Flexural Buckling Analysis of Thin Walled T Cross Section Beams with Variable Geometry

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Abstract: **Thin walled structure is a structure whose thickness is small compared to its other dimensions but which is capable of resisting bending in addition to membrane forces. Which is basic part of an aircraft structure, the structural components of an aircraft consist mainly of thin plates stiffened by arrangements of ribs and stringers. Thin plates (or thin sections or thin walled structures) under relatively small compressive loads are prone to buckle and so must be stiffened to prevent this. The determination of buckling loads for thin plates in isolation is relatively straightforward but when stiffened by ribs and stringers, the problem becomes complex and frequently relies on an empirical solution. The buckling of the thin plates is a phenomenon which could lead to destabilizing and failure of the aircraft; in this paper it is considered T cross section with variable geometry and length. The critical buckling stresses have been studied for several combinations of the geometry parameters of the beam with the help of ANSYS and drawn the result plots**

Keywords: Thin walled beams, buckling analysis, Finite element analysis

I INTRODUCTION

A great deal of attention has been focused on plates subjected to shear loading over the past decades. One main fact in design of such elements, which fall in the category of thin-walled structures, is their buckling behavior. Plate girders and recently shear walls are being widely used by structural engineers, as well as ship and aircraft designers. The role of stiffeners is proved to be vital in design of such structures to minimize their weight and cost.

Xiao-ting et al [1] presented an analytical model for predicting the lateral torsional buckling of thin walled channel section beams restrained by metal sheeting when subjected to an uplift load. And calculated the critical load from critical energy theory and showed that the critical buckling moment in the pure bending case is less than half of the critical moment, it is more effective to use the anti sag bars in the simply supported beams than in the fixed beams, the closer the loading point to the centre the lower the critical load. M.Ma et al [2] developed energy method for analyzing the lateral buckling behavior of the monosymmetric I beams subjected to distributed vertical load, with full allowance for distortion of web. the method assumes that the flanges buckle as rigid the rectangular section beams, but the web distorts as an elastic plate during buckling. it is shown that the disparity between the distortional and classical critical load increases as *h/l* increases and that for short beams the classical method seriously over estimates the critical load. B. W. Schafer [3] worked on cold-formed thin-walled open cross-section steel columns and provided local, distortional, and flexural-torsional buckling. Experimental and numerical studies indicated that post buckling strength in the distortional mode is less than in the local mode. In pin-ended lipped channel and zed columns, local and Euler interaction is well established. A direct strength method is proposed for column design. The method uses separate column curves for local buckling and distortional buckling with the slenderness and maximum capacity in each mode controlled by consideration of Euler equation. Attard Mario et al [4] investigated lateral-torsional buckling behavior of open-section thin-walled beams based on a geometrically nonlinear formulation, which considers the effects of shear deformations, also made Comparisons between the results based on fully nonlinear analysis and linearized buckling analysis in order to illustrate the effects of pre-buckling deformations as well as the shear deformations on the buckling load predictions. Ing. Antonin pistek,[5] analytical method for limit load capacity Calculation Of thin walled aircraft structures focused on description and Comparison of different methods for limit load Capacity calculation of thin walled aircraft Structures – considering all possible forms of Buckling and failures on nonlinear behavior of The structure under gradually increased Loading. Carine Louise Nilsen, et al [6] found that the behavior of thin-walled steel sections, including local buckling, distortional buckling, global buckling and shear buckling have been well understood and appropriate design methods existed. Foudil Mohria et al [7] derived analytical solutions Based on a non-linear stability model, for simply supported beam-column elements with bi-symmetric I sections under combined bending and axial forces. Jaehong Lee et al [8] explained lateral buckling of thin-walled composite beams with monosymmetric sections. A general geometrically nonlinear model for thin walled laminated composites with arbitrary open cross-section and general laminate stacking sequences is given by using systematic variational formulation based on the classical lamination theory. The load capacity of cold-formed thin-walled beams is usually restricted by their stability and post-buckling behaviour. Strength was considered by Cheng and Schafer [9], Trahair [10] Experimental investigations, stress and displacement distribution of cold-formed beams were shown by Paczos et al [11] Other examples of papers directly connected with the subject of this work are Biegus et al [12] Magnucki et al [13] Magnucki et al [14] Paczos, Jeyaragan et al [15] Lawrence W. Rehfield et al [16] design methodology for buckling of thin-walled laminated composite beams shoed buckling by pure bending and bending- torsion coupled modes can occur and that transition among modes are governed by structural parameters. Tomasz kubiak et al [17] presented analysis of local buckling of thin-walled beam-columns, taking account Global pre critical bending within the first order approximation. Marco et al [18] published a paper on buckling of thin-walled structures is presented using the 1D finite element based refined beam theory formulation Deepak et al [19 cold formed c and z sections because of their easy connections but they suffer from certain buckling modes. here the structural behavior of c channel lipped beams due to lateral buckling and load carrying capacity is evaluated. Load Vs deflection curves plotted in comparison with the experimental results attained through fea modeling of the software as part of results. Brad ford [20] lateral-distortional buckling of steel I section members presented how the usual types of buckling of steel members assumed in design are lateral-torsional and local buckling modes. In lateral-torsional buckling, the cross-sections of the member translate and twist as rigid bodies. On the other hand, local buckling is characterized by localized distortions of the cross-section over a short wavelength in the absence of lateral translation.

