Flexural Behavior of Compact Equivalent Foam Cored Sandwich Beams

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Abstract— Flexural behavior of mass equivalent stiff cored sandwich beams were studied numerically using most versatile analysis tool i.e. FEM/ANSYS. To rate the performance of these compact sandwich beams many engineering parameters such as face sheet stress (σf), core shear (τ) beam deflection(y) and the most important beam properties i.e. flexural rigidity (D) and Shear stiffness (S) were considered. Number of mass equivalent sandwich beam models were generated and analyzed under both 3P & 4P bending. The most popular formulae proposed by Kuenzi (ASTM standards; C393) were used to evaluate the mechanical properties of sandwich beams. An attempt was made to replace low density and bulky (large thickness) cores with a mass equivalent compact and high density cores of same material that can perform better under flexural loading. Influence of mass equivalent compact core on the most important mechanical properties such as flexural rigidity (D) and Shear stiffness (S) were studied with great care. Foremost of above, the analyzing tool i.e. FEM/ANSYS was validated using bench marks. From the present investigation it is evident that, for a constant mass of a sandwich beam, equivalent cores of high density foams with proportionate thickness can perform better under flexural load. It was observed tremendous improvements in the mechanical properties with the use of equivalent stiff cores. However as core become more compact there will be no much improvements in the results.

Keywords: High density foams, Mass equivalent stiff cores, Compact sandwich beams, FEM/ANSYS.

1. INTRODUCTION

In today’s space-age generation, sandwich construction has attained broad acceptance and usage due to their wonderful characteristics of high bending stiffness over lightness ratio. This is mainly because of tailorable constituents and indispensable properties of their elements (face sheets, cores and adhesive materials). Sandwich structures find extensive applications in the aerospace, marine and automotive industry [1-3], more exclusively sandwich beams are widely used engineering structures and generally designed to sustain lateral loads. The stresses and deflection induced under the action of external lateral loads and the overall structural performance of sandwich panels is mainly depends on the geometries, physical and mechanical properties of the both facings as well as core materials [4, 5]. In several literatures it is reported that upon varying properties of core materials (exclusively its density) the flexural strength of sandwich beam can be alter to very large extent with a little weight compromise. R.Vijayalakshmi Rao et.al [6] and A Mizapur et.al [7] have studied the effect of core density of foam cored sandwich beams of on flexural strength for limited cases. They concluded that use of high density foam cores for sandwich structures, flexural strength can be improved to a very large extent. However in their studies, effect of foam core density on other engineering parameters such as normal stress distribution in face sheets (σf), core shear (τ) and overall beam deflection (y) was not considered. It is thus clear from the above reviews and references, the flexural behavior of sandwich composites can be characterized by two important factors, foremost the core thickness and other is variation of core density. In view of above fact, the present investigation was aimed at an alternate substitute for existing bulk and low density cores with compact & high density cores without weight compromise by exploiting the advantage of availability of high density foams and to achieve improved structural stability and space saving. To pursue the above task, number of finite element models of sandwich beams with mass equivalent cores was generated and the flexural behavior was studied numerically using FEM/ANSYS.
II. VALIDATION OF FE- MODELING FOR STATIC ANALYSIS OF SANDWICH BEAMS UNDER FLEXURAL LOAD

Finite element method through the medium of general purpose program i.e. FEM/ANSYS offers a powerful tool for engineering analysis. However user of finite element analysis has to validate the elements, meshes and procedure employed by using bench marks. This section is mainly concern with the validation of finite element modeling for static analysis of sandwich beams under three-point bending using analytical solutions of 2D-elasticity of sandwich beams. To pursue this task a sandwich beam model was chosen as per ASTM standards (C323) [9] of dimensions 300mmx50mmx14.2mm loaded in three- point bending with a span length of 150mm as shown in figure-1. It is assume that the composite sandwich beam consists of typically of two thin face sheets of 2.1mm thick made of bi-woven E-glass fiber-epoxy prepreg composite and a light weight thicker polyurethane foam core. The geometric details of chosen sandwich beam model, physical and mechanical properties of both face sheets and core are as follows:

