

Finite Amplitude Binary Reaction-convection in a Rotating Porous Layer with Thermal Non-Equilibrium Effects

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Abstract - This study explores finite amplitude natural convection within a binary-reacting fluid mixture contained within a rotating porous layer. The primary objectives are to evaluate the conditions necessary for the onset of finite amplitude convective motions and to quantify the associated rates of heat and mass transport. This research incorporates two major physical phenomena: first, the use of the local thermal non-equilibrium (LTNE) model, which is utilized to model the distinct temperature fields maintained by the fluid and solid phases; and second, the effect of the Coriolis force resulting from system rotation. We specifically focus on investigating how the reactive component's tendency to precipitate or dissolve significantly influences the characteristics of convection. To determine the qualitative behavior of the finite amplitude solutions and to calculate the quantities of heat and mass transfer, a weakly nonlinear analysis employing the Fourier series method was used. The findings of this current work validate previous research by demonstrating consistency with established results. This investigation ultimately advances the capability to design a control system for convective motions, which is highly relevant for applications in the manufacturing sector.

Keywords - Double-diffusive convection; chemical reaction; modified Darcy model; Rayleigh number; Nusselt number; Sherwood number.

I. INTRODUCTION

The concurrent presence of gradients from two stratifying agents such as heat and salt, which possess distinct diffusivities within a fluid-saturated porous medium, gives rise to fascinating convective phenomena. These complex behaviors are not observed in single-component fluid systems. Over the past several decades, convection within fluid layers governed by two or more stratifying components has been a central focus of extensive theoretical and experimental research. This includes foundational works summarized in excellent reviews by Turner [1-3], Huppert and Turner [4], and Platten and Legros [5]. The appeal of studying two or multi-component convection stems from its stark difference compared to single-component systems. Notably, in contrast to single-component dynamics, convection can initiate even when the fundamental state is hydrostatically stable-meaning the overall density decreases with height. The concept of double diffusive convection holds critical importance across diverse fields, including the manufacturing of high-quality crystals, liquid gas storage, oceanography, pharmaceutical development, the solidification process of molten alloys, and the dynamics within geothermally heated lakes and magmas. Some of the

recent works include Akbar et al. [6], Hu et al.[7], Akbal [8] and the references there in.

In various geological systems, buoyancy forces drive the vertical circulation of a mineral-laden fluid through a porous matrix. During this circulation, the solute concentration in the fluid is often not conserved. This is because the mineral may either dissolve from or precipitate onto the solid porous matrix, as its solubility is dependent on factors like temperature, pressure and the localized rock chemistry. The exact influence of these phase change processes on convective instability remains incompletely understood and has been the subject of limited investigation. Early research into the impact of chemical reactions on the onset of convection in porous media was conducted by Steinberg and Brand[9-10]. Subsequent studies by Gatica et al. [11] and Viljoen et al. [12] explored how exothermic reactions influence the stability of such porous systems. Jupp and Woods [13] also contributed to the field with a study on thermally driven reaction fronts in porous media. The focus has included chemically driven instabilities, such as the work by Malashetty and Gaikwad [14] on binary liquid mixtures with rapid chemical reactions. Specifically, Pritchard and Richardson [15] investigated how temperature-dependent solubility affects the criteria for the onset of convection. The recent important references pertaining to this field are Vaz and Lima [16], Shevchuk et al. [17], Luo et al. [18] and the references there in.

The phenomenon of thermal convection in rotating porous media has been extensively studied. Early work by Friedrich [19] investigated stability using both linear and nonlinear analyses for a bottom-heated rotating layer. Subsequent investigations incorporated additional complexities: Patil and Vaidyanathan [20] included the influence of variable viscosity, while Palm and Tyvand [21] established a valuable analogy between rotating and anisotropic porous layers. Jou and Liaw [22] determined marginal stability conditions for transient heating using the Darcy model. Later research focused on stability criteria. Qin and Kaloni [23] derived sufficient conditions for nonlinear stability in high-porosity media via energy theory. Vadasz [24] performed both linear and weak nonlinear stability analysis, notably finding that, unlike in pure fluids, overstable convection in porous media is not limited by the Prandtl number. Vadasz [25] also provided an excellent overall review. Crucially, studies by Straughan [26-27] utilizing energy stability theory demonstrated key equivalence results. Straughan [26] reported that the global nonlinear stability boundary for Darcy-governed rotating convection matches the linear instability boundary, confirming that

subcritical instabilities are impossible. Extending this, Straughan [27] later proved this equivalence still holds when applying a thermal non-equilibrium model (LTNE) to a layer rotating about a vertical axis. Further studies have been exhaustively reported in the literature. Some of the recent are Siddheshwar and Krishna [28], Rana et al. [29], Rana and Sharma [30] and the references there in.

Most prior investigations have relied on the assumption of Local Thermal Equilibrium (LTE) between the fluid and solid phases within the porous medium. This assumption posits that the temperature difference between the two phases at any given point is negligible. However, for numerous practical scenarios involving rapid heat transfer such as the drying and freezing of materials, or microwave heating, the LTE assumption is insufficient. Consequently, the study of Local Thermal Non-Equilibrium (LTNE) effects becomes highly relevant.

