Published by :
http://www.ijert.org

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 6 Issue 11, November - 2017

Flexural Analysis of Composite Beam using Various Boundary Conditions

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***Abstract* — For the static analysis of composite deep beam a refined beam theory is developed in the present study, considering transverse shear deformation effect. Using the principle of virtual work done governing equations and boundary conditions of the theory are obtained. The results of displacements and stresses obtained from static flexure for various boundary condition of the beam are represented and compared with those of other refined theories and available in literature.**

Keywords: Hyperbolic Shear Deformation Theory, Static Flexure, General Solution of Beam.

I. INTRODUCTION

Many modern technologies require materials with unusual combinations of properties that cannot be met by the conventional metal alloys, ceramics, and polymeric materials alone. The composite materials is the solution to these problems which has various properties such as high strength/stiffness for lower weight, superior fatigue response characteristics, facility to vary the fiber orientation, material and stacking pattern, resistance to electrochemical corrosion and other superior material properties of composites. The wide spread use of shear flexible materials in aircraft, automotive, shipbuilding and other industries has stimulated interest in the accurate prediction of structural behavior of deep beams. The deep beam is basically a two dimensional problem of elasticity theory. The two dimensional theory can be derived by making suitable assumptions concerning the kinematics of deformation or state of stress through the thickness of beam.

The transverse shear deformation effect plays an important role in the structural analysis of shear flexible structures. The flexural analysis of thick beams led to the development of refined theories in order to address the correct structural behavior. Euler-Bernoulli theory of beam (ETB) bending is based on hypothesis that the plane section which is perpendicular to the neutral axis before bending remains plane and perpendicular to the neutral axis after bending. When elementary theory of beam (ETB) is used for the analysis thick beams, deflections are underestimated and natural frequencies and buckling loads are overestimated. This is the consequence of neglecting transverse shear deformations in ETB.

The classical beam theory (ETB) is based on Bernoulli-Euler hypothesis, but this theory is used for analysis of thin beams. As this theory is based on the assumption that the transverse normal to the neutral axis remains so during bending and after bending, implying that the transverse shear strain is zero. As this theory neglects the transverse shear deformation, it under estimates deflections and overestimates the natural frequencies in case of thick beam. As the analysis of thick composite and shear deformable beams is complicated by the two dimensional nature of stress and strain state. The use of elasticity theory is practically unfeasible due to mathematical difficulties and the complexity of shear flexible systems. This led to the development of refined shear deformation theories for beams which approximate the two dimensional solutions with reasonable accuracy. To overcome this drawback First Order Shear Deformation (FSDT) theory was developed by Timoshenko. It was based on the assumption that the normal to the mid-surface before deformation remain straight but not necessary normal to the mid-plane after deformation. In this theory the transverse shear deformation was assumed to be constant through the thickness and thus shear correction factor was required to take into account appropriate strain energy due to shear deformation.

Ghugal and Shimpi [1] showed various methods used for analysis of composite beam right from elementary theory of beam to first order shear deformation theory. Raman and Davalos [2] has used energy equivalence principle, to derive a general expression for the shear correction factor of laminated rectangular beams with arbitrary lay-up configurations. Sayyad [3] has focused on refined shear deformation theory which is developed for static flexural and free vibrati0nal analysis of thick isotropic beams, considering sinusoidal, hyperbolic and exponential functions in terms of thickness co-ordinate associated with transverse shear deformation effect. The results in the paper of displacements and stresses obtained from static flexure and results of free vibration frequencies for simply supported beam are presented and compared with those of other refined theories and exact solution from theory of elasticity. Thuc and Huu [4] have presented static behavior of composite beams with arbitrary lay-ups using various refined shear deformation theories. Bhimaraddi and Chandrashekhara [5] had considered the effect of shear deformation on the static response of beams of rectangular cross section using various distribution functions for shear strain.. Ghugal and Sharma [6] had used hyperbolic shear deformation theory for isotropic beam and by using the general solution they had given results for various boundary conditions. From all the literature available it can be stated that the higher order shear deformation theories with more than three unknown are more in demand. Also use of shear deformation theories using various displacement functions is not explored and there is need to evaluate such theories critically. Refined beam theory

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 6 Issue 11, November - 2017

for non-rectangular cross-section beams as well as beams subjected to load at top and bottom are rarely available. Ghugal and Waghe [7] had used trigonometric shear deformation theory for analysis of thick beams. The number of unknown in the theory is same as that of first order shear deformation theory