II BUCKLING

A Thin plate or a thin walled structure is a structure whose thickness is small compared with its other dimensions but which is capable of resisting bending in addition to membrane forces. Such a plate forms a basic part of an aircraft structure, being, for example, the area of stressed skin bounded by adjacent stringers and frames in a fuselage. The structural components of an aircraft consist mainly of thin plates stiffened by arrangements of ribs and stringers. Thin plates (or thin sections or thin walled structures) under relatively small compressive loads are prone to buckle and so must be stiffened to prevent this. The determination of buckling loads for thin plates in isolation is relatively straight forward but when stiffened by ribs and stringers, the problem becomes complex and frequently relies on an empirical solution. The buckling of the thin plates is a phenomenon which could lead to destabilizing and failure of the aircraft, hence we study the buckling phenomenon on thin plates or thin walled structures with the help of the finite element analysis software ANSYS.

The first significant contribution to the theory of the buckling of columns was made as early as 1744 by Euler. [22] His classical approach is still valid, and likely to remain so, for slender columns possessing a variety of end restraints. Our initial discussion is therefore a presentation of the Euler theory for the small elastic deflection of perfect columns. However, we investigate first the nature of buckling and the difference between theory and practice. It is common experience that if an increasing axial compressive load is applied to a slender column there is a value of the load at which the column will suddenly bow or buckle in some un-predetermined direction. This load is patently the buckling load of the column or something very close to the buckling load. Clearly this displacement implies a degree of asymmetry in the plane of the buckle caused by geometrical and/or material imperfections of the column and its load. However, in our theoretical stipulation of a perfect column in which the load is applied precisely along the perfectly straight centroidal axis, there is perfect symmetry so that, theoretically, there can be no sudden bowing or buckling. We therefore require a precise definition of buckling load which may be used in our analysis of the perfect column. Assume that it is in the displaced state of neutral equilibrium associated with buckling so that the compressive load P has attained the critical value P_{CR}. Simple bending theory

$$EI\frac{d^2v}{dz^2} = -M \quad \text{or} \quad EI\frac{d^2v}{dz^2} = -P_{CR}v$$

so that the differential equation of bending of the column is

$$\frac{d^2v}{dz^2} + \frac{P_{CR}}{EI}v = 0$$

The well-known solution $v = A\cos\mu z + B\sin\mu z$ π

Critical load $P_{CR} = \dfrac{\Pi^2 EI}{l^2}$

Other values of PCR corresponding to n=2, 3,...,

$$P_{CR} = \frac{4\Pi^2 EI}{l^2}, \frac{9\Pi^2 EI}{l^2}, ...$$

The total potential energy of the column in the neutral equilibrium of its buckled state is therefore

$$U + V = \frac{EI}{2}\int_0^l \left(\frac{d^2v}{dz^2}\right)^2 dz - \frac{P_{CR}}{2}\int_0^l \left(\frac{dv}{dz}\right)^2 dz$$

And is capable, within the limits for which it is valid and if suitable values for the constant coefficients are chosen, of representing any continuous curve. We are therefore in a position to find P_{CR} exactly.

$$U + V = \frac{\Pi^4 EI}{4l^3}\sum_{n=1}^{\infty} n^4 A_n^2 - \frac{\Pi^2 P_{CR}}{4l}\sum_{n=1}^{\infty} n^2 A_n^2$$

In general form

$$P_{CR} = \frac{n^2\Pi^2 EI}{l^2}$$

$$P_{CR} = \frac{42EI}{17l^2} = 2.471\frac{EI}{l^2}$$

T section thin walled beam flanges are behave similar to plate buckling, the following equations are valid for thin plate simply supported along all four edges.

$$w = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn} \sin\frac{m\Pi x}{a} \sin\frac{n\Pi y}{b}$$

Also, the total potential energy of the plate is

U+V=

$$\frac{1}{2}\int_0^a\int_0^b\left[D\left\{\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)^2 - 2(1-v)\left[\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2\right]\right\} - N_x\left(\frac{\partial w}{\partial x}\right)^2\right]dxdy$$

