

Fixed Total Particle Loading in Hybrid Nanofluids: Composition Sensitivity and Effective Property Identifiability

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Abstract: Comparing hybrid nanofluids at unequal total solid loading can attribute a loading effect to composition. This analytical study fixes total volume fraction and separates the composition share from the total amount of particles. Mixture rules give exact density and volumetric heat-capacity sensitivities. A Brinkman viscosity closure predicts no composition sensitivity at fixed loading, while a clearly identified illustrative conductivity surrogate has a monotone, explicitly bounded response. The conductivity response scales with total loading in the dilute limit. A dimensionless worked example tabulates the closure outputs and demonstrates their arithmetic without representing measured Al_2O_3 –Cu/water data. We distinguish the structural identifiability of composition from one direct conductivity observation under known constituent properties from its non-identifiability when constituent properties or transport coefficients are free. The result is a reproducible diagnostic for interpreting parametric nanofluid calculations; quantitative predictions for any material pair still require measured effective properties and independently checked closure assumptions.

Keywords: hybrid nanofluid; composition share; fixed total loading; effective conductivity; identifiability; mixture closure

1. INTRODUCTION:

Hybrid suspensions introduce at least two distinct controls: total solid volume fraction and the partition of that total between the particle species. Suresh et al. reported preparation and thermophysical properties of Al_2O_3 –Cu/water suspensions [1], followed by an experimental heat-transfer investigation [2]. Composition and stability have also been measured at differing Cu– Al_2O_3 mixing ratios [3]; an independent Al_2O_3 –CuO mixture-ratio study measured conductivity and viscosity over temperature [4]. These experiments motivate explicit control of loading, composition, temperature, and stability, but their material systems cannot be treated as interchangeable. A computation that varies loading and composition together cannot isolate a composition effect. This paper develops a fixed-loading comparison and states precisely what follows from chosen effective-property rules. Classical mixture and

conductivity models are useful baselines [5, 6], but a model output must not be presented as a measurement.

The analysis is deliberately restricted to algebraic closures. It does not solve a boundary-layer problem or claim heat-transfer enhancement for a physical suspension. The contribution is an exact sensitivity and identifiability audit that can be applied before a numerical flow calculation.

2. DEFINITIONS AND PROPERTY MODEL:

Let subscripts f, 1 and 2 denote base liquid and the two particle species. Let

$0 < \phi < 1$ be fixed and let $0 \leq r \leq 1$ denote the share assigned to species 1:

$$\phi = \phi_1 + \phi_2, \quad r = \frac{\phi_1}{\phi}, \quad \phi_1 = r\phi, \quad \phi_2 = (1 - r)\phi. \dots (1)$$

At fixed ϕ , ordinary volume averaging gives density and volumetric heat capacity $C = (\rho c_p)$:

$$\rho_e = (1 - \phi)\rho_f + \phi\{r\rho_1 + (1 - r)\rho_2\}, \dots (2)$$

$$C_e = (1 - \phi)C_f + \phi\{rC_1 + (1 - r)C_2\}. \dots (3)$$

Mass-specific heat is $c_{p,e} = C_e/\rho_e$; using a direct volume average of the mass-specific c_p would be a different and generally inconsistent rule. For viscosity, adopt the Brinkman baseline [5]:

$$\mu_e = \mu_f(1 - \phi)^{-5/2}. \dots (4)$$

Equation (4) is an assumption about this closure, not a universal rheological law for hybrids. It contains no r . For a transparent conductivity illustration only, first define $k_s(r) = rk_1 + (1 - r)k_2$ and then apply the spherical-inclusion Maxwell form:

$$F(r) := \frac{k_e}{k_f} = \frac{k_s + 2k_f + 2\phi(k_s - k_f)}{k_s + 2k_f - \phi(k_s - k_f)}. \dots (5)$$

The intermediate k_s is an illustrative surrogate. It does not represent a validated two-species microstructure, interfacial resistance, agglomeration, shape distribution, or percolation. Hamilton and Crosser [6] show why inclusion geometry matters even in simpler effective-conductivity descriptions. Experiments on Cu-in-ethylene-glycol nanofluids [7], proposed heat-flow mechanisms [8], interfacial-resistance theory [9], and aggregation kinetics [10] each identify effects absent from (5). These studies concern different systems and are cited to delimit the closure, not to validate its numerical values.