II. METHODOLOGY

A. Beam Under Consideration:

The beam under consideration occupies the region

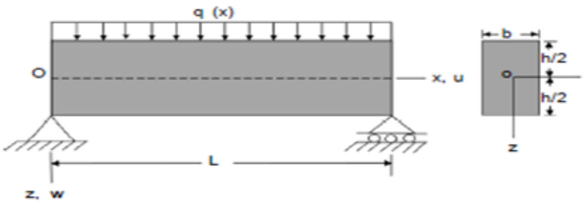

Fig. 1: Composite Beam Subjected To Uniformly Distributed Load

Where x, y, z are Cartesian coordinates, L is the length of beam, b is the width and h is the total depth of beam. The beam is subjected to transverse load of intensity q(x) per unit length of the beam.

B. Assumptions Made in Theoretical Formulation:

1. The axial displacement consists of two parts:

(a) Displacement given by elementary theory of beam bending.

(b) Displacement due to shear deformation, which is assumed to be hyperbolic in nature with respect to thickness coordinate, such that maximum shear stress occurs at neutral axis as predicted by the elementary theory of bending of beam.

2. The axial displacement u is such that the resultant of axial stress , acting over the cross-section should result in only bending moment and should not in force in x direction.
3. The transverse displacement is assumed to be a function of longitudinal (length) co-ordinate 'x' direction.
4. The body forces are ignored in the analysis. (The body forces can be effectively taken into account by adding them to the external forces.)
5. One dimensional constitutive law is used.
6. The beam is subjected to mechanical load.

C. The Displacement Field:

Based on the before mentioned assumptions, the displacement field of the present unified refined beam theory is given as below:

$$u(x,z) = u_0 - z\frac{dw_0}{dx} + f(z)\phi_x \qquad (1)$$

$$w = w_0(x) \qquad (2)$$

Here u and w are the axial and transverse displacements of the beam centre line. The functions f(z) assigned according to the shearing stress distribution through the thickness of the beam are as follows

Present theory: $f(z) = \left[\frac{h}{\pi}sin\frac{\pi z}{h}\right]$

The normal and transverse shear strains are obtained from linear theory of elasticity.

$$\varepsilon_x = \frac{du}{dx} = \frac{du_0}{dx} - z\frac{d^2w_0}{dx^2} + f(z)\phi \qquad (3)$$

$$\gamma_{zx} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = \frac{d}{dz}f(z)\phi \qquad (4)$$

One dimensional law is used to obtained normal bending and transverse shear stresses.

$$\sigma_x^k = E^k\left[\frac{du_0}{dx} - z\frac{d^2w_0}{dx^2} + f(z)\left(\frac{d\phi}{dx}\right)\right] \qquad (5)$$

$$\tau_{zx}^k = G^k\frac{d}{dz}f(z)\phi \qquad (6)$$

D. Governing Equations:

Using the Eqns. (2) through (6) for strains, stresses and principle of virtual work, variationally consistent differential equations for the beam under consideration are obtained. The principle of virtual work when applied to the beam leads to:

$$b\int_{x=0}^{x=L}\int_{z=-h/2}^{z=+h/2}(\sigma_x\delta\varepsilon_x + \tau_{zx}\delta\gamma_{xz})dxdz - \int_{x=0}^{x=L}q(\delta w_b + \delta w_s)dx = 0 \qquad (7)$$

Where the symbol δ denotes the variational operator. Employing the Green's theorem in Eqn. (7) successively and collecting the coefficients of ∂w_b and ∂w_s the governing equations obtained are as follows:

$$D\frac{d^4w_0}{dx^4} - E\frac{d^3\phi}{dx^3} = 0 \qquad (8)$$

$$E\frac{d^3w_0}{dx^3} - F\frac{d^2\phi}{dx^2} + H\phi = 0 \qquad (9)$$

$$C = E_1\int_{-h/2}^{h/2} f(z)dz; D = E_1\int_{-h/2}^{h/2} z^2dz;$$

$$E = E_1\int_{-h/2}^{h/2} zf(z)dz; F = E_1\int_{-h/2}^{h/2}[f(z)]^2dz;$$

$$H = G\int_{-h/2}^{h/2}[f'(z)]^2dz$$

E. General Solution Scheme for Analysis of Composite Beam:

The general solution for transverse displacement w_0 and warping function ϕ is obtained using equations 8 and 9by solution of linear differential equations.

$$\frac{d^3w_0}{dx^3} = \frac{E}{D}\frac{d^2\phi}{dx^2} + \frac{Q_{(x)}}{D} \qquad (10)$$

Where $Q_{(x)}$ is the generalized shear force for beam and it is given by $Q_{(x)} = \int_0^x qdx + C_1$.

