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Finite Element Analysis for Control of Lateral and Torsional Vibrations in Drilling Directional and Multi-Lateral Wells

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Abstract **- Vibration during drilling operations has a large effect on both the bottomhole assembly which in turn has significant effect on the drilling efficiency. Drillstring vibration causes damage to the drillstring and BHA components, premature bit failure, wear on tool joints and stabilizers, twist-offs where the drillstring breaks down hole due to fatigue or excessive torque and poor control of the deviation due to inadequate understanding of the tool face position and stresses especially in deviated and multilateral wells.**

The primary causes of drillstring vibrations are bit/formation and drillstring/borehole interactions. Large vibration levels cause reduced rates of penetration and catastrophic failures while lower levels may lead to a reduced operating life. The benefits of addressing this problem are obvious and include reduced drilling time and costs, reduced maintenance, and lower equipment turnover.

This research was carried out to analyse the effect of torsional and lateral vibrations on drilling efficiency when drilling directional wells. Harmonic and modal finite element analyses were adopted for the obvious advantage that it is able to approximate the real structure with a finite number of degrees of freedom. In the cause of the analysis, different critical load, critical speed for the finite element of the drillstring and the BHA and saturation points were developed for the different components of the drillstring from the Euler model to determine the crippling loads that will cause each component to buckle and propagate lateral and torsional vibration.

The results shows that it is safer to drill below the critical speed of the finite elements of the drillstring and BHA to avoid resonance and to ensure the axial forces acting on the BHA is below the critical load.

1.0. INTRODUCTION

The drill-string vibrations are induced by the characteristics of the bit-rock interaction and by the impacts that might occur between the column and the borehole. If not controlled, vibrations are harmful to the drilling process causing:

1. Premature wear and consequent damage of the drilling equipment and BHA components - bit, motor / RSS, MWD etc. resulting many times in failures, especially due to fatigue.
2. Decrease of the rate of penetration (ROP), increasing the well cost
3. Interferences on the measurements performed during the drilling process and damage of the measurement equipment and even failure to acquire evaluation data
4. Significant waste of energy due to increased Tripping Times and inability to run & set casing, torque and drag
5. BHA instability, reducing the directional control.
6. Wellbore instability occasioned by the fracturing effect on the wellbore due to BHA whirl.

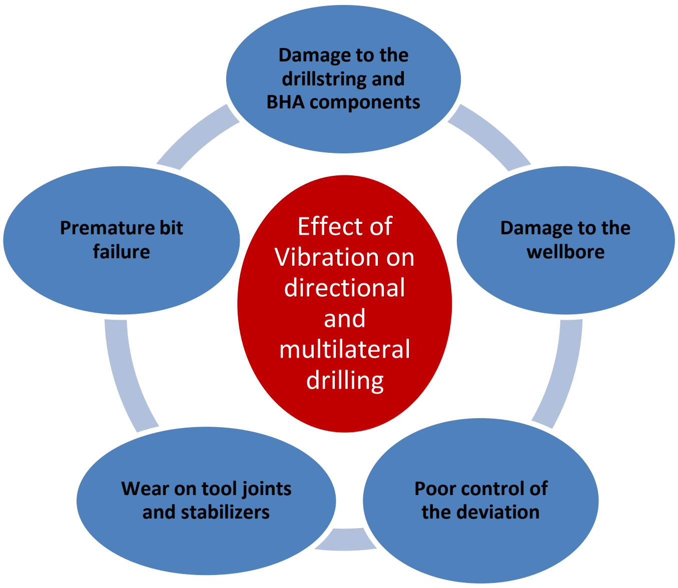

Fig. 1. Effect of vibration on directional and multilateral drilling

Factors that affect torsional and lateral drillstring vibrations

1. Material of the drillstring (shear modulus). For the steel with increasing torque, shear stress increases linearly with shear strain until plastic region is reached.
2. Drilling fluid
3. Well geometry (Hole angle)
4. Difference in friction between the static and dynamic friction of the drill bit and bit face
5. The nature of vibration in drillstrings depends on the type of bit, among other factors. PDC bits work by shearing the rock rather than crush the rock. This results in a bit-rock interaction mechanism

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characterized by cutting forces and frictional forces. The torque on bit and the weight on bit have both the cutting component and the frictional component when resolved in horizontal and vertical direction.