A comprehensive review of research utilizing the LTNE model is provided in the literature [31], with more recent significant contributions found in [32-35]. The concept of a two-field energy model was initially proposed by Borjes [38] and was adopted by Nield and Bejan [31] and Nield and Kuznetsov [39] as the simplest approach for modeling LTNE. Rather than using a single energy equation for a common porous media temperature, this model employs two distinct energy equations to describe the fluid and solid phase temperatures separately. These two equations are necessarily coupled via terms that mathematically account for the heat transfer occurring between the phases.

While significant progress has been made in understanding the individual effects of rotation, chemical reaction and thermal non-equilibrium within porous media, a comprehensive study that integrates all these complexities is currently lacking. Specifically, the combined influence of the Coriolis force, solute precipitation/dissolution, and Local Thermal Non-Equilibrium (LTNE) on the stability and heat/mass transport of double-diffusive convection has not yet been fully investigated.

Therefore, the present work aims to fill this critical gap by performing a nonlinear stability analysis of finite amplitude double-diffusive convection in a binary-reacting fluid mixture saturated in a rotating porous layer under the governing framework of the LTNE model. The primary objectives are to (i) determine the conditions required for the occurrence of finite amplitude convective motions, (ii) quantify the resulting heat and mass transfer rates, and (iii) understand how the parameters associated with rotation, chemical reaction, and LTNE interact to control the onset and magnitude of convection.

II. FORMULATION OF THE PROBLEM

The present study analyzes a long, horizontal layer of porous medium saturated with a binary-reacting fluid mixture. The system is confined between two surfaces located at $z=0$ and $z=d$. Both boundaries are assumed to be stress-free, impermeable, and isothermal. The layer is subjected to a constant gravitational field, $\mathbf{g} \equiv (0, 0, -g)$, acting in the negative z -direction. To establish the driving forces for convection, the lower boundary is uniformly heated and salted to create a destabilizing thermal gradient and a stabilizing solute gradient relative to the upper surface. Furthermore, the

entire porous layer rotates uniformly about the vertical z -axis with an angular velocity, $\boldsymbol{\Omega} \equiv (0, 0, \Omega)$. Due to the assumed significance of heat exchange between the solid and fluid components within the porous medium, the Local Thermal Non-Equilibrium (LTNE) model is adopted, requiring separate energy equations for each phase. The effects of variable fluid density are incorporated through the Boussinesq approximation. Regarding the binary reaction, chemical equilibrium is maintained at the boundaries. We invoke a linear relationship to describe the dependence of the equilibrium concentration, $C_{eq}(T_f) = C_0 + \phi(T_f - T_0)$ where $\phi = (\Delta C / \Delta T_f)$ is a constant. This relationship is constrained by the physical premise that the solubility of the reactive component increases with temperature ($\phi > 0$). The physical phenomena described above are governed by a set of coupled partial differential equations, which constitute the mathematical model, including the equations,

$$\nabla \cdot \mathbf{q} = 0, \quad (1)$$

$$\frac{\rho_0}{\varepsilon} \left(\frac{\partial \mathbf{q}}{\partial t} + \mathbf{q} \cdot \nabla \mathbf{q} + 2\Omega \times \mathbf{q} \right) + \frac{\mu}{K} \mathbf{q} + \nabla p - \rho_f \mathbf{g} = 0, \quad (2)$$

$$\left((\rho_0 c)_f \left(\varepsilon \frac{\partial}{\partial t} + \mathbf{q} \cdot \nabla \right) - \varepsilon k_f \nabla^2 \right) T_f + h(T_f - T_s) = 0, \quad (3)$$

$$(1 - \varepsilon) \left((\rho_0 c)_s \frac{\partial}{\partial t} - k_s \nabla^2 \right) T_s + h(T_s - T_f) = 0, \quad (4)$$

$$\left(\varepsilon \frac{\partial}{\partial t} + \mathbf{q} \cdot \nabla - \varepsilon \kappa_c \nabla^2 + k \right) C - \tilde{k} C_{eq}(T_f) = 0, \quad (5)$$

$$\rho = \rho_0 (1 - \beta_T (T_f - T_0) + \beta_C (C - C_0)), \quad (6)$$

and the boundary conditions

$$\begin{aligned} w = \partial^2 w / \partial z^2 = 0 \text{ at } z = 0, d, \\ T_f = T_s = T_0, C = C_0 \text{ at } z = 0, \\ T_f = T_s = T_u, C = C_u \text{ at } z = d. \end{aligned} \quad (7)$$

Here h denotes the coefficient of inter-phase heat transfer, $\tilde{k}$, the lumped effective reaction rate. There is no motion and inter-phase heat transfer in the basic state, and all the unknown variables of above equations are homogeneous in the horizontal direction. To analyze the system's stability the finite amplitude perturbations are enforced on to the static state. For the ease of analysis, the focus is made on two-dimensional rolls (perturbations). Thus, all physical quantities become free from the y -coordinate. Then, it is convenient to introduce the stream function ψ , defined by $u = \partial \psi / \partial z$ and $w = -\partial \psi / \partial x$. Finally, up on rescaling the length, time, velocity, temperatures, and concentration with division by d , d^2/κ_f , κ_f/d , ΔT and ΔC respectively the Eqs. (1)-(6) take dimensionless form

$$\left(\frac{1}{\text{Pr}_D} \frac{\partial}{\partial t} + 1 \right) \nabla^2 \psi - \sqrt{\text{Ta}} \frac{\partial \omega_z}{\partial z} + \text{Ra}_T \frac{\partial T_f}{\partial x} - \text{Ra}_C \frac{\partial C}{\partial x} = 0, \quad (8)$$