Now the equation number 9 is rearranged in the following form

$$\frac{d^3 w_0}{dx^3} = \frac{F}{E}\frac{d^2\phi}{dx^2} - \beta\phi \qquad (11)$$

A single equation in terms of ϕ is obtained by using equation 10 and 11 as:

$$\frac{d^2\phi}{dx^2} - \lambda^2\phi = \frac{Q_{(x)}}{\alpha D_0} \qquad (12)$$

Where $\alpha = \frac{F_0}{E_0} - \frac{E_0}{D_0}, \beta = \frac{H_0}{E_0} and \lambda^2 = \frac{\beta}{\alpha}$

The general solution of equation 12 is given by

$$\phi(x) = C_2\cosh\lambda x + C_3\sin\lambda x - \frac{Q_{(x)}}{\beta D_0} \qquad (13)$$

Transverse displacement w(x) can be obtained by substituting the value of $\phi(x)$ in equation 11

$$w(x) = \iiint\int q\,dxdxdxdx$$
$$+\frac{C_1x^3}{6} + \frac{A_0}{\lambda D_0}\left[C_2\cosh\lambda x + C_3\sin\lambda x\right] + \frac{C_4x^2}{2} + C_5x + C_6 \quad (14)$$

Where C_1-C_6 are the arbitrary constants of integration and can be obtained by imposing natural (forced) and kinematic (geometric) boundary conditions of beam.

F. Illustrative Examples:

As shown in figure a simply supported beam uniform beam of rectangular cross-section occupying the region given by figure1 is considered for detailed numerical study.

Example: 1

A simply supported beam with rectangular cross-section (b × h) is subjected to uniformly distributed load (UDL) q over the span L at surface z = −h/2 acting in the downward z direction. The origin of beam is taken at left end support, i.e. at x = 0. The boundary conditions associated with simply supported beam are as follow.

$$\frac{d^3w}{dx^3} = \frac{d^2\phi}{dx^2} = \frac{dw}{dx} = \phi = 0 \qquad at\ \ x = L/2 \qquad (15)$$

$$\frac{d^2w}{dx^2} = \frac{d\phi}{dx} = w = 0 \qquad at\ \ x = 0 \qquad (16)$$

The boundary condition, $\phi = 0$ at x = L/2 is used from the condition of symmetry of deformation, in which the middle cross-section of the beam must remain plane without warping [Gere and Timoshenko (1986)]. From the general solution of beam, expressions for ϕ and w are obtained as follows:

$$\phi = \frac{qL}{2\beta D_0}\left[\left(1 - \frac{2x}{L}\right) - \frac{\sinh\lambda(L/2 - x)}{(\lambda L/2)\cosh(\lambda L/2)}\right] \qquad (17)$$

$$w = \frac{qL^4}{24D_0}\left[\left(\frac{x}{L}\right)^4 - 2\left(\frac{x}{L}\right)^3 + \left(\frac{x}{L}\right)\right]$$
$$+\frac{qL^2E^2}{2D^2{}_0H}\left[\begin{array}{l}\frac{x}{L}\left(1 - \frac{x}{L}\right) \\ -\frac{2}{(\lambda L)^2}\left(1 - \frac{\cosh\lambda(L/2 - x)}{\cosh(\lambda L/2)}\right)\end{array}\right] \qquad (18)$$

Example: 2

A fixed supported beam with rectangular cross-section (b × h) is subjected to uniformly distributed load (UDL) q over the span L at surface z = −h/2 acting in the downward z direction. The origin of beam is taken at left end support, i.e. at x = 0. The boundary conditions associated with fixed supported beam are as follow.

$$\frac{d^3w}{dx^3} = \frac{d^2\phi}{dx^2} = \frac{dw}{dx} = 0 \qquad at\ \ x = L/2 \qquad (19)$$

$$w = \phi = \frac{dw}{dx} = 0 \qquad at\ \ x = 0, L \qquad (20)$$

$$\frac{d^2w}{d^2x} = \frac{d\phi}{dx} = \frac{qL^2}{12} \qquad at\ \ x = 0 \qquad (21)$$

