Finite Element Analysis of the Dish Multi-Point Forming Process

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Abstract - This study focuses on simulation of multi-point die to produce the dish uses Bezier function for surface modeling to get the best modeling points for dish. The mathematical functions required to generate curves and surfaces are investigated together with complete algorithms using MatLap program for their achievement. Finite element method software (ANSYS 15.0) was used to simulate the dish. A multi-point die was built with the same dimensions suggested by Wang and Li [1], quarter of the die in three dimensional model is used due to symmetry of the required product. Copper plate has been chosen with 5KN blank holder force, three rubber’s thickness (elastic cushion) are used (0.5,0.7,2)mm to avoid dimples also three radii of punch group (3,4,6)mm are investigated. It is concluded that the punch element with radius 4mm and elastic cushion (rubber) 2mm is the best design to avoid the dimples defect and have the homogenously distributed stress.

Keywords: Multi-point forming, Bezier surface, Dish, MATLAB program, control points, Finite Element Method, ANSYS, Rubber, Stress analysis.

1. INTRODUCTION

Multi-point forming (MPF) is a flexible forming technology in which the fixed shape of conventional dies is replaced via movable elements call “punch group”. Any complex design need to represent its lines, curves and surface mathematically, the accurate design of any complex part depending on the curves & surfaces and its degree of the curve equation. The most used surfaces to describe the complex parts are the third degree surfaces because of their high accuracy, but the disadvantage of using this degree of surfaces is its limitation to represent the desired parts [2],[3]. The curves that caused using Beziers method are called Bezier curves [4]. The Bézier curve, referred to the French researcher Pierre Bézier. The Bézier surface is the extended of the Bezier curve. It is consist of dragging Bezier curved via spaced to generate the [5]. Analyzed the deformation mechanism of the multi-point dieless forming and positioning technology with commercial FEM codes [6]. Bezier surface was used to the surface of Dish in (3D) model by using MATLAB software to determine the coordinate of all points along curve and locate them on the surface[7],[8]. Qian et al.,[9] (2007) studied the analysis of multi-point forming for dish head and concluded that the forming quality is improved by MPF, Wardhani et al., [10] (2014) studied the numerical simulation of multipoint forming used two model the first one was pins of die MPF arranged in hexagonal packing and the other one was squared and found that the hexagonal packing is more efficient. Zhong. et al.,[11] described approach the incremental displacement on the basis of Lagrangian formulation and elastic-plastic material model for finite element method of MPF process. analyzed MPF by using a new finite element code. The results of numerical shown good agreement with those of the experiments and simulation.

In this study MATLAB software program, is used to find the coordinate of all dish points that are situated on the Bezier curve and surface to create the forming dish surface by adjusting the height of the every punch by ANSYS according to resulted points coordinates of the MatLap software and different parameters of punch group and rubber thickness are used to find the optimum case.

2. MATHEMATICAL REPRESENTATION OF BEZIER SURFACES

The Bezier surface is a direct extension of a Bezier curve. The simple extension for three dimensional free–form curve is by incorporating another parameter (s) to the vector equation of the curve to obtain the surface equation:

\[ P(t,s) = [x(t,s), y(t,s), z(t,s)] \]  \( ... (1) \)

where: \( 0 \leq (t,s) \leq 1 \) and they are independent variables. Bezier surface is an extended of a Bezier curve[12]:

\[ P(t,s) = \sum_{i=0}^{n} \sum_{j=0}^{m} b_{ij} B_{i,n}(t) B_{j,m}(s) \]  \( ... ... (2) \)

\[ 0 \leq (t,s) \leq 1 ; \quad ij = 0, ..., n,m. \]

The \( b_{ij} \) rectangular array of control points, \( B_{i,n}(t) \) and \( B_{j,m}(s) \) are the basis functions, defined in the same way as for "Bezier curves". The general equation of a Bezier surface is [13]:

\[ P(t,s) = T_{1*n} M_{B,N*n} b_{n*m} M_{B,m*1} S_{B,m+1} \]  \( ... (3) \)

where the size of the matrices depend on the dimensions of the control point array defined by a (4x4) array of control points. [2].

