bending moment on the box girder structure as they involve pure bending and pure rotation. Only ψ_3 strain mode generates distortional bending moment which can be evaluated using the distortion diagram for the relevant cross section. Consequently the relevant expression for the coefficient becomes:

$$s_{hk} = s_{kh} = s_{33} = \frac{1}{E}\int_s \frac{M_3(s)M_3(s)}{EI_s} \qquad (19)$$

where $M_3(s)$ is the distortional bending moment of the relevant cross section due to strain mode 3.

The procedure for evaluation of distortional bending moments is given in literatures [1], [17]. Fig. 3 shows the distortional bending moment for evaluation of S_{hk} for the double cell mono symmetric frame of Fig. 2(a). The computed value of $s_{hk} = s_{33}$ for the single cell mono symmetric frame example was : $S_{33} = 0.723 * I_S$.

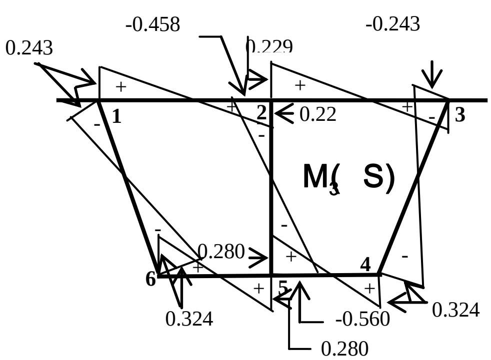


Fig. 3 Distortional bending moment for double cell mono-symmetric frame

7. Flexural-distortional equilibrium equations

The relevant coefficients for flexural-distortional equilibrium are those involving strain modes 2 and 3 as shown in the computation of Vlasov's coefficients. These are:

$a_{22},\ a_{23}, a_{33},\ b_{22},\ b_{23},\ b_{33},\ c_{23},\ c_{33},\ r_{22},\ r_{23},\ r_{33}$

and s_{33}. All other coefficients are zero.

Substituting these into eqns. [14] and [15] and adopting matrix notation of the equations we obtain:

$$\kappa\begin{bmatrix}0&0&0\\0&a_{22}&a_{23}\\0&a_{32}&a_{33}\end{bmatrix}\begin{bmatrix}U_1''\\U_2''\\U_3''\end{bmatrix}-\begin{bmatrix}0&0&0\\0&b_{22}&b_{23}\\0&b_{32}&b_{33}\end{bmatrix}\begin{bmatrix}U_1\\U_2\\U_3\end{bmatrix}-$$

$$\begin{bmatrix}0&0&0&0\\0&c_{22}&c_{23}&0\\0&c_{32}&c_{33}&0\end{bmatrix}\begin{bmatrix}V_1'\\V_2'\\V_3'\\V_4'\end{bmatrix}=0 \qquad (16)$$

$$\begin{bmatrix}0&0&0\\0&c_{22}&c_{23}\\0&c_{32}&c_{33}\\0&0&0\end{bmatrix}\begin{bmatrix}U_1'\\U_2'\\U_3'\end{bmatrix}-$$

$$-\ \kappa\begin{bmatrix}0&0&0&0\\0&0&0&0\\0&0&s_{33}&0\\0&0&0&0\end{bmatrix}\begin{Bmatrix}V_1\\V_2\\V_3\\V_4\end{Bmatrix}+$$

$$+\begin{bmatrix}0&0&0&0\\0&r_{22}&r_{23}&0\\0&r_{32}&r_{33}&0\\0&0&0&0\end{bmatrix}\begin{bmatrix}V_1''\\V_2''\\V_3''\\V_4''\end{bmatrix}+\frac{1}{G}\begin{bmatrix}q_1\\q_2\\q_3\\q_4\end{bmatrix}=0 \qquad (17)$$

Multiplying out we obtain:

$$ka_{22}U_2'' + ka_{23}U_3'' - b_{22}U_2 - b_{23}U_3 - - c_{22}V_2' - c_{23}V_3' = 0 \qquad (18)$$

$$ka_{32}U_2'' + ka_{33}U_3'' - b_{32}U_2 - b_{33}U_3 - c_{32}V_2' - c_{33}V_3' = 0 \quad (19)$$

$$c_{22}U_2' + c_{23}U_3' + r_{22}V_2'' + r_{23}V_3'' = -\frac{q_2}{G} \quad (20)$$

$$c_{32}U_2' + c_{33}U_3' - ks_{33}V_3 + r_{32}V_2'' + r_{33}V_3'' = -\frac{q_3}{G} \quad (21)$$