A common mode of failure of PDC bits in hard rock drilling is that of catastrophic breakage caused by the various modes of drillstring dynamics. Typically this mode of failure takes place in advance of any appreciable wear that may dictate cutter replacement.

To obtain required drilling performances, it is necessary to adjust features such as profile shape, gage and mainly cutter characteristics (shape, type and orientation). To optimize the drilling parameters in order to increase the ROP, you need to know the real drill bit response which is a direct function of the cutter rock interaction. Cutter rock interaction model is a critical feature in the design process. But previously used models considered only three forces on a cutter based on the cutter-rock contact area: drag force, normal force and side force. Such models are no longer valid with the introduction of PDC cutters with chamfer and special shape (Gerbaud et al, 2006).Chamfer was introduced to avoid diamond chipping when drilling hard formations. Two different mechanisms take place at the chamfer with respect to the depth of cut. If the depth of cut is greater than the chamfer height, crushed rock is trapped between the cutting face and the rock and additional forces are generated in the same way than for the cutting face crushed material.

Now, if depth of cut is lower than chamfer height, the chamfer becomes the cutting face with higher back rake angle and the chamfer forces are the cutting face forces. For
example, at 45° chamfer angle and 15° back rake angle, the real back rake angle for small depth of cut becomes 60°.

2.0. HARMONIC FINITE ELEMENT ANALYSIS OF DRILLSTRING LATERAL VIBRATION

Lateral vibration will cause fatigue and failure of drillstring, broaden hole and change bending angle of bit. The major difficulty however encountered in controlling lateral vibration amplitude and impact intensity is the fact that lateral vibration and its consequent drillstring impact cannot be observed at the surface without the expensive MWD apparatus.

Bit whirl is predominant with PDC bits because tricone bits penetrate the bottom of the borehole more and do not allow sideways movement of the bit. Whirl generation is caused by two factors: a centrifugal force generated as a result of the high rotary speed. The added force creates more friction which further reinforces whirl. The second is the center of rotation which is no longer the centre of the bit. This fact however contradicts the basic bit design assumption that the geometric center of the bit is the center of rotation. The impact loads associated with this motion cause PDC cutters to chip, which in turn, accelerates wear.

In this analytical model, we would be able to control lateral vibration by controlling some factors such as under-gauge stabilizer, initial phase angle, initial deformation, WOB and ROP.

The rotation of the cross section as measured by $\theta \approx \frac{dy}{dx}$ is less than 1.0, one radian

At any section XX distant X from the fixed point B, for the curved beam, bending moment is expressed as given as

$$\frac{M}{EI_a} = \frac{d2y}{dx2} \qquad \text{(Leonhard Euler equation)}$$

Recall,
M = Force x distance

$$EI_a \frac{d2y}{dx2} = P(\delta - y)$$

$$EI_a \frac{d2y}{dx2} + Py = P\delta$$

$$\therefore \frac{d2y}{dx2} + \frac{P}{EI_a} y = \frac{P\delta}{EI_a}$$

Solving the differential equation,

$$y = C_1 \cos(x\sqrt{\frac{P}{EI_a}}) + C_2 Sin\left(x\sqrt{\frac{P}{EI_a}}\right) + \delta \qquad (1)$$

Where
E = Modulus of elasticity for steel expressed in pa (from 200,000 to 220,000MPa)
I_a = Moment of inertia of the straight section of the drillstring expressed in m^4
l = Free buckling length which depends on the actual length of the pipe and the way the end s are fixed, expressed in m
C_1C_2 = Arbitrary constants of integration

At the fixed point, B, the deflection is zero
Boundary conditions:
$y = 0 @ x = 0$
$y = \delta @ x = l$