**Geometric details of sandwich beam model**
- Overall length = 300mm
- Span length (l1) = 150mm
- Width of beam (b) = 50mm
- Thickness of face sheet (t) = 2.1mm
- Thickness of core (c) = 10mm

**Mechanical properties of face sheets (Bi-woven glass/epoxy)**
- $E_x = E_y = 16.74$ GPa
- $E_z = 7.85$ GPa
- $G_{xy} = 2.45$ GPa
- $G_{yz} = G_{zx} = 2.30$ GPa
- $\nu_{xy} = 0.5$
- $\nu_{yz} = \nu_{zx} = 0.49$

**Mechanical properties of core (PUF)**
- $\rho = 250$ Kg/m$^3$
- $E_c = 75$ MPa
- $\nu = 0.35$

![Fig. 1: Geometry of a sandwich beam under three-point bending](image)

The face sheets and core were modeled using nonlinear SHELL 91(7 layered) and SHELL 93 elements respectively. The modeled structure was considered as a simply supported sandwich beam with overhanging loaded in three point bending. The relevant mechanical properties of both face sheet and core were carefully pre-processed. The schematic view of FE-modeling of sandwich beam so generated is as shown in figure-2 with necessary boundary conditions. After successful run of finite element program, various contour plots were extracted from the post files are as shown in figure-3 for reference. The various post processing results such as bending stress in face sheet ($\sigma_f$), shear stress in core ($\tau$) and maximum beam deflection (y), obtained from numerical analysis are tabulated in table-1.

<table>
<thead>
<tr>
<th>Bending stress ($\sigma_f$)</th>
<th>Shear stress in core ($\tau$)</th>
<th>Max deflection (y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.1 MPa</td>
<td>0.133 MPa</td>
<td>0.253 mm</td>
</tr>
</tbody>
</table>

![Fig. 2: Finite element model of sandwich beam](image)

![Fig. 3(a): Bending stress distribution in Face Sheets](image)

![Fig. 3(b): Shear stress distribution in Core](image)

![Fig. 3(c): Resultant deflection of sandwich beam](image)

Fig. 3: Contours of post processing results under 3-point bending

**Table 1**: Post processing results of FEM/ANSYS under 3-point bending

The theoretical calculations were obtained using 2D elasticity solutions proposed by H.G.Allen [5] for the above sandwich beam model is as shown in table-2. Comparison between theoretical & numerical results obtained using FEM/ANSYS are shown in table-3. The agreement was generally good and hence it can be applicable for the study of flexural behavior of sandwich panels. The validated numerical tool was successfully applied to study the flexural behavior of sandwich beams of high density polyurethane foam cores.

$F=200N  \quad l=150mm  \quad b=50mm  \quad t=2.1mm  \quad h=14.2mm  
E_f=16740$ MPa $E_c=75$ MPa $\nu = 0.35$ $G_{ce}=26$ MPa

**Table 2**: Theoretical results from analytical formulæ proposed by Allen H.G [5]

<table>
<thead>
<tr>
<th>$M_b$ = $Ft/4$ (N-mm)</th>
<th>$\sigma_f$ = $M_b/l$ (MPa)</th>
<th>$d_0$ = (c+1) (mm)</th>
<th>$D_c = E_cbd_c^2$ (N-mm$^2$)</th>
<th>$S_c = hbd_c$ (N)</th>
<th>$\tau = F/2bd$ (MPa)</th>
<th>$y = FL^2/48D$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7500</td>
<td>5.9</td>
<td>12.1</td>
<td>$128.67 \times 10^6$</td>
<td>15730</td>
<td>0.16</td>
<td>0.286</td>
</tr>
</tbody>
</table>

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### Table 3: Comparison of FEM/ANSYS & Analytical results

<table>
<thead>
<tr>
<th>Particulars</th>
<th>FEM/ANSYS (MPa)</th>
<th>Theoretical MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bending Stress (σf)</td>
<td>6.1</td>
<td>5.9</td>
</tr>
<tr>
<td>Shear stress (τ)</td>
<td>0.133</td>
<td>0.16</td>
</tr>
<tr>
<td>Max deflection (y)</td>
<td>0.253</td>
<td>0.286</td>
</tr>
</tbody>
</table>