The total potential energy of the plate has a stationary value in the neutral equilibrium of its buckled state i.e. $Nx = Nx_{,CR}$

$$N_{x,CR} = \frac{k\Pi^2 D}{b^2}$$

where the plate buckling coefficient k is given by the minimum value of

$$k = \left(\frac{mb}{a} + \frac{a}{mb}\right)^2$$

Where "a" is length of the plate, "b" is width of the plate, m and n are the number of half-waves in the x and y directions,

The critical stress of the plate is given by the equation

$$\sigma_{CR} = \frac{k\Pi^2 E}{12(1-v^2)}\left(\frac{t}{b}\right)^2$$

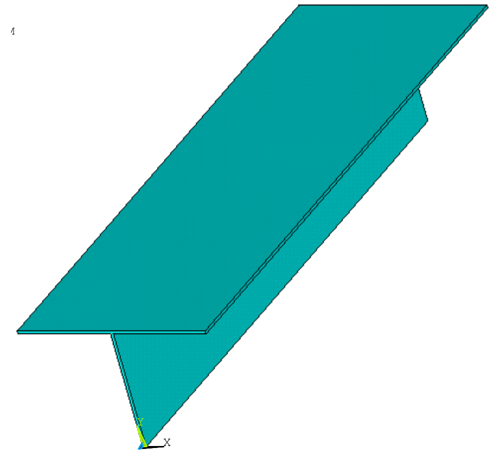
The local failure stress in longitudinally stiffened panels was determined by Gerard using a slightly modified form

$$\frac{\overline{\sigma}_f}{\sigma_{cy}} = \beta_g\left[\frac{gt_{sk}t_{st}}{A}\left(\frac{E}{\overline{\sigma}_{cy}}\right)^{\frac{1}{2}}\right]^m$$

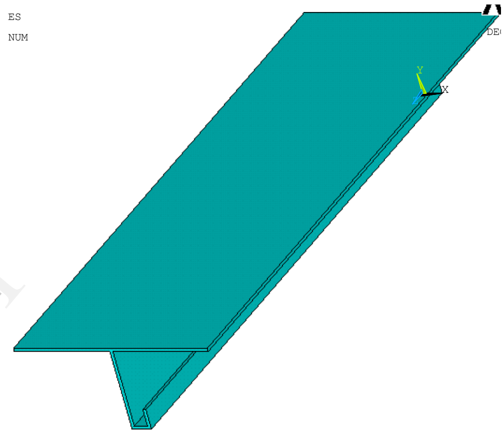
Where 'g' is number of cuts + Flanges

III MODELING

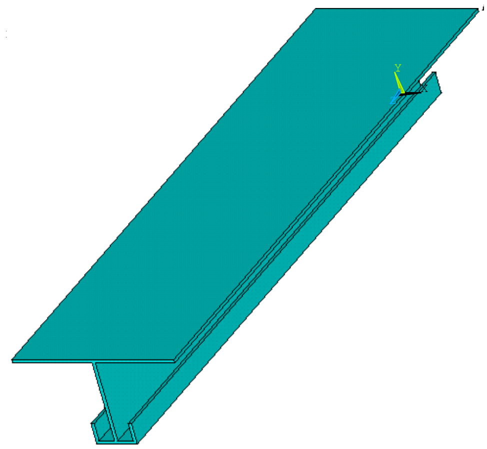
Modeled the six types of thin walled T cross sections by using ANSYS, all the cases are with constant area of cross sections (63.36 mm^2) by changing the length, thickness parameters