$$\frac{\partial T_f}{\partial t} - \nabla^2 T_f - \frac{\partial(\psi, T_f)}{\partial(x, z)} + \frac{\partial \psi}{\partial x} - H(T_s - T_f) = 0, \quad (9)$$

$$\alpha \frac{\partial T_s}{\partial t} - \nabla^2 T_s - \gamma H(T_f - T_s) = 0, \quad (10)$$

$$\frac{\partial C}{\partial t} - \frac{\partial(\psi, C)}{\partial(x, z)} - \frac{1}{Le} \nabla^2 C + \frac{\partial \psi}{\partial x} - \chi(T_f - C) = 0, \quad (11)$$

$$\frac{1}{Pr_D} \frac{\partial}{\partial t} \left(\frac{\partial \omega_z}{\partial z} \right) + \frac{\partial \omega_z}{\partial z} + \sqrt{Ta} \frac{\partial^2 \psi}{\partial z^2} = 0. \quad (12)$$

Here ω_z represents z -component of vorticity vector $Pr_D = d^2 \varepsilon \nu (\rho_0 c)_f / K k_f$, the Darcy-Prandtl number, $Ta = (2K\Omega / \varepsilon \nu)^2$, the Taylor number, $Ra_T = \beta_T g \Delta T d K (\rho_0 c)_f / \varepsilon k_f \nu$, the thermal Rayleigh number, $Ra_C = \beta_C g \Delta C d K (\rho_0 c)_f / \varepsilon k_f \nu$, the solutal Rayleigh number, $H = d^2 h / \varepsilon k_f$, non-dimensional inter-phase heat transfer coefficient, $\alpha = \kappa_f / \kappa_s$, the diffusivity ratio, $\gamma = \varepsilon k_f / (1 - \varepsilon) k_s$, the porosity modified conductivity ratio, $Le = \kappa_f / \kappa_C$, the Lewis number and $\chi = d^2 \tilde{k} / \varepsilon \kappa_f$, the Damkohler number (non-dimensional reaction-rate).

III. NON-LINEAR STABILITY ANALYSIS

We now analyze the onset of finite amplitude convection, amplitude of convective motions, quantity of heat and mass transports. The methodology we adopted is based on double Fourier series representation of stream function, temperature, concentration and vorticity component in the form

$$\psi = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn}(t) \sin(max) \sin(n\pi z), \quad (13)$$

$$T_f = \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} B_{mn}(t) \cos(max) \sin(n\pi z), \quad (14)$$

$$T_s = \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} C_{mn}(t) \cos(max) \sin(n\pi z), \quad (15)$$

$$C = \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} D_{mn}(t) \cos(max) \sin(n\pi z), \quad (16)$$

$$\omega_z = \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} E_{mn}(t) \sin(max) \cos(n\pi z), \quad (17)$$

It is observed that in the laboratory setups and practical contexts the flows are dominated by few spatial harmonics. Thus, one can truncate the above series as

$$\psi = A_{11}(t) \sin(ax) \sin(\pi z), \quad (18)$$

$$T_f = B_{11}(t) \cos(ax) \sin(\pi z) + B_{02}(t) \sin(2\pi z), \quad (19)$$

$$T_s = C_{11}(t) \cos(ax) \sin(\pi z) + C_{02}(t) \sin(2\pi z), \quad (20)$$

$$C = D_{11}(t) \cos(ax) \sin(\pi z) + D_{02}(t) \sin(2\pi z), \quad (21)$$

$$\omega_z = E_{11}(t) \sin(ax) \cos(\pi z) + E_{20}(t) \sin(2\pi x). \quad (22)$$

This gives the initial values for solving a full non-linear convection problem. Our first intention is to find the finite

amplitudes $A_{11}, B_{11}, \dots$ using the dynamics of the system. The following eighth order Lorenz model is obtained on using Eqs. (18)-(22) into the Eqs. (8)-(12)

$$\frac{dA_{11}(t)}{dt} = \frac{Pr_D}{\delta^2} \left(-\delta^2 A_{11}(t) + \sqrt{Ta} (\pi E_{11}(t)) - Ra_T (aB_{11}(t)) + Ra_C (aD_{11}(t)) \right) \quad (23)$$

$$\frac{\partial B_{11}(t)}{\partial t} = -\delta^2 B_{11}(t) - aA_{11}(t) - 2\pi aA_{11}(t)B_{02}(t) + H(C_{11}(t) - B_{11}(t)) \quad (24)$$

$$\frac{\partial B_{02}(t)}{\partial t} = -4\pi^2 B_{02}(t) + \frac{\pi a}{2} A_{11}(t)B_{11}(t) + H(C_{02}(t) - B_{02}(t)) \quad (25)$$

$$\frac{\partial C_{11}(t)}{\partial t} = -\frac{1}{\alpha} \left(\delta^2 C_{11}(t) - \gamma H(B_{11}(t) - C_{11}(t)) \right) \quad (26)$$

$$\frac{dD_{11}(t)}{dt} = -\frac{\delta^2}{Le} D_{11}(t) - aA_{11}(t) - 2\pi aA_{11}(t)D_{02}(t) + \chi(B_{11}(t) - D_{11}(t)) \quad (28)$$

$$\frac{dD_{02}(t)}{dt} = -\frac{4\pi^2}{Le} D_{02}(t) + \frac{\pi a}{2} A_{11}(t)D_{11}(t) + \chi(B_{02}(t) - D_{02}(t)) \quad (29)$$

$$\frac{dE_{11}(t)}{dt} = -Pr_D \left(E_{11}(t) + \pi \sqrt{Ta} A_{11}(t) \right) \quad (30)$$