3. EXACT SENSITIVITIES AND BOUNDS:

Differentiation of (2)–(4), holding ϕ and constituents fixed, yields

$$\frac{d\rho_e}{dr} = \phi(\rho_1 - \rho_2), \quad \frac{dC_e}{dr} = \phi(C_1 - C_2), \quad \frac{d\mu_e}{dr} = 0. \dots\dots\dots (6)$$

The mass-specific heat sensitivity requires the quotient rule:

$$\frac{dc_{p,e}}{dr} = \frac{\phi\{(C_1 - C_2)\rho_e - C_e(\rho_1 - \rho_2)\}}{\rho_e^2}. \dots\dots\dots (7)$$

Write $D = (1 - \phi)k_s + (2 + \phi)k_f$, the strictly positive denominator for positive conductivities and $0 < \phi < 1$. Direct differentiation of (5) gives

$$\frac{dF}{dr} = \frac{9\phi k_f(k_1 - k_2)}{D^2}. \dots\dots\dots (8)$$

Thus F is strictly increasing if $k_1 > k_2$, decreasing if $k_1 < k_2$, and independent of composition if $k_1 = k_2$. Its endpoint bound is $\min\{F(0), F(1)\} \leq F(r) \leq \max\{F(0), F(1)\}$. At small ϕ , (8) is $O(\phi)$: composition sensitivity necessarily vanishes as particle loading tends to zero under this closure. Likewise the density and heat-capacity differences are $O(\phi)$.

For $k_1 \neq k_2$ and known k_f, k_1, k_2, ϕ , an exact direct measurement of F in the model range gives one unique r . To make this explicit, let $A = F - 1$ and $B = F + 2$. Rearrangement of (5) gives

$$k_s = k_f \frac{\phi B + 2A}{\phi B - A}, \quad r = \frac{k_s - k_2}{k_1 - k_2}, \dots\dots\dots (9)$$

provided the denominator is nonzero and the recovered r lies in $[0, 1]$. Near $\phi = 0$ or $k_1 = k_2$, inversion becomes ill-conditioned or impossible. If k_1 and k_2 are unknown, one conductivity measurement does not separately determine them and r . A single Nusselt number additionally depends on the flow, thermal boundary conditions, and material properties, so it cannot generally identify composition without independent constraints.

3.1 Two-parameter identifiability and measurement scale

Suppose the total loading is also unknown, but normalized density $R = \rho_e/\rho_f$ and conductivity $F = k_e/k_f$ are both observed. Put $h(r) = r\rho_1/\rho_f + (1 - r)\rho_2/\rho_f$, so $R = 1 + \phi[h(r) - 1]$. The second partial derivative of (5), in addition to (8), is

$$\frac{\partial F}{\partial \phi} = \frac{3(k_s - k_f)(k_s + 2k_f)}{D^2}, \quad D = (1 - \phi)k_s + (2 + \phi)k_f. \dots\dots\dots (10)$$

For the map $(r, \phi) \mapsto (R, F)$, its local Jacobian determinant is

$$\det J = \phi \left(\frac{\rho_1 - \rho_2}{\rho_f} \right) \frac{\partial F}{\partial \phi} - [h(r) - 1] \frac{\partial F}{\partial r}. \dots\dots\dots (11)$$

A nonzero determinant is sufficient for local recovery of (r, ϕ) under the adopted closures and known constituent properties. It does not ensure practical precision: near a singular Jacobian, small measurement errors may produce large parameter changes. This calculation also explains why a single conductivity ratio, one scalar observation for two unknowns, cannot determine both r and ϕ .