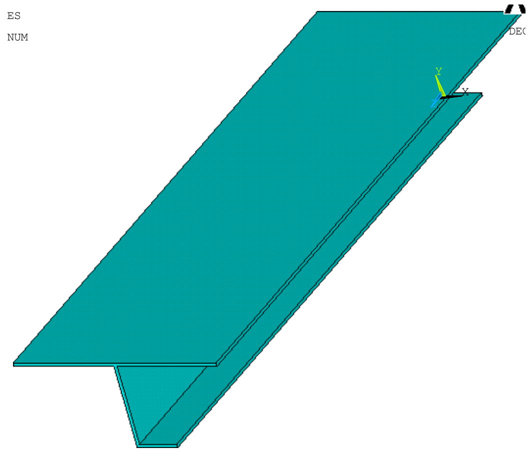
T section case 1



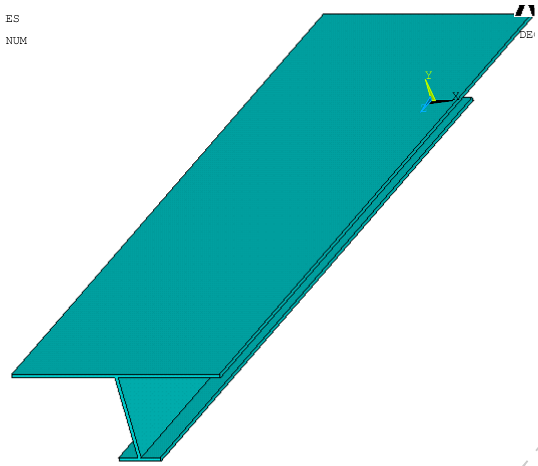
T section case 2



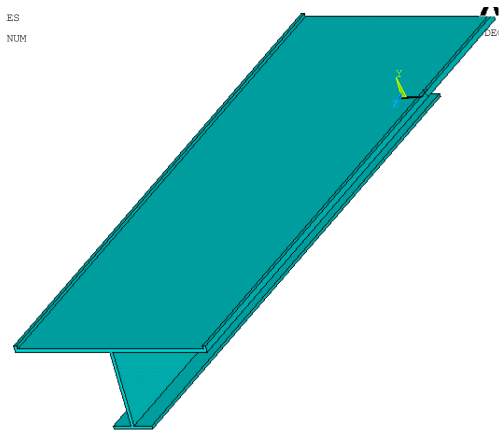
T section case 3



T section case 4

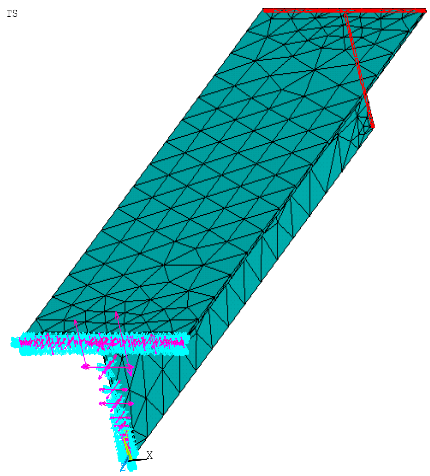


T section case 5

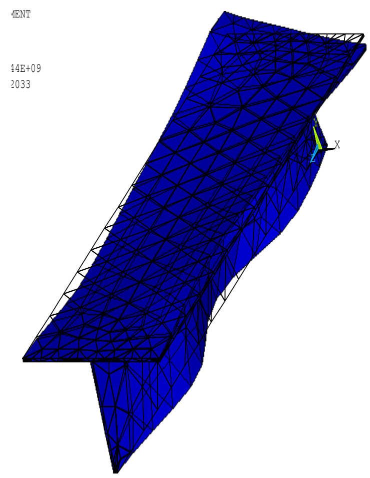


T section case 6

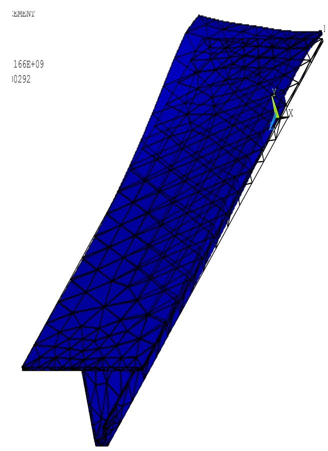
Fig.1 the different shapes of T sections considered for analysis in this paper



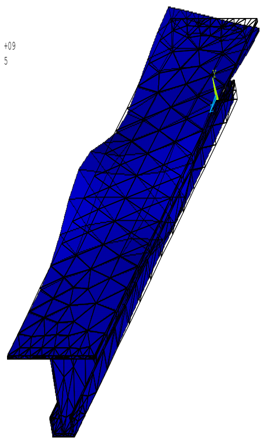
(a)



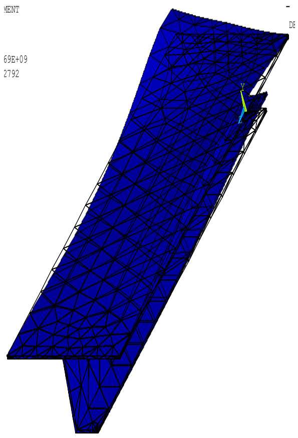
(b)



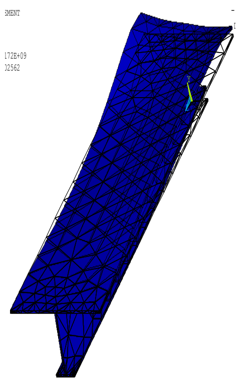
(c)



(d)