A. Steady Finite Amplitude Convection

The derivation of onset threshold for steady finite amplitude convection, namely, Ra_{Tc}^F , the critical finite amplitude Rayleigh number is the major point of emphasis. This is achieved by first evaluating the finite amplitudes $A_{11}, B_{11}, \dots$, from the system of homogeneous non-linear algebraic equations which arise from the Eqs. (23)-(30), upon setting $d/dt(\cdot) = 0$. Since there are nine unknowns with only eight equations, it is required to express, in terms the first unknown A_{11} , all the remaining amplitudes. This yields the following relation, where $x = A_{11}^2$,

$$\left(\frac{a_1 x^2 + b_1 x + c_1}{d_1} \right) A_{11} = 0. \quad (31)$$

The expressions of the coefficients omitted for the sake of conciseness. There exist two possibilities from Eq. (31), either $A_{11} = 0$ or $(a_1 x^2 + b_1 x + c_1) / d_1 = 0$. The first one corresponds to the non-convective case, i.e., a purely conduction state. On the other hand the later possibility yields the expression for convective amplitude A_{11} , in the form of the non-negative double root of quadratic equation

$$a_1 x^2 + b_1 x + c_1 = 0. \quad (32)$$

The condition for existence of double root provides the quadratic equation in Ra_T^F .

$$a_2 \left(Ra_T^F \right)^2 + b_2 \left(Ra_T^F \right) + c_2 = 0. \quad (33)$$

The real, positive, minimum root of this equation represents the onset threshold of finite amplitude convection, i.e., Ra_{Tc}^F . Further, up on using this for the computation of a non-negative double root of Eq. (32) one can obtain the value of the amplitude of steady convective motions, namely, A_{11} . Consequently, the values of all other unknowns $B_{11}, B_{02}, \dots$, are obtained.

B. Steady Finite Amplitude Convection

It is impossible to tackle the above non-linear system of differential equations (23)-(30), representing unsteady finite amplitude convection, by the analytical approach. Thus a powerful numerical technique is to be employed to do so. However in the realm of unsteady finite amplitude analysis, one can derive several important qualitative predictions. Let us calculate the divergence of velocity field represented by the above system in the form

$$\sum_{A=A_{11}}^{E_{11}} \frac{\partial}{\partial A} \left(\frac{dA}{dt} \right) = - \left(2 \left(Pr_D + H + \chi + \frac{H\gamma}{\alpha} \right) + (\delta^2 + 4\pi^2) \left(1 + \frac{1}{Le} + \frac{1}{\alpha} \right) \right). \quad (34)$$

The negative divergence of velocity field signifies the contraction of volume in the phase space. Consequently, in the phase space the trajectories tend to a set of measure zero or in particular to a fixed point. The Eq. (34) demonstrates that if $V(0)$ is the volume of the region in the phase space occupied by the initial points of the trajectories at $t=0$, then after the time t seconds the volume occupied by the final points of the trajectories is

$$V(t) = V(0) \exp \left(- \left(2 \left(Pr_D + H + \chi + \frac{H\gamma}{\alpha} \right) + (\delta^2 + 4\pi^2) \left(1 + \frac{1}{Le} + \frac{1}{\alpha} \right) \right) t \right). \quad (35)$$

Thus, there is an exponential decay in the volume with time. This dissipation is enhanced by the governing parameters like Pr_D, χ, H and γ . Further as the above system (23)-(30) preserves the dissipative nature of the original equations (8)-(12), one can expect that this system continue to hold many properties of the full non-linear problem. The above system of non-linear differential equations is solved numerically by Runge-Kutta-Gill method to obtain the values of the amplitudes $A_{11}, B_{11}, \dots$, as the functions of time t . Among these $B_{02}(t)$ and .. are used for the computation of unsteady heat and mass transfer rates (the derivation of formulae is shown in the following Section).

C. Heat and Mass Transports

The estimation of heat and mass transports is of great emphasis in the study of convection. The onset of convection significantly influences the heat and mass transfer rates. Before the convection sets in the heat and mass transfer take place only through the conduction mode. The non-dimensional coefficient of heat and mass transfer due to the onset of convection are denoted by the Nusselt and Sherwood numbers respectively. They are defined as

$$Nu = \text{Heat transport via} \left\{ \frac{\text{Conduction} + \text{Convection}}{\text{Conduction}} \right\} = 1 + \left(\frac{\int_0^{2\pi/a_c} (\partial T_f / \partial z) dx}{\int_0^{2\pi/a_c} (\partial T_{fb} / \partial z) dx} \right)_{z=0} \quad (36)$$

$$Sh = \text{Mass transport via} \left\{ \frac{\text{Conduction} + \text{Convection}}{\text{Conduction}} \right\} = 1 + \left(\frac{\int_0^{2\pi/a_c} (\partial C / \partial z) dx}{\int_0^{2\pi/a_c} (\partial C_b / \partial z) dx} \right)_{z=0} \quad (37)$$