4. REPRODUCIBLE DIMENSIONLESS EXAMPLE

The following ratios are selected for arithmetic illustration and are **not measured properties**: $k_1/k_f = 5$, $k_2/k_f = 2$, $\rho_1/\rho_f = 4$, $\rho_2/\rho_f = 3$, $C_1/C_f = 2$, $C_2/C_f = 1.5$, and $\phi = 0.03$. Values below follow directly from (2)–(5); $\mu_e/\mu_f = (0.97)^{-5/2}$ for every row.

Table 1a. Fixed-loading composition and conductivity inputs and outputs.

r	ϕ_1	ϕ_2	k_s/k_f	k_e/k_f
0.00	0.0000	0.0300	2.00	1.022670
0.25	0.0075	0.0225	2.75	1.033528
0.50	0.0150	0.0150	3.50	1.041475
0.75	0.0225	0.0075	4.25	1.047542
1.00	0.0300	0.0000	5.00	1.052326

Table 1b. Fixed-loading density, heat capacity and viscosity outputs.

r	ρ_e/ρ_f	C_e/C_f	μ_e/μ_f
0.00	1.0600	1.01500	1.07912
0.25	1.0675	1.01875	1.07912
0.50	1.0750	1.02250	1.07912
0.75	1.0825	1.02625	1.07912
1.00	1.0900	1.03000	1.07912

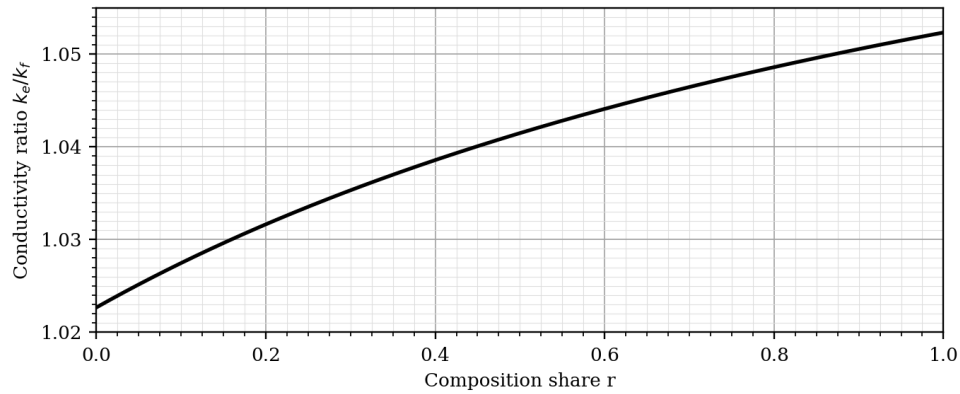


Figure 1. Illustrative conductivity closure at fixed total loading; the graph-paper grid is a display aid, not experimental data.

The example has a larger k_e/k_f at larger r because the stipulated k_1 exceeds k_2 . This arithmetic trend alone establishes neither a larger Nusselt number nor lower wall temperature in a flow problem. Those outcomes require a specified momentum/energy model and boundary conditions. The constant viscosity column is a useful check against a code that silently changes μ_e when r changes under (4).

4.1 Sensitivity to loading and inverse conditioning

At $r = 0.5$ the derivative $\partial F / \partial r$ and the two-observable Jacobian follow directly from (8), (10), and (11). The condition number below uses the ordinary Euclidean norm of unweighted dimensionless observables; it is a numerical diagnostic, not a measurement uncertainty or a confidence interval.

Table 2a. Conductivity ratio and composition sensitivity at $r = 0.5$.

ϕ	F	$\partial F / \partial r$
0.01	1.013699	0.009007
0.02	1.027523	0.018180
0.03	1.041475	0.027522
0.04	1.055556	0.037037
0.05	1.069767	0.046728

Table 2b. Loading sensitivity and inverse conditioning at $r = 0.5$.