Simplifying further we obtain the coupled differential equations of flexural-distortional equilibrium for mono symmetric sections as follows:

$$\begin{aligned} &\alpha_1 V_2^{iv} + \alpha_2 V_3^{iv} - \beta_1 V_3'' = K_3 \quad (a) \\ &\alpha_3 V_2^{iv} + \alpha_4 V_3^{iv} - \beta_2 V_3'' - \gamma_1 V_3 = -K_4 \quad (b) \end{aligned} \qquad (22)$$

where, $\alpha_2 = \frac{r_{44}}{c_{43}}, \quad \gamma_1 = c_{43}ks_{33}$

$$\beta_1 = r_{34}c_{43} - c_{33}r_{44}; \qquad \beta_2 = \frac{b_{33}r_{44} - c_{34}c_{43}}{ka_{33}c_{43}},$$

$$K_1 = -\left(\frac{c_{22}}{c_{33}c_{22} - c_{32}^2}\right)\frac{\bar{q}_3}{G}$$

$$K_2 = -\left(\frac{c_{23}}{c_{33}c_{22} - c_{32}^2}\right)\frac{\bar{q}_3}{G}$$

$$K_3 = b_{23}K_1 - b_{22}K_2, \qquad K_4 = b_{32}K_2 + b_{33}K_1$$

8. Flexural-distortional analysis of double cell mono symmetric section

In this section the solutions of the differential equations of equilibrium eqns. (22) are obtained for the double cell mono symmetric box girder structure whose cross section is shown in Fig. 1(a). Live loads are considered according to AASHTO-LRFD [21], following the HL-93 loading: uniform lane load of 9.3N/mm distributed over a 3m width plus tandem load of two 110 KN axles. The loads are positioned at the outermost possible location to generate the maximum torsional effects. A 50m span simply supported bridge deck structure is considered. The obtained torsional loads are ; $\bar{q}_2 = 0.00KN, \quad \bar{q}_3 = 196.46KN$.

The governing equations of equilibrium are:

$$\begin{aligned} &\alpha_1 V_2^{iv} + \alpha_2 V_3^{iv} - \beta_1 V_3'' = K_3 \quad (a) \\ &\alpha_3 V_2^{iv} + \alpha_4 V_3^{iv} - \beta_2 V_3'' - \gamma_1 V_3 = -K_4 \quad (b) \end{aligned} \qquad (22)$$

The relevant coefficients are as follows:

$a_{22} = 25.05; \quad a_{23} = a_{32} = -0.270, \quad a_{33} = 0.757$

$b_{22} = c_{22} = r_{22} = 2.982 \quad b_{33} = c_{33} = r_{33} = 1.407$

$b_{23} = b_{32} = c_{23} = c_{32} = r_{23} = r_{32} = -0.153$

$r_{44} = 14.616; \quad s_{33} = 0.261*6.9712*10^{-4} = 1.8195*10^{-4}$

$E = 24*10^9\ N/m^2; \quad G = 9.6*10^9\ N/m^2, \quad k = 2.5$

The coefficients of the governing equations are as follow:

$\alpha_1 = ka_{22} = 62.6825; \quad \alpha_2 = ka_{23} = 1.0625$

$\alpha_3 = Ka_{32} = 1.0625; \quad \alpha_4 = Ka_{33} = 1.875$

$$\beta_1 = \frac{\left(a_{23}c_{22} - a_{22}c_{23}\right)k^2 s_{33}}{c_{33}c_{22} - c_{32}^2} = 0.0283$$

$$\beta_2 = \frac{k^2 s_{33}\left(a_{33}c_{22} - a_{32}c_{23}\right)}{c_{33}c_{22} - c_{32}^2} = 1.750*10^{-3}$$

$$\gamma_1 = \frac{\left(a_{32}c_{23} - b_{33}c_{22}\right)ks_{33}}{c_{33}c_{22} - c_{32}^2} = -1.260*10^{-3}$$

$$K_1 = -\left(\frac{c_{22}}{c_{33}c_{22} - c_{32}^2}\right)\frac{q_3}{G} = -1.115*10^{-5}$$