Using the boundary conditions above equations for w(x) and $\phi(x)$ can be obtained as follow

$$\phi = \frac{qL}{2\beta D_0}\left[\frac{\sinh\lambda(x - L/2)}{\sinh(\lambda L/2)} + 2\frac{x}{L} - 1\right] \qquad (22)$$

$$w = \frac{qL^4}{24D_0}\left\{\begin{array}{l}\left(\frac{12}{L^2}\left[\frac{F}{H} - \frac{1}{\lambda^2}\right]\right)\left[\frac{\cosh\lambda L/2 - \cosh\lambda(L/2 - x)}{\lambda L\sinh(\lambda L/2)}\right] + \\ \left[-\left(\frac{x}{L}\right)^4 + 2\left(\frac{x}{L}\right)^3 - \left(\frac{x}{L}\right)^2\right] + \\ \left[\frac{F}{H} - \frac{1}{\lambda^2}\right]\left[\frac{6x^2\lambda}{L^3}\frac{\cosh(\lambda L/2)}{\sinh(\lambda L/2)} - \frac{12x}{L^3}\right]\end{array}\right\} \qquad (23)$$

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ISSN: 2278-0181
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Example 3:

A cantilever beam with rectangular cross-section (b × h) is subjected to uniformly distributed load (UDL) q over the span L at surface z = −h/2 acting in the downward z direction. The origin of beam is taken at left end support, i.e. at x = 0. The boundary conditions associated with cantilever beam are as follow.

$$\frac{d^3w}{dx^3} = \frac{d^2\phi}{dx^2} = \frac{dw}{dx} = \frac{d\phi}{dx} = 0 \qquad at \;\; x = L \qquad (24)$$

$$\frac{dw}{dx} = \phi = w = 0 \qquad at \;\; x = 0 \qquad (25)$$

Using the boundary conditions above equations for w(x) and $\phi(x)$ can be obtained as follow

$$\phi = \frac{qL}{\beta D_0}\left[\begin{array}{l} -\cosh\lambda Lx + \dfrac{\sinh\lambda x}{(\lambda L)\cosh(\lambda L/2)} \\ \sinh\lambda x - \dfrac{x}{L} + 1 \end{array}\right] \qquad (26)$$

$$w = \frac{qL^4}{24D_0}\left[\left(\frac{x}{L}\right)^4 - 4\left(\frac{x}{L}\right)^3 - 6\left(\frac{x}{L}\right)\right] + \frac{qL^2E^2}{HD_0^2}\left[\begin{array}{l} \dfrac{x}{L}\left(1-\dfrac{x}{2L}\right) \\ +\left(\dfrac{\cosh\lambda x - \sinh\lambda x - 1}{\lambda L}\right) + \left(\dfrac{\cosh\lambda x - 1}{\lambda^2 L^2\cosh(\lambda L)}\right) \end{array}\right] \qquad (27)$$

III. RESULT AND DISCUSSIONS

The result for of transverse displacement ($\bar{w}$), axial stress σ_x and transverse shear stress τ_{zx} for composite beam subjected uniform distributed load are presented in non-dimentional form for purpose of presenting the results in this paper.

$$\bar{w} = \frac{10Ebh^3w}{ql^4}, \quad \bar{\sigma}_x = \frac{\sigma_x b}{q}, \quad \tau_{zx} = \frac{b\tau_{zx}}{q}$$

1. *Simply Supported Beam:*

Table 1: Comparison of transverse displacement $(\bar{w})$ axial stress (σ_x) and transverse shear stress (τ_{zx}) for simply supported beam subjected to uniformly distributed load.

A.R	Theory	Model	$\bar{w}$	$\bar{\sigma}_x$	τ_{xz}^{CR}
10	Present	HSBT	1.6016	76.0503	7.6580
	Ghughal and Sharma[6]	HPSDT	1.6020	75.2580	7.5600
	Timoshenko	FSDT	1.5950	75.0000	4.999
	Euler and Bernoulli	EBT	1.5630	75.0000	----