So

$$0 = C_1 + \delta \text{ or } C_1 = -\delta$$

The slope at any section is given by

$$\frac{dy}{dx} = -C_1\sqrt{\frac{P}{EI_a}}\sin(x\sqrt{\frac{P}{EI_a}}) + -C_2\sqrt{\frac{P}{EI_a}}\cos(x\sqrt{\frac{P}{EI_a}})$$

At the fixed point, B, slope is zero

$$\therefore x = 0, \frac{dy}{dx} = 0$$

So

$$0 = C_2\sqrt{\frac{P}{EI_a}}$$

$$\therefore C_2 = 0$$

At A, the deflection is δ

$$\therefore x = l, y = \delta$$

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$$y = C_1 \cos(x \sqrt{\frac{P}{EI_a}}) + C_2 Sin\left(x\sqrt{\frac{P}{EI_a}}\right) + \delta$$

$$\therefore \delta = -\delta \cos(l \sqrt{\frac{P}{EI_a}}) + \delta$$

$$\cos(l \sqrt{\frac{P}{EI_a}}) = 0$$

$$l \sqrt{\frac{P}{EI_a}} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \ldots$$

Where K = 1, 3, 5, …

Considering the first critical value,

$$l \sqrt{\frac{P}{EI_a}} = \frac{\pi}{2}$$

$$p = \frac{\pi^2 EI_a}{4l^2} \quad \text{(Euler formula)}$$

$$Ia = \frac{\pi}{4}(R_e^4 - R_i^4)$$

Where
R_e: outside radius of the pipe expressed in m
R_i: inside radius of the pipe expressed in m

$$Ia = \frac{\pi}{64}(D_e^4 - D_i^4)$$

D_e: outside diameter of the pipe expressed in m
D_i: inside diameter of the pipe expressed in m

P, the crippling load is the maximum limiting load at which the column tends to have lateral displacement. Buckling occurs about the axis having least moment of inertia.

From above, the factors affecting the vibration/deflection are critical load, length of drillstring, young modulus and moment of inertia.

3.0. HARMONIC ANALYSIS OF NATURAL FREQUENCY OF DRILLSTRING LATERAL VIBRATIONS

To determine the natural frequency of free lateral vibrations, consider a drillstring whose end is fixed and the other end carries a body of weight, W as seen in the figure below

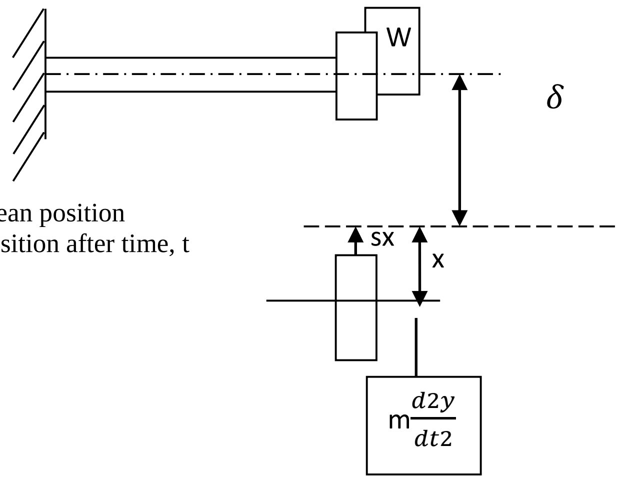

Fig 2. Natural frequency of free lateral vibration

Where

k = stiffness of the drillstring (N/m)

m = mass (Kg)

W = weight of drillstring = mg

δ = static deflection due to weight of the body

x= displacement of the body from the equilibrium position

w_n = circular natural frequency (rad/s)

f_n = natural frequency, Hz

f= frequency of mass body

A = Cross-sectional area

In equilibrium position, the gravitational pull, W = mg and is balanced by force of spring, such that W = k. δ

Since the mass is displaced from its equilibrium position by a distance of x, as shown in Fig 2. Above and is then released, therefore after time t,

Restoring force = $W - k(\delta + x) = W - k\delta - kx$

Recall $W = K\delta$

$$\therefore K\delta - k\delta - kx = -kx \; (taking\ upward\ force\ as\ negative) \quad 2$$