### III. EVALUATION OF MECHANICAL PROPERTIES OF SANDWICH BEAMS

In this section the most essential mechanical properties of the sandwich beams under flexural load i.e. flexural rigidity (D) and shear stiffness (S) were calculated for a sandwich beam model as shown in figure-4 of dimensions as per ASTM standards (C393). The most popular formulae proposed by Kuenzi (ASTM Standards, C393) were modified in terms of slopes of loads v/s deflections (θ₁ & θ₂) and ratio of span lengths (l₁ & l₂) to get an average effect. These equations (1-3) are effectively used in calculations. Finite element models were generated by discretization of geometric modeling of the beam model. The relevant mechanical properties of both face sheet and core were pre-processed carefully. The schematic view of FE-modeling of sandwich beam model for 3P-bending and 4P-bending is as shown in figure-5. The contours of deflections of beams obtained from post processing results at constant load of 200N are as shown in figure-6. Deflections of beams at different loads were recorded by successfully running the finite element programs. A graph of load v/s deflection from the results of 3-Point bending and 4-Point bending tests using finite element analysis was plotted as shown in figure-7. The various beam properties obtained from FEM/ANSYS is tabulated in table-4. The theoretical calculations were also obtained using the formulae proposed by Allen H G [5] as tabulated in table-5. Finally a comparison between FEM/ANSYS and theoretical results are made in table-6.

\[
\begin{align*}
E_f &= 16740 \text{MPa} \\
E_c &= 60 \text{MPa} \\
l_1 &= 150 \text{mm} \\
l_2 &= 250 \text{mm} \\
b &= 50 \text{mm} \\
c &= 30 \text{mm} \\
t &= 2.1 \text{mm} \\
d &= 32.1 \text{mm} \\
h &= 34.2 \text{mm}
\end{align*}
\]

\[
\begin{align*}
D &= \left(\frac{(l_1^3 \theta_1)}{48}\right) \times 2.821 / \{2.4 \left(\theta_1 / \theta_2\right)\}^{-1} \quad (1) \\
G &= \left[3 \left(\frac{\theta_1 c}{(h+c)^2}\right) \{0.7383/ [1-0.6284 (\theta_1 / \theta_2)]\}\right] \quad (2) \\
S &= bdG \quad (3) \\
\theta_1 &= \frac{\Delta F}{\Delta w_1} \quad (4) \\
\theta_2 &= \frac{\Delta F}{\Delta w_2} \quad (5)
\end{align*}
\]
Table 4: Beam properties using FEM/ANSYS results

<table>
<thead>
<tr>
<th>$b_1$ (N/mm)</th>
<th>$b_2$ (N/mm)</th>
<th>$D_1$ (N-mm²)</th>
<th>$D_2$ (N-mm²)</th>
<th>$S_1$ (N)</th>
<th>$S_2$ (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1197</td>
<td>1227</td>
<td>0.076</td>
<td>0.176 x 10⁵</td>
<td>49.93</td>
<td>0.8 x 10⁵</td>
</tr>
</tbody>
</table>

Table 5: Theoretical estimation of beam properties

<table>
<thead>
<tr>
<th>$D_1$ = $E_1 b_1^3 t_1^2$ (N-mm²)</th>
<th>$G = E_1 / (2(1+\nu_1))$ (MPa)</th>
<th>$S = b_1 d G$ (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.905 x 10⁶</td>
<td>22.23</td>
<td>0.358 x 10⁵</td>
</tr>
</tbody>
</table>

Table 6: Comparison of Beam properties using various solutions

<table>
<thead>
<tr>
<th>Mechanical properties</th>
<th>Theoretical</th>
<th>Experimental</th>
<th>Numerical</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flexural rigidity (D) (N-mm²)</td>
<td>0.905 x 10⁶</td>
<td>0.225 x 10⁷</td>
<td>0.176 x 10⁹</td>
</tr>
<tr>
<td>Rigidity modulus (G) (MPa)</td>
<td>22.23</td>
<td>12.93</td>
<td>49.93</td>
</tr>
<tr>
<td>Shear stiffness (S) (N)</td>
<td>0.358 x 10⁵</td>
<td>0.21 x 10⁵</td>
<td>0.801 x 10⁵</td>
</tr>
</tbody>
</table>