Up on using the basic-conduction state temperature and concentration distributions $T_{fb}(z) = T_0 - \Delta T(z/d)$ and $C_b(z) = C_0 - \Delta C(z/d)$ along with the Eqs. (19) and (21) in (36) and (37) respectively, one can obtain

$$Nu = 1 + 2\pi(-B_{02}) \quad (38)$$

$$Sh = 1 + 2\pi(-D_{02}) \quad (39)$$

In the above expressions the second terms, viz., $2\pi(-B_{02})$ and $2\pi(-D_{02})$ signify the contribution of finite amplitude convective motions towards the heat and mass transports respectively. Since both B_{02} and D_{02} decrease from zero to negative values as the thermal Rayleigh number is elevated above its critical value, it is observed that at the onset of convection both Nu and Sh take their conduction state value viz., one and then increase with Ra_T^F . Further in the limiting case as $Ra_T^F \rightarrow \infty$, the rate of heat and mass transfer attain independence from Ra_T^F . Its prime reason is our choice of truncated Fourier series expansion with only two terms (see Eqs. (18)-(22)).

IV. RESULTS AND DISCUSSION

The combined influence of the Coriolis force, solute precipitation/dissolution and LTNE on the double-diffusive convection is investigated. This work mainly focused on a nonlinear stability analysis of finite amplitude double-diffusive convection in a binary-reacting fluid mixture saturated in a rotating porous layer under the governing framework of the LTNE model. As a result of this study we determined the conditions for the occurrence of finite amplitude convective motions, quantified the resulting heat and mass transfer rates, and understood how the parameters associated with rotation, chemical reaction and LTNE interact to control the onset and magnitude of convection.

The onset threshold in the form of critical Rayleigh number for finite amplitude convective motions is computed as a function of various governing parameters. The impact of LTNE and chemical reaction on the stability boundary is revealed in Figures 1 (a-b). These figures show the dependence of Ra_T^F on interphase heat transfer coefficient. The influence of interphase heat transfer coefficient is to elevate the value of Ra_{Tc}^F .

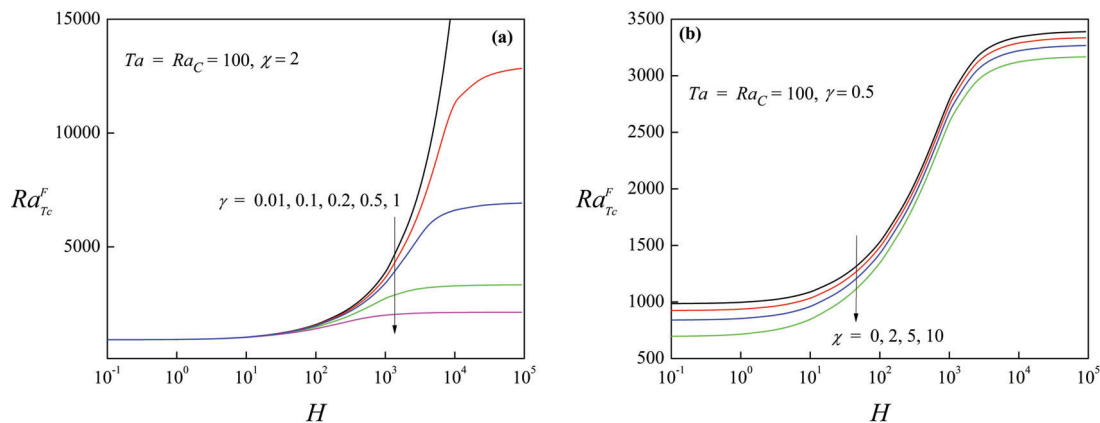


Figure 1. Finite amplitude Rayleigh number versus H for different values of (a) γ and (b) χ .

Thus, due to the LTNE effect the onset of finite amplitude convection is significantly delayed. This stabilizing effect of H is confined to specific moderate range. However, when H is very small, i.e., the difference between fluid and solid temperature is almost zero, the curves become flat. This reveals that the onset of convection is independent of H and hence corresponds to the LTE case. On the other hand when H is very large, i.e., fluid and solid temperature fields are not comparable, the impact of solid phase temperature field becomes insignificant and thus once again the curves become horizontal and approach those of LTE case.

Furthermore, Fig. 1 (a) reports that increasing the porosity-modified conductivity ratio (γ) from 0.01 to 1 leads to a consistent decrease in Ra_{Te}^F , suggesting that a more thermally conductive solid phase relative to the fluid phase destabilizes the system. Figure 1(b) illustrates that increasing the chemical reaction parameter (χ) results in a decrease in Ra_{Te}^F . This indicates that the reaction (precipitation/dissolution) acts as a destabilizing agent, advancing the onset of finite amplitude convective motions.

The influence of the Coriolis force is depicted in Figure 2. Increasing the Taylor number leads to a sharp increase in Ra_{Te}^F , confirming that rotation has a strong stabilizing effect

on the double-diffusive system. Higher values of solutal Rayleigh number Ra_C elevate the stability threshold (see Fig. 2a), while an increase in the Lewis number—representing a decrease in solutal diffusivity relative to thermal diffusivity—further stabilizes the system against finite amplitude perturbations (see Fig. 2b).