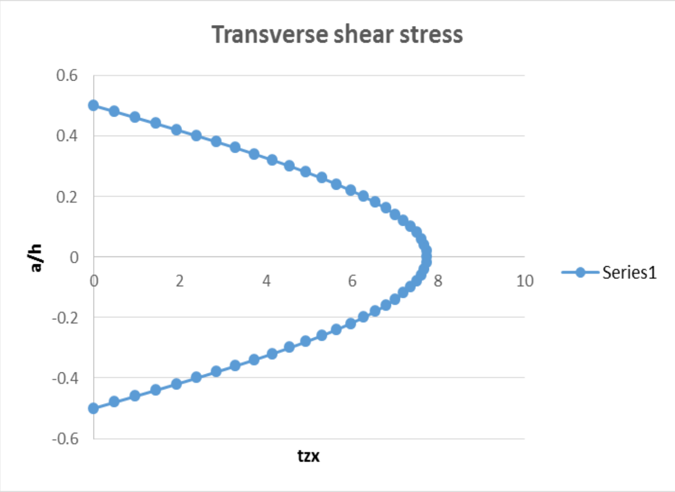

Fig. 2: Variation Of Transverse Shear Stress Through The Thickness Of Simply Supported Beam Subjected To Uniformly Distributed Load And Obtained Using Constritutive Relation For Aspect Ratio 10.

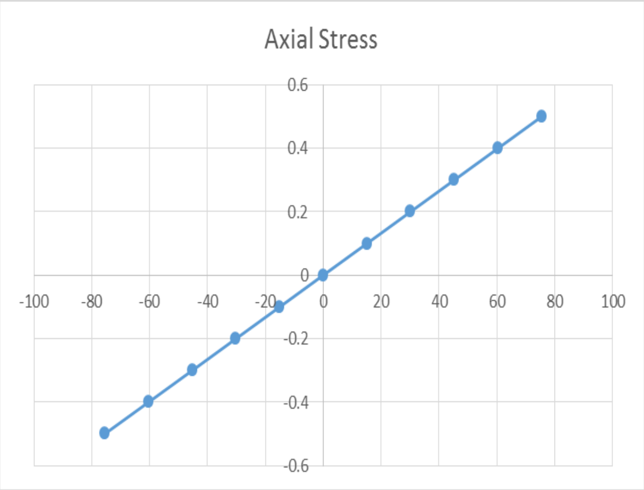

Fig. 3: Variation Of Axial Stress Through The Thickness Of Simply Supported Beam Subjected To Uniformly Distributed Load For Aspect Ratio 10.

2. *Fixed Supported Beam:*

Table 2: Comparison of transverse displacement $(\bar{w})$ axial stress (σ_x) and transverse shear stress (τ_{zx}) for fixed supported beam subjected to uniformly distributed load.

A.R	Theory	Model	$\bar{w}$	$\bar{\sigma}_x$	τ_{xz}^{CR}
4	Present	HSBT	0.5845	15.1321	6.0763
	Ghughal and Sharma[6]	HPSDT	0.5412	16.7540	6.3503
	Timoshenko	FSDT	0.5565	8.0000	3.0000
	Euler and Bernoulli	EBT	0.3125	8.0000	----

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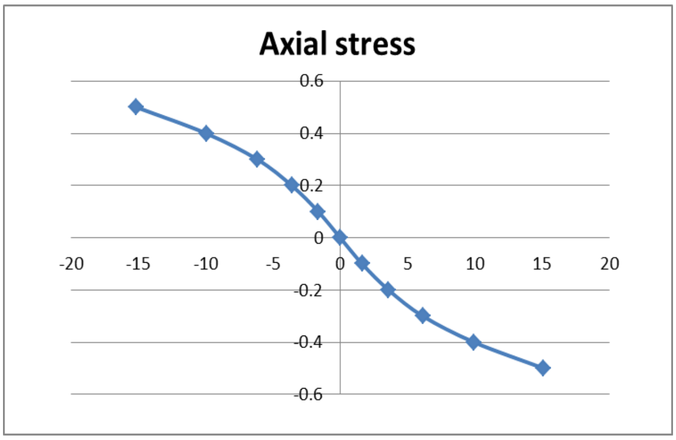

Fig. 4: Variation Of Axial Stress Through The Thickness Of Fixed Supported Beam Subjected To Uniformly Distributed Load For Aspect Ratio 4.

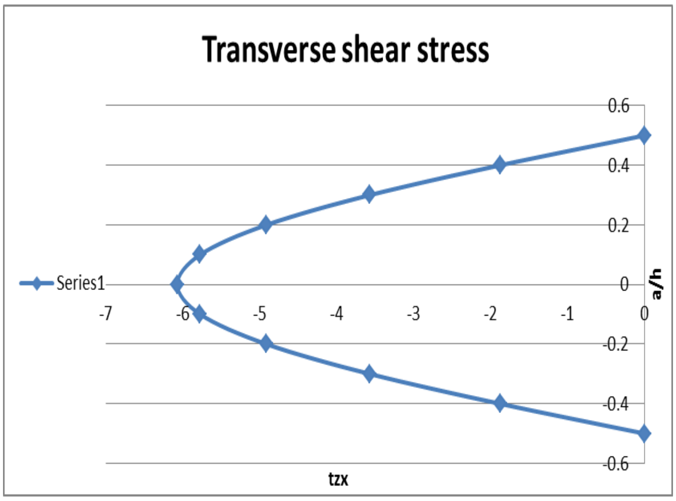

Fig. 5: Variation of Transverse Shear Stress Through The Thickness Of Fixed Supported Beam Subjected To Uniformly Distributed Load For Aspect Ratio 4.