IV. FLEXURAL BEHAVIOR OF MASS EQUIVALENT STIFF CORES SANDWICH BEAMS

The mechanical response in general and flexural behavior in particular of a sandwich beam depends upon various parameters such as constituents of face sheets, geometric and material properties of cores, the adhesive bonding the cores to the skins and type of external load. For sandwich construction usually thick core of low density foams are used in order to have higher values of flexural & shear stiffness. However these derived beam properties are not only depend on geometry of beam but also depend upon physical properties of the foam material. In view of above fact an attempt was made to understand the flexural behavior of sandwich beams of mass equivalent cores by replacing the thick and low density foams with a mass equivalent compact and high density foams of same foam material. To pursue this task, foams of different densities ranged from 50 Kg/m³ to 250 Kg/m³ were considered and corresponding core thickness of equivalent cores were obtained. Figure-8 shows geometric comparison of two mass equivalent cores.

By selecting foams of different densities (in increasing order) it is always possible to obtain the proportionate thickness of an equivalent core from above relation, the other dimensions such as width & span length of the two cores remain unchanged. For the present study polyurethane foams of density ranged from 50 Kg/m³ to 250 Kg/m³ were considered and corresponding core thickness of equivalent cores were obtained. As per model development scheme finite element models were generated and analyzed under both three-point bending and four-point bending. Various post processing results such as face sheet stress ($\sigma_f$), core shear ($\tau$), maximum deflection ($w$) etc were recorded. The most essential mechanical properties of sandwich beams i.e. flexural rigidity (D) and Shear stiffness (S) were calculated for all the equivalent models considered for study. Similar procedure was adopted as discussed in previous section. An attempt was made to identify the most feasible equivalent sandwich beam as compared to existing bulky sandwich beams. The deflection contours of FEM/ANSYS results for first & last equivalent model is as shown in figure 9 and 10 respectively.

$$D = E_b b t_1^2 / 2$$ (N-mm²)

$$G = E_b / (2(1+\nu))$$ (MPa)

$$S = b_1 d G$$ (N)

Fig. 8: Comparison of two mass equivalent cores

Fig. 8(a): Sandwich with bulk core ($c_o$) (Low density ($\rho_o$) foam)

Fig. 8(b): Sandwich with compact core ($c_e$) (High density ($\rho_e$) foam)

Fig. 9: Deflection contours of Equivalent model-1 at load (F) = 200N

Fig. 9(a): 3P-bending

Fig. 9(b): 4P-bending

Fig. 10: Deflection contours of Equivalent model-2 at load (F) = 200N

Fig. 10(a): 3P-bending

Fig. 10(b): 4P-bending

Note: $\rho_o$-soft core (low density) & $\rho_e$-equivalent stiff core (high density)
Comparison of Mechanical properties of mass equivalent sandwich beams

\[ \rho_o = 50\text{Kg/m}^3 \quad E_c = 15\text{MPa} \quad C_o = 50\text{mm} \quad b = 50\text{mm} \quad t = 2.1\text{mm} \]

Table 7: Mechanical properties of sandwich beams of mass equivalent cores

<table>
<thead>
<tr>
<th>Equivalent Cores ((c_e))</th>
<th>Specifications</th>
<th>Mechanical properties</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(p_e/p_o)</td>
<td>(c_e) (mm)</td>
</tr>
<tr>
<td>Core-1 ((e=1))</td>
<td>1</td>
<td>50</td>
</tr>
<tr>
<td>Core-2 ((e=2))</td>
<td>2</td>
<td>25</td>
</tr>
<tr>
<td>Core-3 ((e=3))</td>
<td>3</td>
<td>16.67</td>
</tr>
<tr>
<td>Core-4 ((e=4))</td>
<td>4</td>
<td>12.5</td>
</tr>
<tr>
<td>Core-5 ((e=5))</td>
<td>5</td>
<td>10</td>
</tr>
</tbody>
</table>