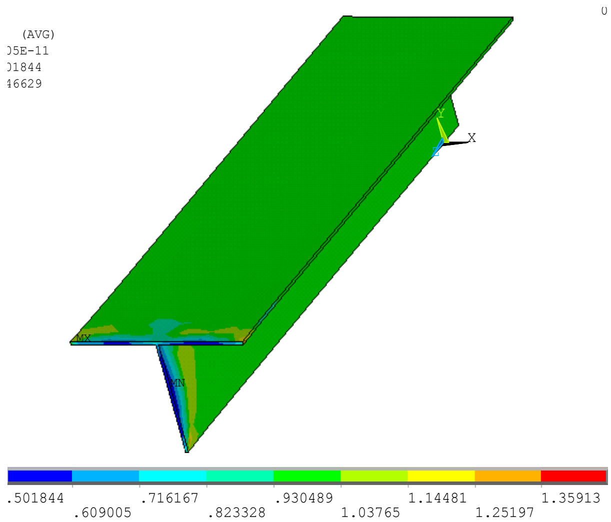


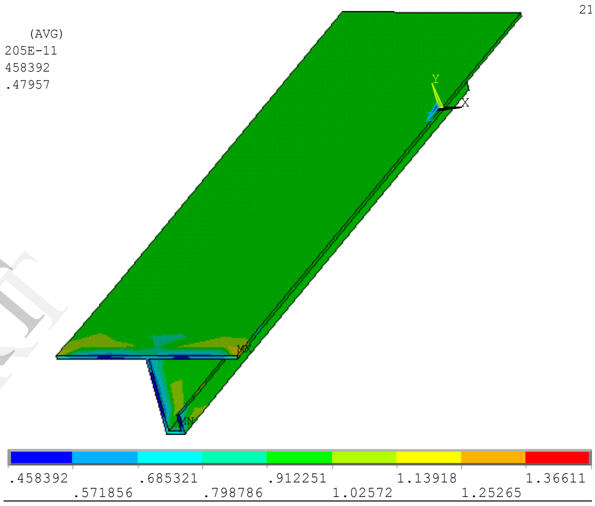
(e)



(f)

Fig.2 (a) Meshed model (b)(c)(d)(e)(f) are Some of the mode shapes of modal analysis





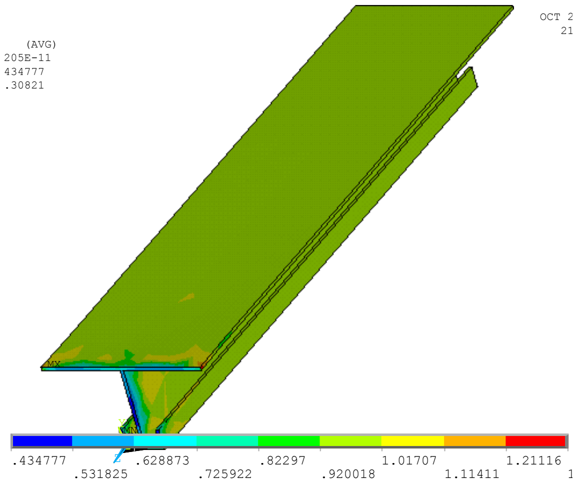


Fig.3 some of the stress figures

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IV RESULT

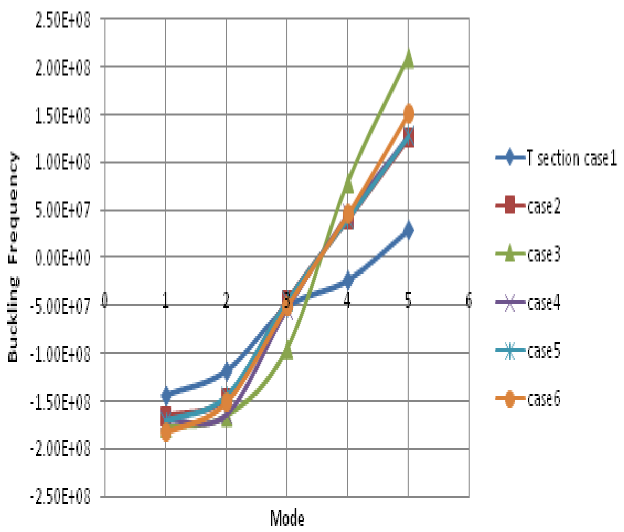
Combined graph for all cases of T section, length150 mm:

Mode	Case1 Buckling	Case2 Buckling	Case3 Buckling	Case4 Buckling	Case5 Buckling	Case6 Buckling
1	-1.44E+08	-1.66E+08	-1.78E+08	-1.69E+08	-1.72E+08	-1.83E+08
2	-1.19E+08	-1.47E+08	-1.66E+08	-1.63E+08	-1.46E+08	-1.51E+08
3	-5.19E+07	-4.42E+07	-9.53E+07	-5.27E+07	-4.58E+07	-5.00E+07
4	-2.41E+07	4.01E+07	7.80E+07	4.53E+07	3.98E+07	4.50E+07
5	2.79E+07	1.26E+08	2.10E+08	1.27E+08	1.27E+08	1.50E+08