Figure 3 display the steady-state heat and mass transfer rates are quantified by the Nusselt number (Nu) and Sherwood number (Sh) as a function of the Rayleigh number ratio (Ra_T / Ra_{Te}^F). In all cases shown in Figure 3, the broken curves (Sh) lie above the solid curves (Nu), indicating that mass transfer is consistently more efficient than heat transfer in this binary-reacting mixture. Increasing H or γ leads to an increase in both Nu and Sh , as a stronger thermal coupling and higher solid conductivity facilitate more vigorous convection. An increase in the Taylor number suppresses the transport rates (decreasing Nu and Sh), while an increase in the chemical reaction parameter (χ) enhances them. Higher values of Le significantly enhance the Sherwood number while having a more moderate effect on the Nusselt number, highlighting the solutal sensitivity of the system.

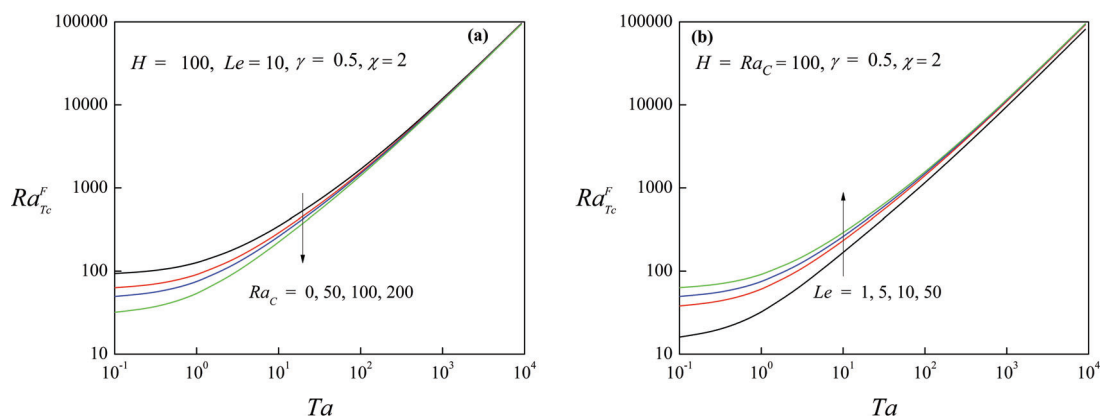


Figure 2. Finite amplitude Rayleigh number versus Ta for different values of (a) Ra_C and (b) Le .