Accelerating force = mass x acceleration

$$= m\frac{d2y}{dt2} (taking\ downward\ force\ as\ positive) \quad 3$$

Equating equation 2 and 3, the equation of motion becomes

$$m\frac{d2y}{dt2} = -kx$$

$$m\frac{d2y}{dt2} + kx = 0$$

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$$\frac{d2y}{dt2} + \frac{k}{m}x = 0 \qquad 4$$

Recall that the fundamental equation of simple harmonic equation is

$$\frac{d2y}{dt2} + \omega x = 0 \qquad 5$$

Comparing equation 4 and 5, we have,

$$\omega^2 = \frac{k}{m}$$

$$\therefore \omega = \sqrt{\frac{k}{m}}$$

The periodic time of the vibration is

$$t_p = \frac{2\pi}{\omega_n}$$

And the natural frequency $= \frac{1}{t_p} = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}}$

Since the static deflection due to gravity, $\delta = \frac{mg}{k}$

Taking the value of g as 9.81m/s^2 and δ in metres

Therefore, the Natural Frequency, f_n

$$f_n = \frac{1}{2\pi}\sqrt{\frac{9.81}{\delta}} = \frac{0.4985}{\sqrt{\delta}} Hz$$

The value of static deflection can be obtained from the relation

$$E = \frac{Stress}{strain} = \frac{W\ x\ L}{A\ X\ \delta}$$

Implication

The external forces causing vibration to the drillstring should not operate at this natural frequency of the drillstring to avoid resonance.

4.0. MODAL FINITE ELEMENT ANALYSIS OF DRILLSTRING VIBRATION

4.1.CRITICAL ROTARY SPEED

Drillstrings develop vibrations, excessive wear and fatigue when run at critical rotary speeds. Drilling deeper in hard rock induces severe vibrations in drillstring which causes resonance, wear, fatigue and reduced rates of penetration and consequent premature failure of the equipment. The current means of controlling vibration by varying conditions such as reducing the rotary speed or WOB often ends up reducing the drilling efficiency.

Similarly, using shock subs are not a universal solution as they are designed for one set of conditions. When the drillstring environment changes as it often does, shock subs become ineffective and often result in increased drilling vibrations, making the situation more complex.

Fundamentally, the rotary table is driven by means of a sprocket and chain by the drawworks. It can however also be driven by an electric motor independent of the drawworks transmission on heavyweight rigs.

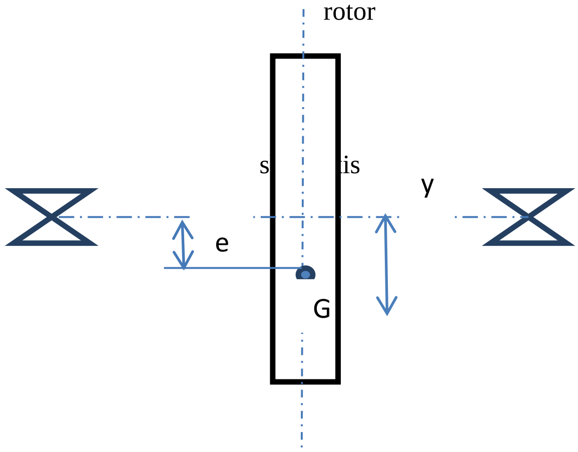

Fig 3. Rotary table when shaft is stationary

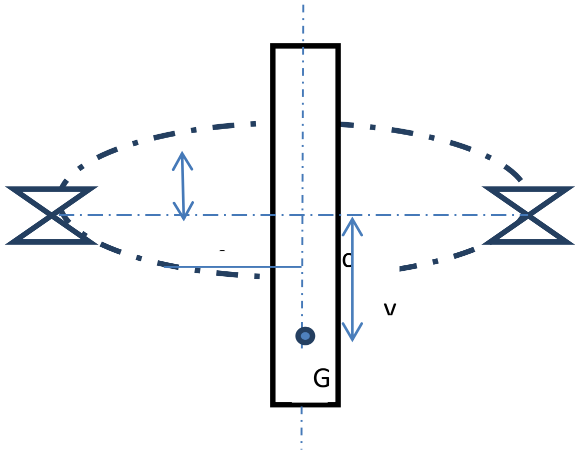

Fig 4. Rotary table when shaft is rotating

Let's consider a shaft of negligible mass carrying a rotor as seen in Fig 3 where G is the centre of gravity of the shaft.