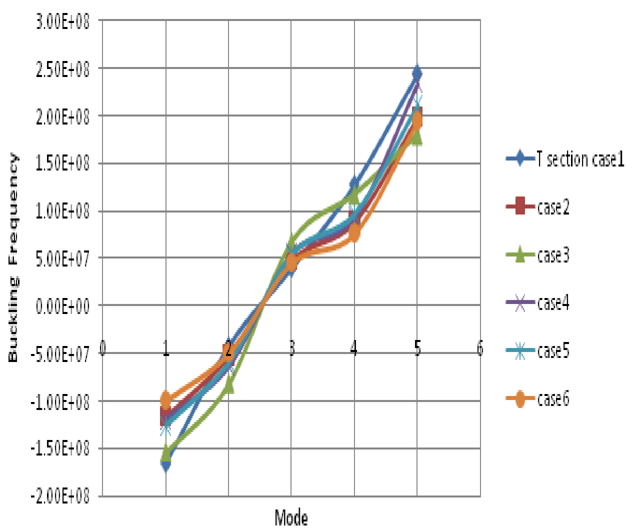
Combined graph for all cases of T section, length : 300 mm

Mode	Case1 Buckling	Case2 Buckling	Case3 Buckling	Case4 Buckling	Case5 Buckling	Case6 Buckling
1	-1.66E+08	-1.16E+08	-1.54E+08	-1.19E+08	-1.25E+08	-9.94E+07
2	-4.24E+07	-5.24E+07	-8.17E+07	-6.29E+07	-5.94E+07	-4.97E+07
3	4.01E+07	4.77E+07	6.78E+07	5.55E+07	5.44E+07	4.53E+07
4	1.28E+08	8.88E+07	1.18E+08	9.31E+07	9.69E+07	7.64E+07
5	2.45E+08	1.98E+08	1.80E+08	2.34E+08	2.10E+08	1.94E+08

Combined graph for all cases of T section, length 150 mm:



Combined graph for all T sections, length 300 mm:



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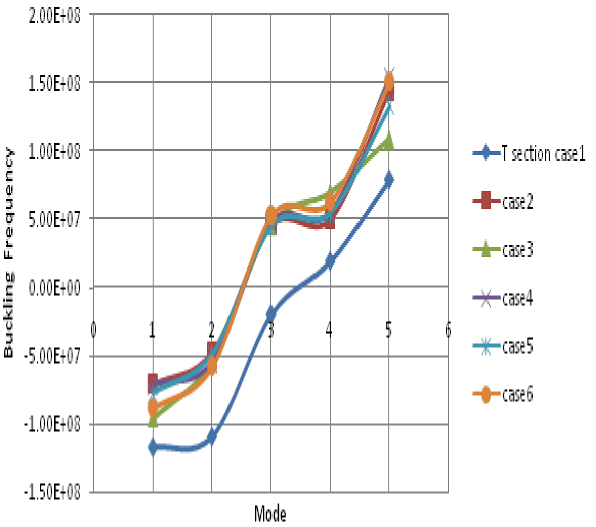
Combined graph for all cases of T section Length -600 mm:

Mode	Case1 Buckling	Case2 Buckling	Case3 Buckling	Case4 Buckling	Case5 Buckling	Case6 Buckling
1	-1.17E+08	-7.10E+07	-9.58E+07	-7.13E+07	-7.70E+07	-8.83E+07
2	-1.10E+08	-4.75E+07	-5.35E+07	-5.46E+07	-4.83E+07	-5.77E+07
3	-1.95E+07	4.52E+07	4.56E+07	4.75E+07	4.56E+07	5.26E+07
4	1.94E+07	4.98E+07	6.93E+07	5.49E+07	5.54E+07	6.27E+07
5	7.92E+07	1.44E+08	1.09E+08	1.54E+08	1.33E+08	1.51E+08

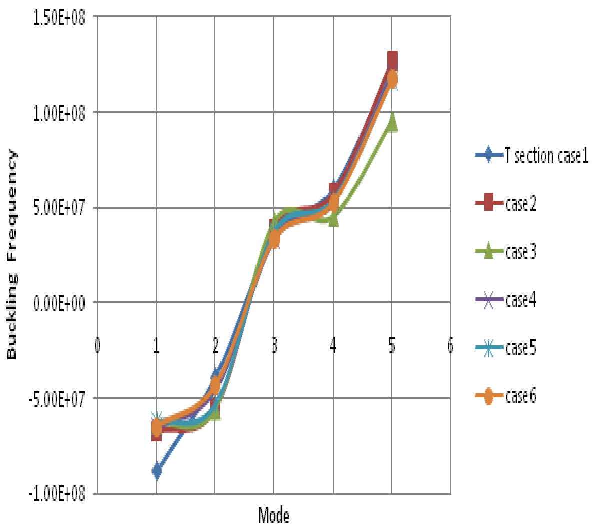
Combined graph for all cases of T section Length-450mm:

Mode	Case1 Buckling	Case2 Buckling	Case3 Buckling	Case4 Buckling	Case5 Buckling	Case6 Buckling
1	-8.86E+07	-6.73E+07	-6.37E+07	-6.57E+07	-6.22E+07	-6.49E+07
2	-3.92E+07	-5.54E+07	-5.59E+07	-4.54E+07	-5.31E+07	-4.33E+07
3	3.43E+07	3.84E+07	4.25E+07	3.34E+07	3.71E+07	3.34E+07
4	5.93E+07	5.69E+07	4.53E+07	5.38E+07	5.34E+07	5.23E+07
5	1.24E+08	1.26E+08	9.50E+07	1.20E+08	1.16E+08	1.17E+08

Combined graph for all T sections Length -600 mm:



Combined graph for all T sections Length-450mm:



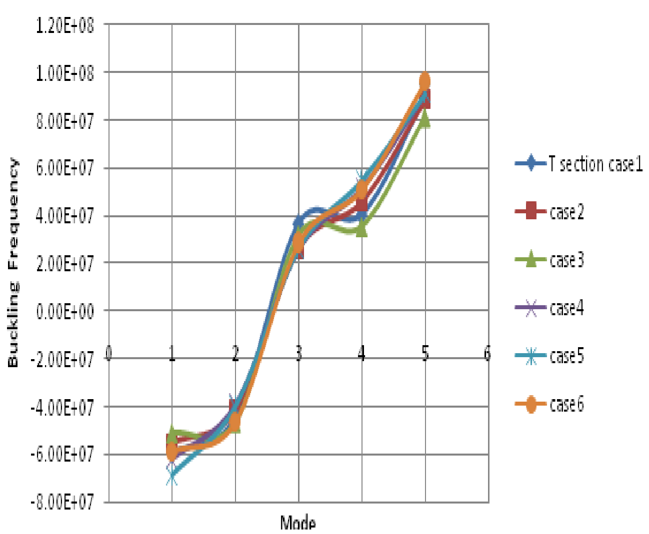
Combined graph for all cases of T section Length750 mm:

Mode	Case1 Buckling	Case2 Buckling	Case3 Buckling	Case4 Buckling	Case5 Buckling	Case6 Buckling
1	-5.93E+07	-5.54E+07	-5.12E+07	-6.21E+07	-6.88E+07	-5.91E+07
2	-4.43E+07	-4.18E+07	-4.67E+07	-3.93E+07	-4.00E+07	-4.67E+07
3	3.69E+07	2.62E+07	3.19E+07	2.61E+07	2.61E+07	2.83E+07
4	4.05E+07	4.59E+07	3.54E+07	5.18E+07	5.47E+07	5.09E+07
5	8.97E+07	8.83E+07	8.14E+07	9.13E+07	9.08E+07	9.58E+07

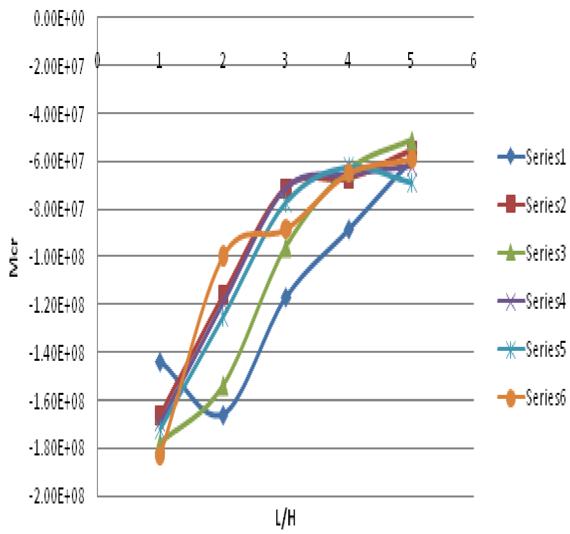
Critical moment Vs length to height ratio:

L/H	Mcr	Mcr	Mcr	Mcr	Mcr	Mcr
1	-1.44E+08	-1.66E+08	-1.78E+08	-1.69E+08	-1.72E+08	- 1.83E+08
2	-1.66E+08	-1.16E+08	-1.54E+08	-1.19E+08	-1.25E+08	- 9.96E+07
3	-1.17E+08	-7.10E+07	-9.58E+07	-7.13E+07	-7.70E+07	- 8.83E+07
4	-8.86E+07	-6.73E+07	-6.37E+07	-6.57E+07	-6.22E+07	- 6.49E+07
5	-5.93E+07	-5.54E+07	-5.12E+07	-6.21E+07	-6.88E+07	- 5.91E+07

Combined graph for all T sections:



Critical moment Vs length to height ratio:



V CONCLUSIONS

- Thin-walled T sections with different cross-sections have been considered for buckling analysis in this paper.
- It is observed that in buckling analysis, at mode 1, the value of buckling value is higher for normal T section whereas at mode 4, the T section with lipped flanges is giving higher buckling value than all other cases.

