

# FEM Study Of Elliptical Coreoptical Fibres

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## Abstract

*Elliptical core optical fibres have the distinct property of geometry birefringence which enables them to exhibit polarization maintaining characteristics. These specialty fibres are typically used to guide linearly polarised light from point to point thereby finding many specialised applications in optical sensors as well as telecommunications and sensor research. The 2D-FEM modal analysis presented in this paper relies on solving Maxwell's equations of electromagnetic wave propagation using COMSOL Multiphysics®. The model developed in this study is adapted for FEM study of any elliptical core waveguide based on its geometry and doping concentration. The implementation of the FEM method in COMSOL enhances an insight into the numerical methodology and analyses factors that affect its performance. As a result, computational stability, convergence rate, modelling accuracy together with the influence of time and space step lengths can all be examined.*

**Keywords** – elliptical core optical fibre, specialty fibre, birefringence, 2D-FEM, propagation mode

## 1. Introduction

Conventional single mode optical fibres used in communication systems ideally have a perfect cylindrical core, with uniform diameter. In an ideal single mode fibre, the fundamental propagation mode ( $HE_{11}$ ) is a degenerated combination of two orthogonal propagation modes ( $H_{11}^x, H_{11}^y$ ). Thus, if linearly polarized light is launched in such a fibre, the polarization state of the light beam should not change as it propagates. However, practically it is found that the state of polarization changes as light propagates along the fibre and hence the output state of polarization (SOP) is in general arbitrary [1,2]. The change of SOP of the light is caused by many factors, such as slight ellipticity of the core, uneven stress distributions in the fibre when the fibres are manufactured, or bends and twists when the fibre is laid on ground. In an elliptical core fibre, if light that is polarized along the major or minor axis is launched, the output from the fibre

has the same polarization as the input and hence the fibre maintains this polarization [3]. Elliptical core fibres are used in applications where the transmission and delivery of polarized light is required. These include: interferometry [4], fibre optic gyroscopes [5,6], coherent communications, integrated optics [6], Optical Coherence Tomography [7], Laser Doppler Anemometry and Velocimetry [8]. In this present study we developed a 2D-FEM model of an elliptical core optical waveguide. Using the 2D-FEM modal analysis technique, the guided fibre modes are resolved and identified in terms of the obtained mode intensity profiles.

## 2. Analysis Technique and Model

Optical mode in a fibre is a general concept in optics that also occurs in the theory of lasers. Mode analysis in optical fibres can be accomplished more rigorously by solving Maxwell's equations and applying appropriate boundary conditions defined by fibre geometries and parameters [9]. An optical mode refers to a specific solution of the Maxwell's equations that satisfy the proper boundary conditions at the core cladding interface. The mode has the property that its spatial distribution does not change with propagation. The fibre modes can be classified as guided modes, leaky modes, and radiation modes [9-11]. In fibre optic communication systems signal propagation takes place through the guided modes only [10-12].

The optical modal analysis is carried out assuming that the wave propagates along the  $z$ -direction and the electric field of the wave has the form:

$$\mathbf{E}(x, y, z, t) = \mathbf{E}(x, y)e^{j(\omega t - \beta z)} \quad (1)$$

where  $\omega$  is the angular frequency and  $\beta$  is the propagation constant. The fibre guide is assumed to be uniform in the direction of wave propagation. An eigenvalue equation in terms of the electric field can be obtained from the Helmholtz equation:

$$\nabla \times (n^{-2} \nabla \times \mathbf{E}) - k_0^2 \mathbf{E} = \mathbf{0} \quad (2)$$

and is solved for modal effective index,

$n_{eff} = \beta/k_0$  as the eigenvalue. The boundary condition for electric field at the outside of the cladding boundary was set to zero. In the COMSOL Multiphysics [14], however, a module based on the

perpendicular hybrid mode wave using transversal fields is used for finding the modal solutions. To do this, the crosssectional domain of the fibre is meshed with the triangular elements while the FEM is used. The birefringence properties and its structural dependence can be obtained easily from the orthogonal mode solutions. The orthogonal modes propagate with different phase velocities and the difference between their effective refractive indices is called the phase birefringence, given by:

$$B = |n_{eff}^x - n_{eff}^y| \quad (3)$$

The fibre birefringence is of great value serving to de-couple the propagation constants and maintain the polarization. If light is injected into the fibre so that both the orthogonal modes are excited, then one will be delayed in phase relative to the other as they propagate. When this phase difference is an integral multiple of  $2\pi$ , the two modes will beat at this point and the input polarization state will be reproduced [15,16]. The length over which this beating occurs is the fibre beat length given by:

$$L_B = \frac{\lambda_0}{B} \quad (4)$$

The basic structure of a step index elliptical core fibre is shown in Figure 1. The fibre is characterized by these parameters: the semi-major radius  $a$ , the semi minor radius  $b$ , and the core cladding refractive index difference,  $\Delta n$ . Using  $n_{core}$  and  $n_{clad}$  for core refractive index and cladding refractive index, respectively, the normalized frequency is defined by:

$$V = \frac{2\pi b}{\lambda} \sqrt{n_{core}^2 - n_{clad}^2} \quad (5)$$

The normalized birefringence  $\Delta(\frac{B}{k})/(\Delta n)^2$  is dependent on the values of  $V$  and the elliptical ratio [9,10,14].

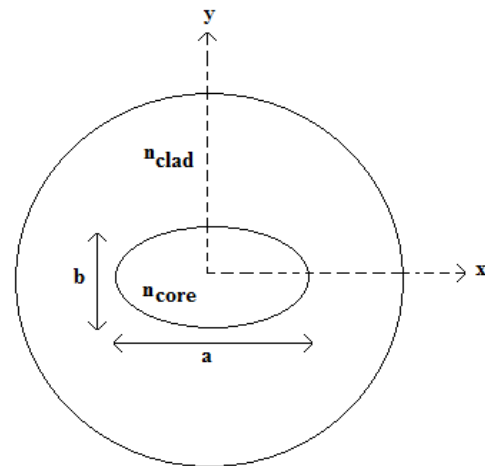


Figure 1. Transverse cross section of an elliptical core fibre

### 3. Modelling and Simulation Procedure

The FEM in COMSOL study employed the RF Module which combines the optics and photonics interfaces. The geometry of the elliptical fibre in Figure 1 was employed. The material properties used in the study are shown in Table 1. Throughout the study, the elliptical parameters  $a$  and  $b$  were fixed at  $6 \mu\text{m}$  and  $2 \mu\text{m}$  respectively. The material properties in Table 1 are valid for the free space wavelength of  $1.55 \mu\text{m}$ . This is the wavelength where the lowest loss is achievable. [11,12,16].

Table 1. Material properties used in the study

	Core	Cadding
Material	Silica Glass	Doped Silica Glass
Refractive Index	$n_{core} = 1.4457$	$n_{clad} = (\text{varied from } 1.4150 \text{ to } 1.4290)$

A modal analysis was performed and the associated parametric sweeps were done in order to investigate the influence of the refractive index difference between core and cladding,  $(\Delta n)$ , on the properties of the elliptical core fibre. The refractive index of the core ( $n_{core}$ ) was fixed at 1.4452 while the cladding refractive index ( $n_{clad}$ ) was varied from 1.4150 to 1.4290 in steps of 0.001. The corresponding  $V$  values, electric field intensities and effective refractive indices were determined for various values of  $\Delta n$ . The standard meshing tool was used with the mesh setting at physics – controlled mesh and element size set to “finer”. Figure 2 shows the meshed geometry of the fibre cross section in 2D. A total of 2924 triangular elements were used in this FEM study.

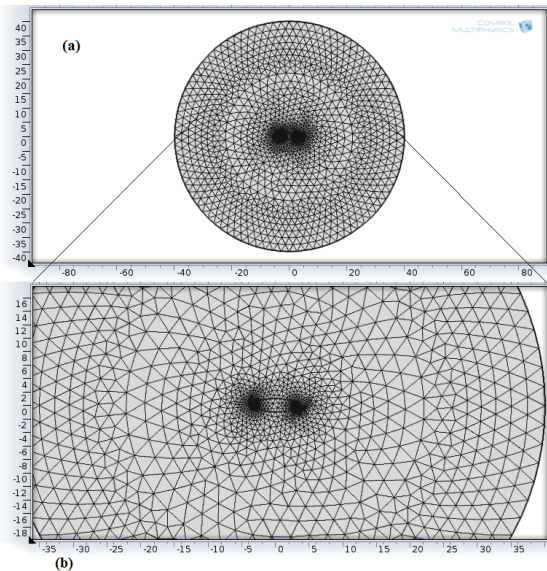


Figure 2. Structure of the triangular finite elements in COMSOL for a single mode step-index fibre