\[ P(t,s) = T_{1*4} M_{B,A*4} b_{4*x} M_{B,A*4} S_{B,A+1} \]  \( (4) \)
That the subscripts of the matrices represent their dimensions. Expanding equation (3) to get:

\[
P(t, s) = \begin{bmatrix} (1 - t)^3 & 3t(1 - t)^2 & 3t^2(1 - t) & t^3 \end{bmatrix} \cdot \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{bmatrix} \cdot \begin{bmatrix} (1 - s)^3 \\ 3s(1 - s)^2 \\ 3s^2(1 - s) \\ s^3 \end{bmatrix}
\]

\[\text{……… (5)}\]

\[
P(t, s) = [t^3 \ t^2 \ t \ 1] \cdot \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{bmatrix} \cdot \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} s^3 \\ s^2 \\ s \\ 1 \end{bmatrix}
\]

\[\text{……… (6)}\]

3. SURFACE MODEL IN THE SIMULATION PROCESS

The mathematical formulation of Bezier techniques have been achieved and programmed with MATLAB program to create the curves and surfaces depending on control points. only one quarter of (3D) model have been established to generate the dish as shown in Fig.(1) [14].

![Fig.(1) control points of the dish](image1)

The control points of the surface represented the Bezier surface with 4x4 control points are:

- \(b_{11}(0,100,0)\)
- \(b_{12}(50,86.6,0)\)
- \(b_{13}(86.5,0,0)\)
- \(b_{14}(100,0,0)\)
- \(b_{21}(0.89,-45)\)
- \(b_{22}(44.77,-45)\)
- \(b_{23}(89.0,-45)\)
- \(b_{24}(89.0,-45)\)
- \(b_{31}(0.58,-80)\)
- \(b_{32}(29.50,-80)\)
- \(b_{33}(50.29,-80)\)
- \(b_{34}(58.0,-80)\)
- \(b_{41}(0.15,-98)\)
- \(b_{42}(7.13,-98)\)
- \(b_{43}(13.7,-98)\)
- \(b_{44}(15.0,-98)\)

![Fig.(2) shown the Bezier surface for the quarter dish using Mat Lap software.](image2)

3.1 MATLAB Program to Generate Dish

The proposed algorithm was coded in MATLAB Ver. (6.5) . Several bi-cubic Bezier patches were designed and tested to generate the dish .

```matlab
clear;
clf;
u=0:0.04:1;
for i=1:25
```

![Fig.(2) Representation the quarter composite Bezier Surface for dish](image3)
"U(1,1)=u(1)^3;"
"U(1,2)=u(1)^2;"
"U(1,3)=u(1);"
"U(1,4)=1;"
"end"
"w=0:0.04:1;"
"for i=1:25
  "W(i,1)=w(i)^3;
  "W(i,2)=w(i)^2;
  "W(i,3)=w(i);"
  "W(i,4)=1;"
"end"
M=[-1 3 -3 1; 3 -6 3 0; -3 3 0 0; 1 0 0 0;];
CVX=[0,50,86.6025,100;0,44.5503,77.1634,89.1007;0,29.3893,50.9037,58.7785;0,7.8217,13.5476,15.6434;];
CVY=[100,86.6025,50,0;89.1007,77.1634,44.5503,0;58.7785,50.9037,29.3893,0;15.6434,13.5476,7.8217,0;];
CVZ=[0,0,0,0;-13.6197,-13.6197,-13.6197,-13.6197;-24.27051,-24.27051,-24.27051,-24.27051;-29.63064,-29.63064,-29.63064,-29.63064;];
for i=1:25
  for j=1:25
    "PX(i,j)=U(i,:)*M*CVX*M'*W(j,:);"
    "PY(i,j)=U(i,:)*M*CVY*M'*W(j,:);"
    "PZ(i,j)=U(i,:)*M*CVZ*M'*W(j,:);"
  "end"
"end"
surf(PX,PY,PZ);grid on;
x=PX
y=PY
z=PZ