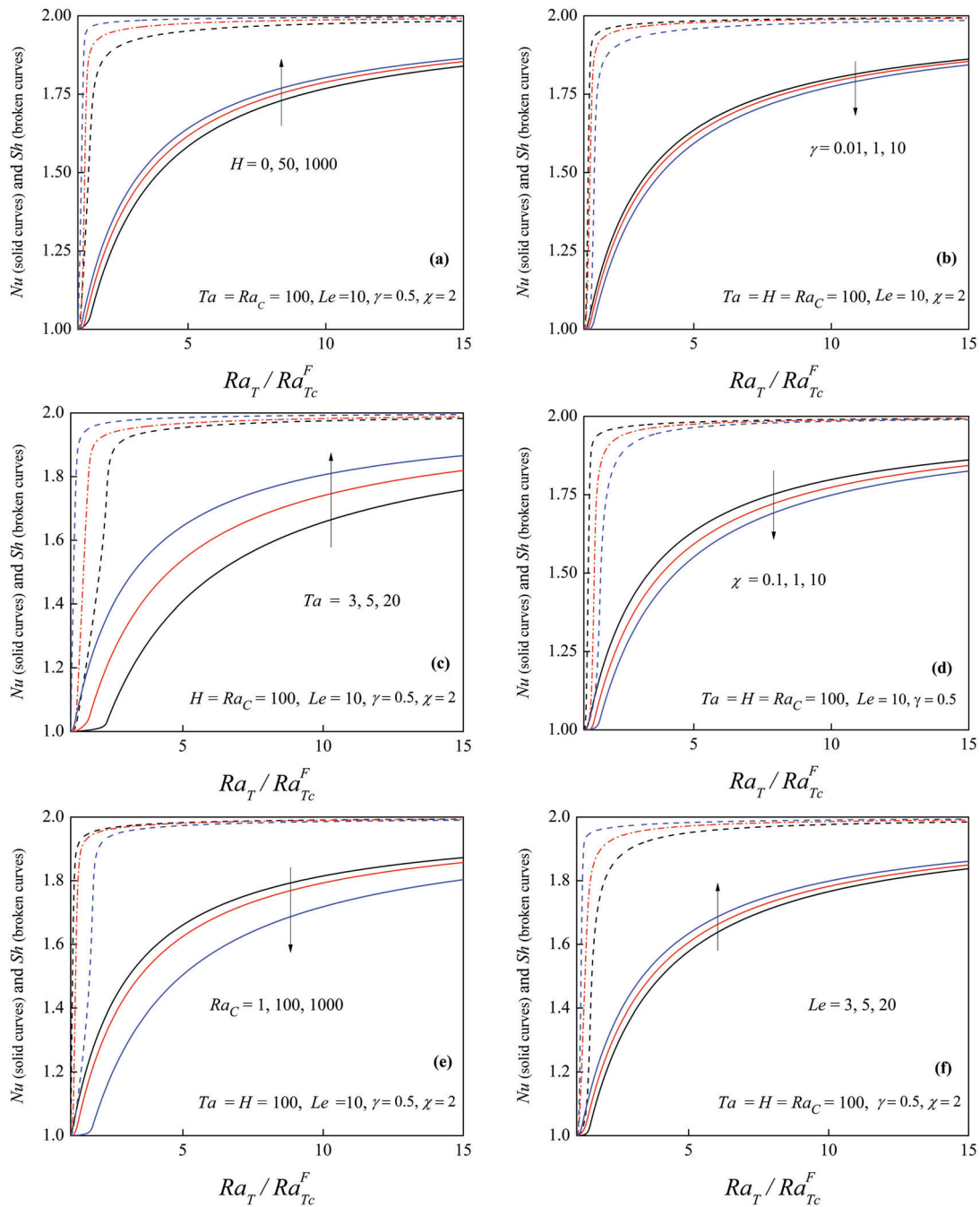


Figure 3. Nu and Sh versus Ra_T / Ra_{Tc}^F for different values of (a) H , (b) γ , (c) Ta , (d) χ , (e) Ra_C , (f) Le

V. CONCLUSIONS

This study investigated the nonlinear stability of double-diffusive convection in a rotating porous layer saturated with a binary-reacting fluid mixture under the framework of Local Thermal Non-Equilibrium. The primary conclusions derived from the weakly nonlinear analyses are as follows:

- The finite amplitude Rayleigh number (Ra_{Tc}^F) is significantly influenced by the thermal coupling

between the fluid and solid phases. An increase in the interphase heat transfer coefficient H delays the onset of convection, demonstrating a stabilizing effect. Conversely, a higher conductivity ratio (γ) and an increase in the chemical reaction parameter (χ) are found to destabilize the system, advancing the onset of convective motions.

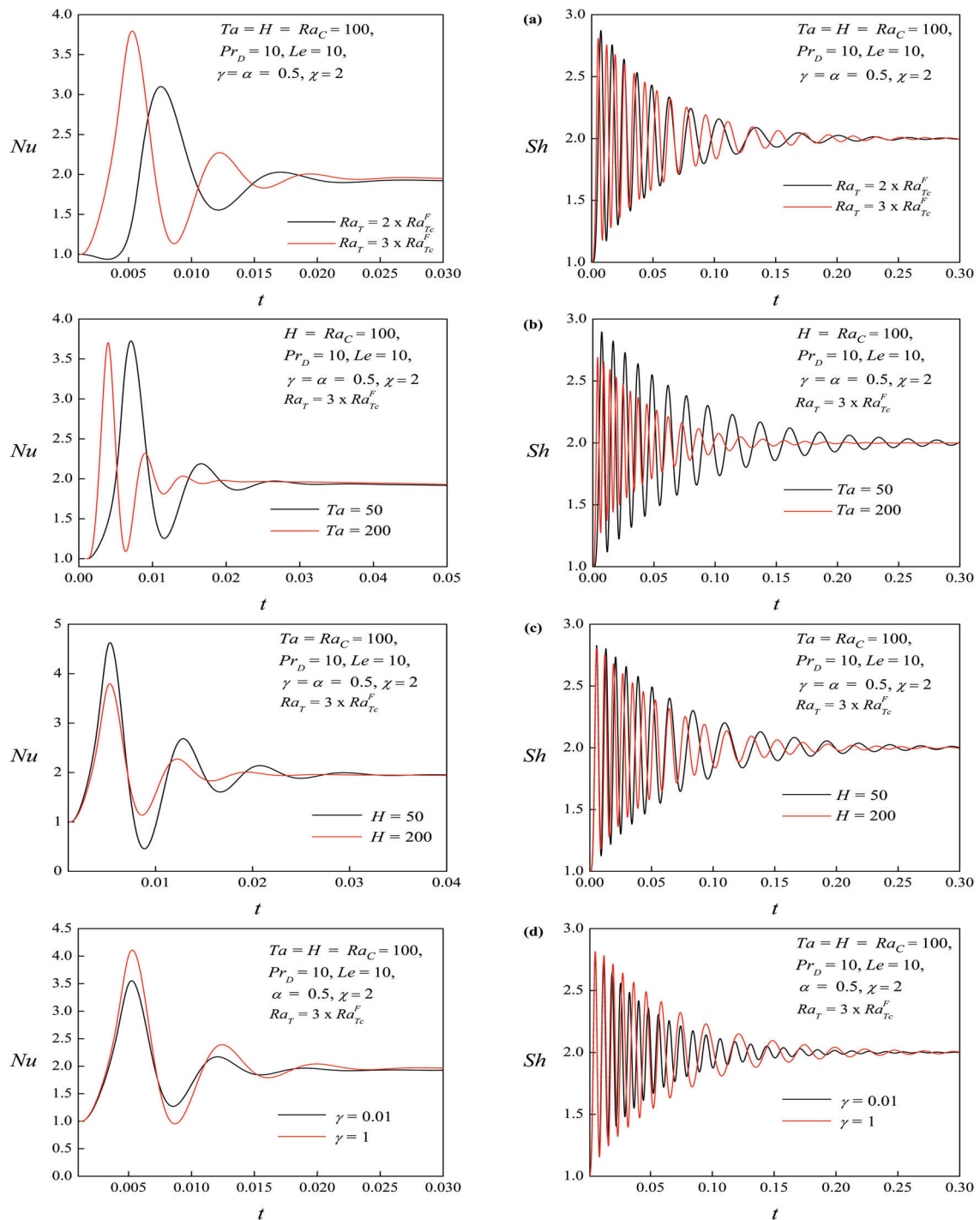


Figure 4. Nu and Sh versus time for different values of (a) Ra_{Tc}^F , (b) Ta , (c) H , (d) γ .