When shaft is stationery, the centre line of the bearing and the axis coincides.
When shaft is rotating with a uniform speed, $\omega\ rad/s$, the centrifugal force acting radially outwards through G causing the shaft to deflect is given as

$$F_c = m\omega^2(y + e)$$

Where
m= mass of the rotor
e = initial distance of centre of gravity of the rotor from the centre line of the bearing or shaft axis, when the shaft is stationery
y = additional deflection of centre of gravity of the rotor when the shaft starts rotating at $\omega\ rad/s$

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The shaft behaves like a spring, therefore the resisting force to deflection f = ky
For the equilibrium position,

$$m\omega^2(y+e) = ky$$

$$m\omega^2 y + m\omega^2 e = ky$$

$$y(k - m\omega^2) = m\omega^2 e$$

$$y = \frac{m\omega^2 e}{k - m\omega^2} = \frac{\omega^2 e}{\frac{k}{m} - \omega^2}$$

We know the circular frequency

$$\omega_n = \sqrt{\left(\frac{k}{m}\right)}$$

Substituting this

$$y = \frac{\omega^2 e}{{\omega_n}^2 - \omega^2}$$

As shown in Fig 4 with the dotted lines, when $w > \omega_n$, the value of y will be negative and the shaft deflects in the opposite direction.

In order to have the value of y always positive, both plus and minus signs are taken

$$y = \pm\frac{\omega^2 e}{{\omega_n}^2 - \omega^2} = \frac{\pm e}{\left(\frac{\omega_n}{w}\right)^2 - 1} =$$

Substituting $w_n = w_c$

$$y = \frac{\pm e}{\left(\frac{\omega_c}{w}\right)^2 - 1} =$$

From the above expression, when $w_n = w_c$, the volume of y becomes infinite
Therefore,
w_c is the critical or whirling speed of the rotary table

$$w_n = w_c = \sqrt{\frac{s}{m}} = \sqrt{\frac{g}{\delta}}\,Hz$$

If Nc is the critical or whirling speed of the rotary in revolutions per second, then

$$2\pi N_c = \sqrt{\frac{g}{\delta}}$$

Therefore,

$$N_c = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}} = \frac{0.4985}{\sqrt{\delta}}\,rev/s$$

**The critical or whirling speed of the rotary is the same as the natural frequency of lateral vibration but its unit will be revolutions per second

Note: when the centre of gravity of the rotor lies between the centre line of the shaft and the centre line of the bearing, e is taken to be negative. On the other hand, if the centre of gravity of the rotor does not lie between the centre line of the shaft and the centre line of the bearing, the value of e is taken to be positive.

4.2. CRITICAL SPEED IN THE DRILLSTRING

The critical speed vary with length and size of drillstring and hole size. Two types of vibrations may occur: nodal vibration as the pipe vibrates in the nodes as a violin string or longitudinal vibration as the pipe may vibrate as a pendulum.

Nodal (lateral or transverse) vibration

$$RPM_n = \frac{33056\sqrt{OD^2 + ID^2}}{L_n} \qquad 5$$

Application: if the drilling RPM is equal to the calculated critical RPM_n, there may be drill pipe, drill collar and HWDP failure. So, the drilling RPM should be less than the calculated RPM_n

Axial (Longitudinal) Vibration

$$RPM_a = \frac{258000}{L} \qquad 6$$

Application: if the drilling RPM is equal to the calculated critical RPM_a, and/or equal to the harmonic vibration, there may be drill pipe, drill collar and HWDP failure.