ϕ	$\partial F / \partial \phi$	$\det J$	$\kappa_2(J)$
0.01	1.376118	-0.008757	930.0
0.02	1.388772	-0.017675	462.8
0.03	1.401601	-0.026758	307.1
0.04	1.414609	-0.036008	229.2
0.05	1.427799	-0.045430	182.5

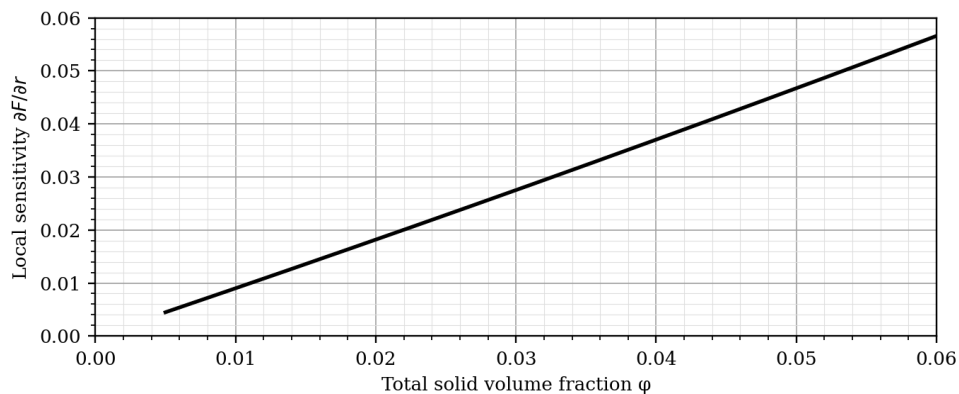


Figure 2. Exact local composition sensitivity at $r = 0.5$ under the illustrative closure.

For example, at $\phi = 0.03$, a local perturbation $\delta r = 0.02$ changes F by about 0.000550, while $\delta \phi = 0.002$ changes it by about 0.002803 when varied separately. These are first-order sensitivities; the actual error budget requires observed precision, correlations, and closure uncertainty. The unweighted condition number improves as ϕ rises over this specified illustrative range, but this trend should not be generalized to arbitrary materials or measurement designs.

5. PRACTICAL INTERPRETATION AND LIMITATIONS

To compare two compositions fairly, record ϕ , r , the constituent data, every property closure, and the imposed thermal and mechanical boundary conditions. First compare algebraic material coefficients using (6)–(8); only then attribute differences in a boundary-layer solution. An uncertainty in r propagates approximately as $\delta F \approx (dF/dr)\delta r$ locally. Uncertainty in constituent properties and model discrepancy must be considered separately; the derivative is not a confidence interval.

The Maxwell-style surrogate neglects interactions between inclusion species. Brinkman viscosity ignores composition-dependent aggregation or rheology; observed composition–stability trade-offs [3] and measured mixing-ratio effects [4] underscore this limitation. Density and heat-capacity averaging presume volume additivity. The formulas are therefore a falsifiable baseline, not a fitted description of Al_2O_3 –Cu/water. Independently measured effective conductivity, viscosity, density, heat capacity, and particle loading are needed before quantitative material claims. Particle-migration transport would require an additional model such as Buongiorno’s [11]; the early nanofluid proposal [12] does not substitute for such calibration.

6. CONCLUSIONS

At fixed total loading, composition enters density and volumetric heat capacity linearly, does not enter the chosen Brinkman viscosity, and produces a strictly monotone conductivity response in the explicitly stated surrogate when constituent conductivities differ. All algebraic composition sensitivities vanish as $\phi \rightarrow 0$. Direct conductivity inversion is conditional on known constituent properties and the correctness of the closure. These constraints provide clear checks for subsequent hybrid nanofluid flow studies.

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