3. *Cantilever Supported Beam:*

Table 3: Comparison of transverse displacement $(\bar{w})$ axial stress (σ_x) and transverse shear stress (τ_{zx}) for cantilever supported beam subjected to uniformly distributed load.

A.R	Theory	Model	$\bar{w}$	$\bar{\sigma}_x$	τ_{xz}^{CR}
4	Present	HSBT	1.5840	65.58	0.7748
	Ghughal and Sharma[6]	HPSDT	1.5944	65.63	
	Timoshenko	FSDT	1.5975	48.00	
	Euler and Bernoulli	EBT	1.5000	48.00	----

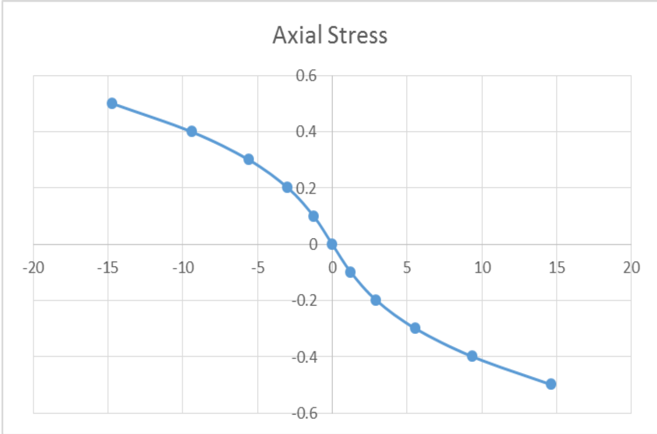

Fig. 7: Variation Of Axial Stress Through The Thickness Of Cantilever Supported Beam Subjected To Uniformly Distributed Load For Aspect Ratio 4.

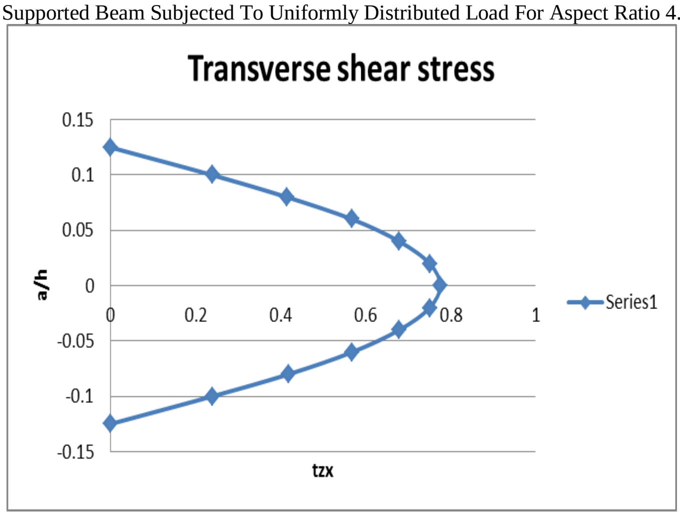

Fig. 8: Variation Of Transverse Shear Stress Through The Thickness of Cantilever Supported Beam Subjected To Uniformly Distributed Load For Aspect Ratio 4.

IV. CONCLUSIONS

From the static flexural analysis of simply composite beams following conclusions are drawn.

1. Transverse deflection predicted by the present theory validates with solutions of the above mentioned theories of EBT, Ghugal & Timoshenko's.
2. Transverse shear stress predicted by the present theory shows excellent results and matches with the exact values.
3. The present theory evaluates the results in such a way that they are consistent variationally.
4. The effect of shear and bending in the present theory is determined and evaluated in an effective manner.

V. SCOPE OF FUTURE WORK

The present beam theory has good scope for future research work. Some of the research areas where this theory can be extended are as follows:

1. Dynamic analysis of shells, plates and beams can be carried out.
2. Non-linear analysis of shells, plates and beams can be carried out.
3. This theory can also be used for analysis of composite shells, plates and beams.

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