For **tri-cone bits,** drill bit displacement frequencies are consistently three cycles for every bit revolution for three-cone bits, in other words, tri-cone bit impacts an excitation frequency of three times the rotary speed

4.3. CRITICAL SPEED IN THE DRILL COLLAR

Natural frequency of longitudinal vibration, f_1

$$f_1 = \frac{4212}{L_{dc}}\,cycles/\sec \qquad 7$$

Natural torsional frequency of drill collars, f_2

$$f_2 = \frac{2662}{L_{dc}}\,cycles/\sec \qquad 8$$

$$Critical\ RotarySpeed, N = 20f_1\ or\ 20f_2 \qquad 9$$

**To prevent vibration with tri-cone bits, bit rotation must be tolerated at a speed less than $20f_1\ or greater\ 20f_2$. The best situation for a drillstring is to operate below its lowest critical speed. In this region, the bit is able to continously make contact with the formation.

4.4. CRITICAL SPEED AT THE BIT

Frequency of vibration at the bit (excitation frequency)

$$f = \frac{3.Nrpm}{60}\,cycles/\sec \qquad 10$$

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5.0. CONCLUSION

The results show that to drill safely,

1. The rotation of the drillstring and BHA should be within the region of speed below the critical speed of the finite elements of the drillstring and BHA to avoid resonance and to ensure the axial forces acting on the BHA is below the critical load
2. Using the modal and harmonic finite element analysis, we were able to set critical limits for different components of the drillstring and BHA, beyond which the drillstring will be subjected to lateral and torsional vibrations.
3. Using the Euler equation, the crippling load that causes each drillstring to begin to buckle and the effect of the mud weight and length of drillstring were analyzed and incorporated in the mathematical model.
4. One of the issues to keep close tap on is the manipulation of the WOB during torsional and lateral vibration control to ensure that the founders point is not exceeded.
5. The greater the moment of inertia of the drillstring and BHA when considered in finite terms, the higher the tolerance and therefore the higher the allowable critical speed of the drillstring

6.0. RECOMMENDATION

Further studies should be carried out on microscopic analysis of transient drillstring vibration and effect of geomechanical stress on vibration.