$$K_2 = -\left(\frac{c_{23}}{c_{33}c_{22} - c_{32}^2}\right)\frac{q_3}{G} = -1.684*10^{-6}$$

$$K_3 = b_{23}K_1 - b_{22}K_2 = 1.003*10^{-5}$$

$$K_4 = b_{32}K_2 + b_{33}K_1 = -1.634*10^{-5}$$

Substituting the coefficients into eqns (22) we obtain

$$62.683V_2^{iv} + 1.0625V_3^{iv} - 0.0283V_3'' = -1.003*10^{-5}$$
$$1.0625V_2^{iv} + 1.875V_3^{iv} - 1.750*10^{-3}V_3'' + 1.26*10^{-3}V_3 = 1.634*10^{-5} \quad (23)$$

Integrating by method of trigonometric series with accelerated convergence we hav:

$$V_2(x) = 8.626*10^{-3} Sin\pi x / 50$$
$$V_3(x) = 1.250*10^{-2} Sin\pi x\pi 50 \quad (24)$$

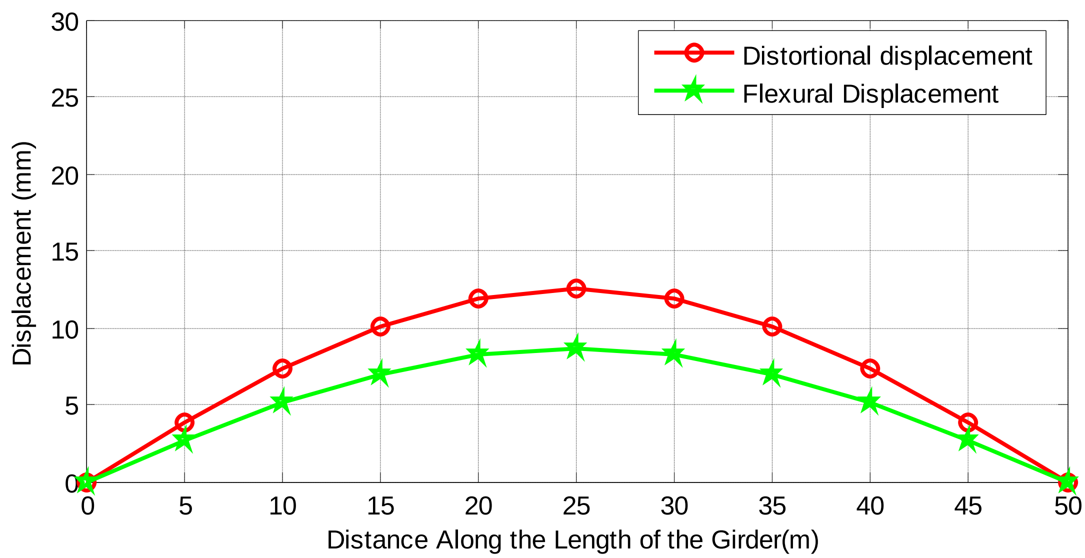


Fig.4: Variation of flexural and distortional displacements along the length of the girder

9. Discussion of Results

The derived governing differential equations of flexural-distortional equilibrium eqn. (22), is applicable to all mono symmetric box girder structures, both single cell and multi cell profiles. Along the axis of symmetry of the box girder structure, bending strain mode 1 does not interact with distortional strain mode 3 hence, there was no relationship between V_1 and V_3 as could be seen from the derived eqn. 22.

Fig.4 shows the variation of flexural and distortional displacements along the length of the girder as described by eqn. (24). It should be recalled that flexural strain mode has interaction with distortional strain mode only on the non symmetric axis of the box girder structure. The results show that on this non symmetric axis, the maximum (mid span) distortional deformation (12.5mm) was one and half times that of flexural deformation (8.5mm), for a simply supported box girder structure of 50m span.

10. Conclusions

In a mono symmetric box girder section flexural strain mode does not interact with distortional strain mode along the axis of symmetry. However, along the non symmetric axis, flexure interacts with distortion giving rise to coupled differential equations of flexural-distortional equilibrium, eqn. (22) which when solved for a particular cross sectional profile yields the flexural and distortional deformations. For the double cell mono symmetric example frame we established that distortional deformation at mid span of the girder was about one and half times that of flexural deformation.

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