- The Coriolis force, represented by the Taylor number, acts as a strong stabilizing agent. Increasing Ta leads to a substantial increase in the critical Rayleigh number, effectively suppressing the transition to convective flow.
- The heat and mass transport rates, quantified by the Nusselt (Nu) and Sherwood (Sh) numbers, increase with

the thermal Rayleigh number. Notably, the Sherwood number is consistently higher than the Nusselt number across all parameter ranges, indicating that mass transfer is the dominant transport mechanism in this reactive system.

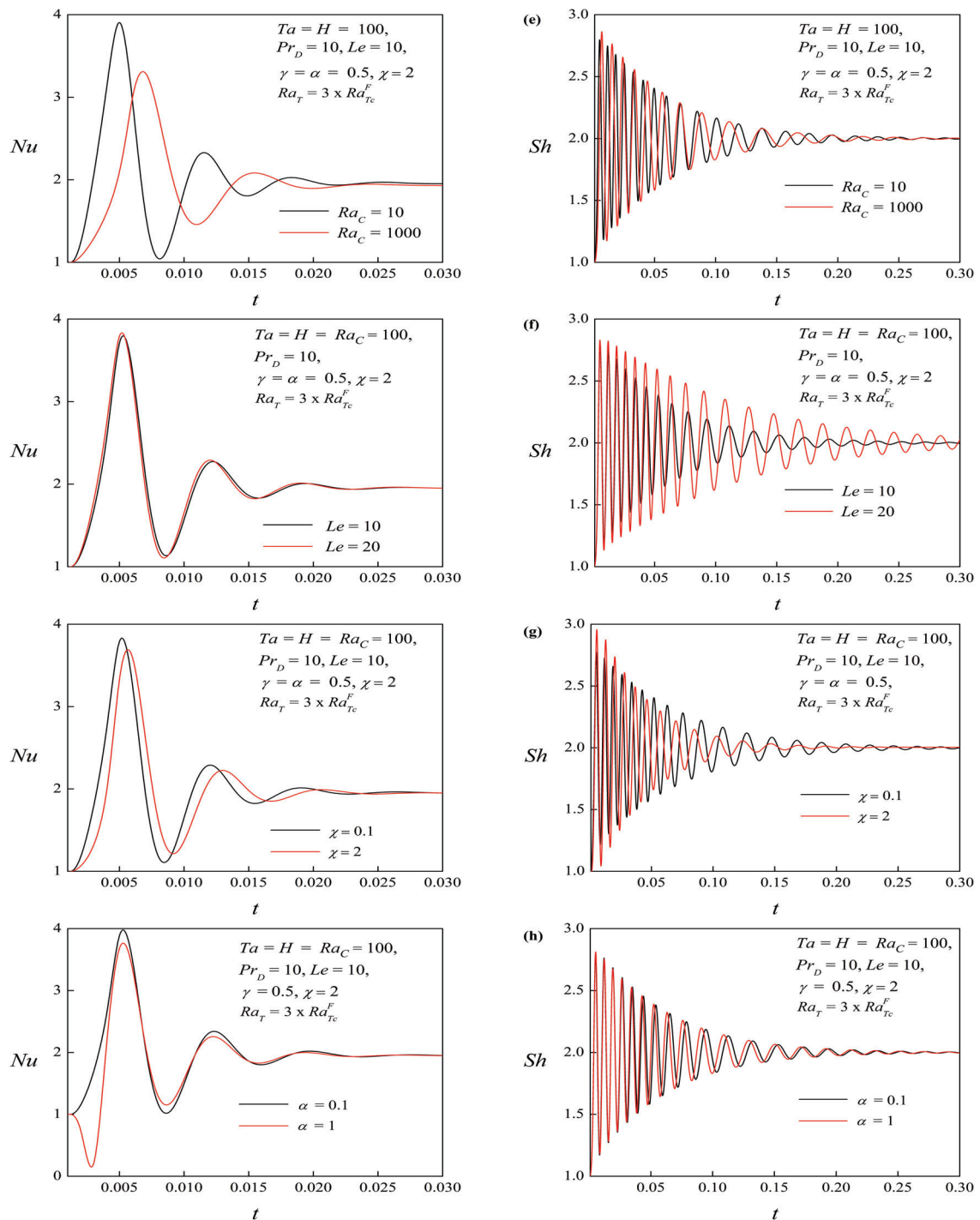


Figure 4. Nu and Sh versus time for different values of (e) Ra_C , (f) Le , (g) χ , (h) α .

- While rotation (Ta) reduces transport efficiency, increasing the interphase heat transfer (H) and the conductivity ratio (γ) enhances both heat and mass transfer rates once convection is established. Furthermore, the Lewis number has a more pronounced effect on the Sherwood number compared to the Nusselt number.
- Transient analysis reveals that the system reaches a steady state following initial oscillatory behavior. The frequency of these oscillations is particularly sensitive to the solutal Rayleigh number (Ra_C), the Lewis number, and the Darcy-Prandtl number, though Pr_D does not significantly impact the final steady-state transport values.

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