REFERENCES

[1] Retrieved from http://www.slb.com/~/media/Files/resources/oilfield_review/ors99/sum99/manage.pdf
[2] Retrieved from http://www.aapg.org/explorer/2012/05may/regsec05.12.cfm
[3] Wikipedia (2015) Approximate Young's modulus for various materials. Retrieved from httm.www.wikipedia.com on 2nd October.
[4] Bailey J.J., and Finnie I. (1960) An Analytical Study of Drill-String Vibration, Trans. ASME, May 1960, 122-128
[5] Brett, J. F. (1991) The Genesis of Bit-Induced Torsional Drillstring Vibrations. SPE/IADC 21943. Paper presented at the SPE/IADC Drilling Conference held in Amsterdam 11-14 March.
[6] Burgess, T.M., MDaniels, G.L. and Das, P.K. (1987) Improving BHA Tool Reliability with Drillstring Vibration Models: Field Experience and Limitations, SPE/IADC Drilling conference, New Orleans, LA, SPE Paper 16109. March 15-18.
[7] Black, H. Paul (1961) Theory of Metal Cutting, McGraw-Hill Book Company, Inc, New York, 1961.
[8] Baryshnikov, A., Calderoni, A., Ligrone, A., Ferrara, P., and Agip, S. P. A. (1998), A New Approach to the Analysis of Drilling String Fatigue Behavior, SPE 30524 presented at the SPE Annual Technical Conference and Exhibition, Dallas, Oct. 22-25.
[9] Besaisow, A. A., Ng F. W. and D. A Close (1990) Application of ADAMS (Advanced Drillstring Analysis and Measurement System) and Improved Drilling Performance. SPE/IADC Drilling Conference, Houston, SPE-19998-MS, 27 February – 2 March.
[10] Besselink B., Van de Wouw N., and Nijmeijer H. (2010) A semi-analytical study of stick-slip oscillations in drilling systems. Journal of computational and Nonlinear Dynamics, published by Design Engineering Division of ASME. April.
[11] Cunningham R. A. (1968) Analysis of downhole measurements of drillstring forces and motions. Journal of Engineers for Industry, pp 208-216, May 13.
[12] Cobern M. E., Carl A.P., Jason A. B., Daniel E. B. and Mark E. W. (2007) Drilling Tests of an active vibration damper. SPE/IADC 105400 SPE/IADC Drilling Conference in Amsterdam, The Netherlands, 20-22 February.
[13] Chen S.S., Nambsganss M.W., Jendrzejczyk (1974) Added Mass and Damping of a Vibrating Rod in Confined Viscious Fluids. Journal of Applied Mechanics, pp 325-329
[14] Coudyzer C., Richard T., (2005) Influence of the back and side rake angles in rock cutting. Paper AADE-05-NTCE-75 presented at the 2005 AADE Technical Conference and Exhibition, Houston, April 5-7
[15] Dawson, R., Lin, Y.Q., Spanos, P.D., (1987). Drill string stick slip oscillations. Proceedings of the Spring Conference of the Society for Experimental Mechanics in Houston, USA, 14–19 June.
[16] Duncan, J.J. (2007) Challenges and developments in direct measurement of down hole forces affecting drilling efficiency. MSc Thesis, Robert Gordon University, Aberdeen.
[17] Dupriest F. E., Witt J.W. and Remmert S. M. (2007) Maximizing ROP with Real-Time Analysis of Digital Data and MSE. International Petroleum Technology Conference, 21-23 November 2005, Doha, Qatar , 2005
[18] Elsayed, M.A., Cherif Aissi and Chaitanya Kancharla Analysis of Vibration Characteristics for Drillstrings Using Active Circuits. University of Louisiana at Lafayette, Lafayette, LA, USA 70504
[19] Elsayed, M. A. and Cherif A. (2009) Analysis of the frequency response function and stability in drillstrings using active circuit model. Journal of Applied Science and Engineering Technology, University of Louisiana, Lafayette.
[20] Elsayed M. A. and Yalamanchili A, "Analysis of mode interaction in drillstrings using active circuits," Paper 124, in *Proc. of the Soc. of Experimental Mechanics (SEM) Conf.*, Orlando, FL
[21] Finnie, I. and Bailey J.J., (1960.) An Experimental Study of Drillstring Vibration. J. Eng for India Trans. ASME, 82: 122 – 135
[22] Glowka D.A., (1989) Use of Single Cutter Data in the Analysis of PDC Bit Designs: Part 1 – Development of a PDC cutting Force Model. SPE Journal of Petroleum Technology, 41 (1989), 797-849.
[23] Ghasemloonia A., Rideout G. & Butt S. (2011) The Effect of Weight on Bit on the Contact Behaviour of Drillstring and Wellbore, Memorial University of Newfoundland, Canada. ISBN: 1-56555-342-X.
[24] Halsey, G.W., Kyllingstad, A., Aarrestad, T.V., Lysne, D., (1986) Drillstring torsional vibrations: comparison between theory and experiment on a full scale research drilling rig. In: SPE 15564, Presented at 61st Annual Technical Conference and Exhibition in New Orleans, USA, Oct 5–8.
[25] Ibrahim, A.A., Musa, T.A. and Fadoul, A.M. (2004) Drill mechanics: consequences and relevance of drillstring vibration on wellbore stability, Journal of Applied Sciences 4(1): 106-109, ISSN 1607-8926, Asian Network for scientific information.
[26] Jamal Z. S. J. H and Gholamreza R. (2011) Finite Element Analysis of Drillstring Lateral Vibration. Scientific Research and Essays, Vol 6(13) pp 2682-2694, July 4.
[27] Jansen J.D. (1993) Nonlinear Dynamics of Oilwell Drillings. Delft: Delft University Press
[28] L.Gerbaud, S.Menand and H.Sellami (2006) PDC Bits: All Comes From the Cutter Rock Interaction. SPE/IADC Drilling Conference, Maimi, United States, SPE Paper 98988. 21-23 February.
[29] Lubinski A. and Woods H.B. (1953) Factors affecting the angle of inclination and dog-leg in rotary boreholes, Drill and Production Practice API 222
[30] Lin Y and Wang Y (1990) New Mechanism in drillstring vibration. Paper presented at the 22nd Annual Offshore Technology Conference, May 7-10.
[31] Leine R. I. and Van Campen D.H. (2002) Stick-slip whirl interaction in drillstring dynamics" Eindhoven University of Technology, The Netherlands
[32] Morten K. J.n and Torgeir M. (2010) Stick-Slip Prevention of Drill Strings Using Nonlinear Model Reduction and Nonlinear Model Predictive Control. MSc Thesis, Norwegian University of Science and Technology
[33] MacDonald, K. A. and Bjune, J. V. (2007) Failure analysis of drillstrings. Engineering Failure Analysis, 14:1641-1666, 2007.