4. FINITE ELEMENT ANALYSIS (FEA)

The finite element Analysis is the numerical method too for predicting the stress-strain etc... of an engineering problem. Later, it was extended to predicting the behavior of complex structure such as to obtain the pressure field and velocity of fluid flow as well as the temperature. The Exact solution of the complex problem had complicated shapes of loadings are difficult and hard to derive and may be impossible to obtain. So FEM is used to do that.

5. NUMERICAL SIMULATION

Because of symmetry, only one half of the real sector is modeled and the following assumptions are used in the analysis presented in this paper:

1. During the process, the plate temperature remains constant.
2. Thermal expansion is neglected.
3. The strain rate has no effect on material stress – strain relation.
4. The material has a bilinear total stress - total strain curve.
5. Pins, Dies and Blank holder are rigid bodies.
6. The friction coefficients are constant

The following steps the general procedure to simulate the problem in "ANSYS code":

5.1 SELECTION OF THE ELEMENT TYPE

1. Forming process by using Multi-point die, the finite element model is consist of five parts: upper discrete pin punches, lower discrete pin punches, plate(blank), blank holder and die. For (3D model). Pins, die and the blank holder are represented as a rigid body and the plate is meshed with (Solid element 185). The contact interface between the die and the blank is represented by (contact element 172 and target 170) [16].
2. Forming process by using Multi-point die (with using elastic cushion), the finite element model is consist of seven parts: upper discrete punches, lower discrete punches, plate, upper (elastic cushion) layer, lower (elastic cushion) layer, blank holder and die. Pins, die and the blank holder are represented as a rigid and the elastic cushion material is modeled with (Hyperelastic Solid element 185). The contact interface between the die and the deformed material is represented by (contact element 172 and target 170). 

5.2 Material properties

Copper is the proposed material for analysis, the mechanical properties of materials as shown in table (1).
### Table (1) Mechanical properties of the copper metal

<table>
<thead>
<tr>
<th>Property</th>
<th>Copper (Cu)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulus of elastic (E)</td>
<td>124 Gpa</td>
</tr>
<tr>
<td>Tangent modulus (E_T)</td>
<td>0.8 Gpa</td>
</tr>
<tr>
<td>Yield stress (σ_y)</td>
<td>45 Mpa</td>
</tr>
<tr>
<td>Poisson’s ratio (υ)</td>
<td>0.34</td>
</tr>
</tbody>
</table>

#### 5.3 Hyperelastic Material Model

In 1951, Rivlin and Sunders developed a hyperelastic material model for large deformations of rubber. This material model is assumed to be incompressible and initially isotropic.

The form of strain energy potential for a Mooney-Rivlin material is given as [17]:

\[
W = c_{10}(I_1 - 3) + c_{01}(I_2 - 3) + 1/d(J - 1)^2 \quad \text{(7)}
\]

Where \( c_{10}, c_{01} \) and \( d \) are material constants.

Mooney-Rivlin (2 parameters) is used to represent the elastic cushion. The hyperelastic material model to describe the behavior of the elastic cushion. In the following analyses, the friction coefficient is assumed as \( 0.1 \).

<table>
<thead>
<tr>
<th>Property</th>
<th>Elastic cushion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulus of elastic (E)</td>
<td>2.87 Mpa</td>
</tr>
<tr>
<td>Poisson’s ratio (υ)</td>
<td>0.499</td>
</tr>
<tr>
<td>Mooney-Rivlin Constants</td>
<td></td>
</tr>
<tr>
<td>( c_{10} )</td>
<td>0.293 Mpa</td>
</tr>
<tr>
<td>( c_{01} )</td>
<td>0.177 Mpa</td>
</tr>
</tbody>
</table>

#### 6. Cases Studies

Three radii of pin punches are used to observed the optimum radius of pin punch used in the MPF process.