- Thin walled T sections consider the following lengths 150 mm, 300mm, 450mm, 600mm, and 750mm. It is observed that the buckling stress decreased gradually with increase lengths of the thin walled beams.

REFERENCES

1) Xiao-ting Chu, Roger Kettle, Long-yuan Li. *"Lateral-torsion buckling analysis of partial-laterally restrained thin-walled channel-section beams."* Journal of Constructional Steel Research 60 (2004) 1159-1175.

2) M.Ma&O. Hughes. *"Lateral Distortional Buckling of Monosymmetric I-Beams under Distributed Vertical Load."* Thin-Walled Structures Vol. 28, No. 2, pp.123-145, 1996.

3) B. W. Schafer. *"Local, Distortional, and Euler Buckling of Thin-Walled Columns"* Journal of Structural Engineering, Vol. 128, No. 3, March 1, 2002

4) R. Emre Erkmen, Mario M. Attard. *"Lateral–torsional buckling analysis of thin-walled beams including shear and pre-buckling deformation effects."* International Journal of Mechanical Sciences - INT J MECH SCI 01/2011; 53(10):918-925.

5) Prof. Ing. Antonin pistek. *"Analytical Method for Limit Load Capacity Calculation of Thin Walled Aircraft Structures"* 28th International Congress of the Aeronautical Sciences

6) Carine Louise Nilsen, Md Azree Othuman Mydin and Mahyuddin Ramli 2012*" Performance of lightweight thin-walled steel sections: theoretical and mathematical considerations."* Advances in Applied Science Research, 2012, 3 (5):2847-2859.

7) Foudil Mohria, Cherif Bouzerirab, Michel Potier-Ferryc of France in the year 2008*" Lateral buckling of thin-walled beam-column elements under combined axial and bending loads."* Thin-Walled Structures 46 (2008) 290–302.

8) Jaehong Lee. *"Lateral buckling analysis of thin-walled laminated composite beams with monosymmetric sections"* Engineering Structures 01/2006.

9) Cheng Yu, Benjamin W. Schafer. *"Simulation of cold-formed steel beams in local and distortional buckling with applications to the direct strength method."* Journal of Constructional Steel Research 63 (2007) 581-590.

10) N. S. Trahair. *"Buckling Analysis Design of Steel Frames."* Journal of Constructional Steel Research, 65(7), 1459-1463.

11) K. Magnucki, P. Paczos. *"Theoretical shape optimization of cold-formed thin-walled channel beams with drop flanges in pure bending."* Journal of Constructional Steel Research 65 (2009) 1731-1737.

12) A. Biegus, D. Czepiżak. *"Experimental investigations on combined resistance of corrugated sheets with strengthened cross-sections under bending and concentrated load."* Thin-Walled Structures (Impact Factor: 1.23). 01/2008; 46(3):303-309.

13) K. Magnucki, M. Rodak, J. Lewinski. *"Optimization of mono and anti-symmetrical I-sections of cold-formed thin-walled beams"* Thin-Walled Structures 44 (2006) 832-836.

14) P. Paczos, P. Wasilewicz. *"Experimental investigations of buckling of lipped, cold-formed thin-walled beams with I-section."* Thin-Walled Structures 47 (2009) 1354-1362.

15) S. Jeyaragan and M. Mahendran. *"Experimental Investigation of the New Built-up Litesteel Beams."* Fifth International Conference on Thin-Walled Structures Brisbane, Australia, 2008.
16) Lawrence W. Rehfield and Ulrich Mueller. *"Design Methodology for Buckling of Thin-Walled Laminated Composite Beams."* ICCM - 12 Europe 1999.

17) Tomasz Kubiak. *"Interactive Buckling in Thin-walled Beam-columns with Widthwise Varying Orthotropy."* Journal of Theoretical and Applied Mechanics 44, 1, pp. 75-90, Warsaw 2006.
18) Syed Muhammad Ibrahim, Erasmo Carrera, Marco Petrolo, Enrico Zappino 2012. *"Buckling of thin-walled beams by a refined theory."* J Zhejiang Univ-Sci A (Appl Phys & Eng) 2012 13(10):747-759.

19) M.S.Deepak, R.Kandasamy, Dr R.Thenmozhi. *"Investigation on Lateral Torsional Buckling Performance of Cold-formed Steel C-Channel Sections."* International Journal of Emerging Trends in Engineering and Development Issue 2, Vol. 4 (May-2012).

20) M.A. Bradford. *"Lateral-Distortional buckling of steel I-Section members"* Journal of Constructional Steel Research 23, 97-116.

21) Sudhir Sastry YB, Y Krishna, Pattabhi R. Budarapu *" Parametric studies on buckling of thin walled channel beams"* Manuscript submitted to Journal of Computational Materials Science, Elsevier Editorial System

22) T. H. G. Megson, "Aircraft-Structures-for-Engineering-Students-Fourth-Edition"