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[34] Paslay, P. R. and Bogey D. B. (1963). Drillstring vibrations due to intermittent contact of Bit Teeth. Trans. ASME., 82: 153-165.
[35] Pavonne, D. R. and Desplans, J. P. (1994) Application of high sampling rate downhole measurements for analysis and cure of stick-slip in drillstring. SPE 28324. Paper presented at the SPE 69th Annual Technical Conference and Exhibition held in New Orleans, 25-28 September.
[36] R.I. Leine (1997) Literature Survey on Torsional Drillstring Vibrations. Internal Report number: WFW 97.069, Division of Computational and Experimental Mechanics, Eindhoven University of Technology, The Netherlands
[37] R. K. Rajput (2008) Strength of Materials. Published by Rajendra Ravindra Printers, New Delhi, India
[38] Reckmann, Jogi , Herbig, (2007), Using Dynamics Measurements While Drilling To Detect Lithology Changes And To Model Drilling Dynamics. Paper 29710, OMAE, 26th International Conference
[39] Ramkamal Bhagavatula (2004) Analysis of PDC Bit Wobbling And Drilling String Buckling. MSc Thesis In Petroleum Engineering, Texas Tech University
[40] Richard T., Germany C and Detournay E. (2007) A simplified Model to Explore the Root cause of Stick-Slip Vibrations in Drilling Systems with Drag Bits. J. Sound Vib., 305, pp432-456
[41] Sellami H., Fairhurst C., Deliac E., Delbast B., (1989) The role of in situ rock stresses and mud pressure on the penetration rate of PDC Bits. Presented at the 1989 International Symposium Rock at Great Depth, Pau, France, Vol. 2, pp. 769-777
[42] Teale, R. (1965) The Concept of Specific Energy in Rock Drilling. Intl. J. Rock Mech. Mining Sci. (1965) 2, 57-73
[43] Thiago Ritto G (2010) Numerical analysis of the nonlinear dynamics of a drill-string with uncertainty modeling. PhD Thesis Université Paris-Est
[44] Yigit, A.S. & Christoferou, P., (1996) Coupled Axial and Transverse Vibrations of Oilwell Drillstring. Journal of Sound and Vibration, 195: 617 – 627

APPENDIX

SYMBOLS

T_1 = torque on bit
T_2 = surface torque
J_1J_2 = equivalent moment of inertia at bottom end and top end respectively
C_1C_2 = mud damping
C_S = structural damping
Ω_1 = bit speed
Ω_2 = top drive speed (surface RPM)
φ_1 φ_2=rotational displacement (angle) of the bit starting and top drive respectively with zero at time t = 0
k = equivalent stiffness of the drillpipe
The equivalent mass moment of inertia
$J_1 = \rho_{BHA} I_{BHA} L_{BHA}$
ξ = the damping factor
T' =vector with efforts
q = state-space coordinate
m = Mass

k = stiffness

x = displacement of m from the equilibrium position

E = Modulus of elasticity for steel expressed in pa (from 200,000 to 220,000MPa)
I_a = Moment of inertia of the straight section of the drillstring expressed in m^4
$l =$ Free buckling length which depends on the actual length of the pipe and the way the end s are fixed, expressed in m
C_1C_2 = constants of integration
L = length of drillstring in inches

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