A- Radius of pin punch = 4mm

The effect of radius punch is various by various thickness of cushion on dimple suppression, are the using without rubber, (0.5mm) rubber thickness, (0.7mm) rubber thickness and (2mm) rubber thickness elastic cushion.

1-Without Rubber

Firstly no rubber as cushion is used, the pin punches contact directly with blank that caused wrinkling and dimpling which are the most defects in multi-point forming (MPF) of sheet metal. Those phenomenon affect the shape quality. Dimple happens due to discontinuous contact between the sheet and pin punches. As shown in Figure (1).
2- Blank cover from top and bottom by (0.5mm) thickness Rubber.

When using the rubber with thickness 0.5 mm as a cover to the forming area, the result illustrated the appearance of the smallest dimple. The pin punch causes the concentrated force on the small contact area between the metal plate, therefore the plastic deformation are caused, that called "dimple" as shown in Figure 2.
3- Blank cover from top and bottom by 0.7mm thickness Rubber.
Used rubber with thickness 0.7 mm upper and lower plate and the result showed the wrinkling are least as before, as shown in figure (3)

4- Blank cover from top and bottom by 2mm thickness Rubber.
When used the thickness of rubber 2mm upper and lower plate the result illustrated that the surface of the dish is free wrinkling and high surface quality. Figure (4) observed the results of 2mm thickness of rubber.
In this case the radius of pin punch is taken 3mm, and the thickness of the rubber is 2mm, as a result of the previous case, which can be concluded that 2mm is the best rubber thickness. Figure(5) shown the results.

B- Radius of pin punch = 3mm

In this case the radius of pin punch is taken 3mm, and the thickness of the rubber is 2mm, as a result of the previous case, which can be concluded that 2mm is the best rubber thickness. Figure(5) shown the results.
Figure(5) the deflection, equivalent strain, equivalent stress of dish with 2mm rubber

C- Radius of pin punch = 6mm
The other case when the end of hemispherical end is (6mm), the changing is more stable, and these observed the dimples are smaller on the formed part. From the result, it can be concluded that the larger end radius of punch element is given smaller dimples. Figure (6) shown the results.

Figure(6) the deflection, equivalent strain, equivalent stress of dish with 2mm rubber

When the punch radius increasing the contact area is increased with reducing the curvature, and dimple will be smaller.

The ANSYS results shown that the larger hemispherical end radius should be selected to insure that the contact between the blank and punch element is good.

But in our case (production dish via MPF), the results shown that the pin punches radius (4mm) product the dish without dimple. So it can be deduced that the 4mm radius is the optimum radius for MPF dish with 2mm thickness rubber.

7. FORMING LOAD – DISPLACEMENT RELATION
The punches force – displacement curves of the dish during MPF are determined in this paper for all cases done and it seem that load increased with the displacement firstly then decreasing when the material goes to plastic deformation as shown in Figure(7).
Figure (7) punch force – displacement curve with different punch pin radii (the force in KN - Displacement in mm)

8. CONCLUSIONS

This paper is to develop an algorithm that generate the surface of dish. A designed surface is represented by control points the surface has been represented depending on Bezier curve to generate the dish surface. The control points generation for a disk is developed. Applying Bezier surface controls points for an existing set of collected three dimensional data points is introduced to describe such dish. The proposed algorithm for dish generation was developed and implemented successfully through ANSYS software. Copper plate has been chosen with three rubber’s thickness (elastic cushion) (0.5, 0.7, 2) mm to avoid dimples also three radii of punch group (3, 4, 6) mm are investigated. It is concluded that the punch element with radius 4 mm and elastic cushion (rubber) 2 mm is the best design to avoid the dimples defect and have the homogenously distributed stress.

9. REFERENCES