V. RESULTS AND DISCUSSIONS

The present investigation was mainly focused on the flexural behavior of mass equivalent sandwich beams made up of compact and stiff polyurethane foams and composite face sheets (Bi-woven E-glass/epoxy resin) using FEM/ANSYS. To rate the performance of these structures, the most important beam properties i.e. flexural rigidity \(D\) and Shear stiffness \(S\) and many engineering parameters such as face sheet stress \((\sigma_f)\), core shear \((t)\), beam deflection \((y)\) of the sandwich beam were considered. Foremost to this, the numerical procedure (FEM/ANSYS) was validated using 2D-elasticity solutions. To pursue this task a sandwich beam model was chosen as per ASTM standards (C323) [9] of dimensions 300mm\(\times\)50mm\(\times\)14.2mm loaded in three-point bending with a span length of 150mm as shown in figure-1. Finite element analysis was performed and analyzed under three-point bending with a constant mid span load of 200N. To be more confident on the FE-modeling and its results, analytical verification using 2D-elasticity of sandwich beam were carried out as shown in table-2. A comparison of FE-analysis results and analytical solutions are tabulated in Table-3. The agreement between numerical and theoretical results were generally good.

After successful validation of finite element analysis for flexural loading of sandwich beams, the most essential mechanical properties of the sandwich beams under flexural load i.e. flexural rigidity \(D\) and shear stiffness \(S\) were calculated from the results of both 3-Point and 4-point bending analysis performed using validated numerical tool i.e. FEM/ANSYS for a sandwich beam model as shown in figure-4. The most popular formulae proposed by Kuenzi (ASTM Standards, C393) were modified in terms of slopes of loads v/s deflections \((\theta_1 \& \theta_2)\) and ratio of span lengths \((l_1 \& l_2)\) to get an average effect. These equations (1-3) are effectively used in calculations. The results so obtain were compared using theoretical & experimental results [11]. The agreement between numerical and experimental results was generally good. Furthermore tremendous improvements on mechanical properties of the sandwich beams can be obtained by the use of high density foams for cores, but however the total weight of the structure increases which is undesirable for sandwich construction. To overcome this problem, sandwich beams were modeled using mass equivalent cores (cores of same material & weight with variable density & thickness) as discussed in previous section. Number of mass equivalent sandwich beams were obtained as per model generation scheme and analyzed them under both 3-point & 4-point bending. The results of both three-point and four point bending were used to estimate the mechanical properties of these beams in order to study the effect of equivalent compact cores under flexural load. The most popular formulae proposed by Kuenzi (ASTM standards) were used to calculate the Flexural rigidity \(D\) and Shear stiffness \(S\) of the sandwich beams. The results so obtained for various mass equivalent beams are represented graphically in figure-11.

![Graph of D v/s Equivalent cores](image)

![Graph of G v/s Equivalent cores](image)

The various post processing results such as normal stress in face sheet \(\sigma_f\) transverse shear stress in core \(t\) and...
maximum beam deflection (y) for all the equivalent models were obtained under 3-point bending at a load of 200N is as shown in figure-12. It was observed that when the beam become more and more compact i.e. Label 4 and 5 of figure-12 the performance parameters ($\sigma_f$, $\tau$) are better under flexural loading. However as core become more compact (Label-5) there will be no much improvements in performance parameters ($\sigma_f$, $\tau$, $\omega$), as shown in figure-12. It was observed that when the beam thickness.

**Fig. 12(a): Effect of equivalent cores on Face sheet stress ($\sigma_f$)**

**Fig. 12(b): Effect of equivalent cores on Core shear**

**Fig. 12(c): Effect of equivalent cores on Maximum beam deflection (y)**

**VI. CONCLUSION**

Flexural behavior of stiff and compact foam cored sandwich beams were studied numerically using general purpose program i.e. FEM/ANSYS. The performance of mass equivalent stiff and compact cores is better than bulky and low density soft foams under flexural load. Polyurethane foam cores of density range 250Kg/m$^3$ to 300Kg/m$^3$ with proportionate thickness are suitable as compared to bulky and low density soft foams of same mass for sandwich construction. No such remarkable performance was observed with use of cores of its density higher than 300Kg/m$^3$ because the results are nearly asymptotic.

**ACKNOWLEDGEMENT**

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**REFERENCES**

1. S.D.Yu, W.L.Cleghorn, J Sound vib 284(1-2) pp189-204 2005
4. Zenkert D An Introduction to sandwich construction Sheffield, UK, Engineering Material Advisory Services,1995