## 4. Results and Discussion

### 3.1 Normalized Birefringence, $V$ and $\Delta n$

The first part of the modal study was a parametric analysis of the effect of  $\Delta n$  on the normalised birefringence for the fundamental mode. Figure 3 shows that an increase in  $\Delta n$  from around 0.016 to 0.03 corresponds to a linear decrease in the normalized birefringence from around 0.0230 to 0.144.

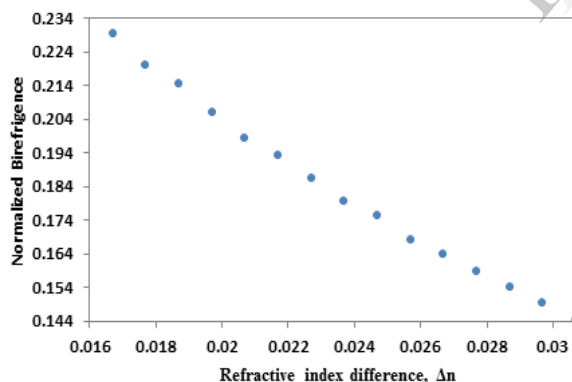


Figure 3. Normalized birefringence as a function of normalized frequency  $V$  for the fundamental mode

Figure 4 shows the dependence of normalized birefringence on the normalized frequency  $V$  for the fundamental mode. Figure 5 shows the dependence of electric field difference (i.e.  $\Delta E = E^x - E^y$ ) on  $\Delta n$  for the fundamental mode. From Figure 4,  $\Delta n$  is accompanied by a general increase in  $\Delta E$ . In other words the difference between the electric field intensity associated with the  $H_{11}^x$  polarisation state and the  $H_{11}^y$  polarisation state increases as  $\Delta n$  is increased. Table 2 summaries results of the dependence of NA,  $V$  parameter and  $\Delta n_{eff}$  on the

refractive index difference  $\Delta n$ . This may be expected since the higher  $\Delta n$  corresponds to a higher value of the  $V$  parameter.

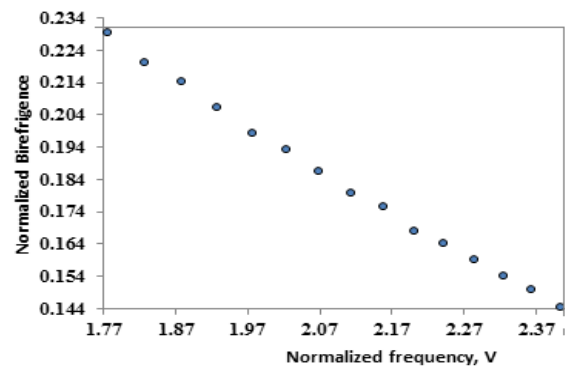


Figure 4. Normalized birefringence as a function of normalized frequency  $V$  for the fundamental mode

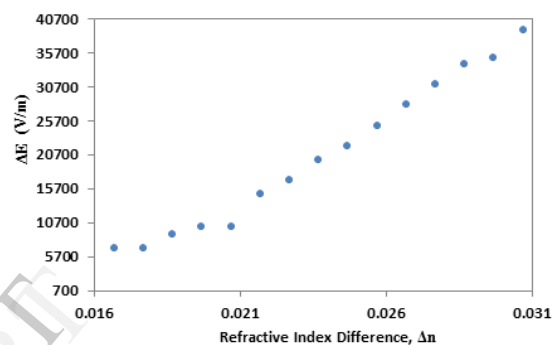


Figure 5. Dependence of electric field difference ( $\Delta E = E^x - E^y$ ) on  $\Delta n$  for the fundamental mode

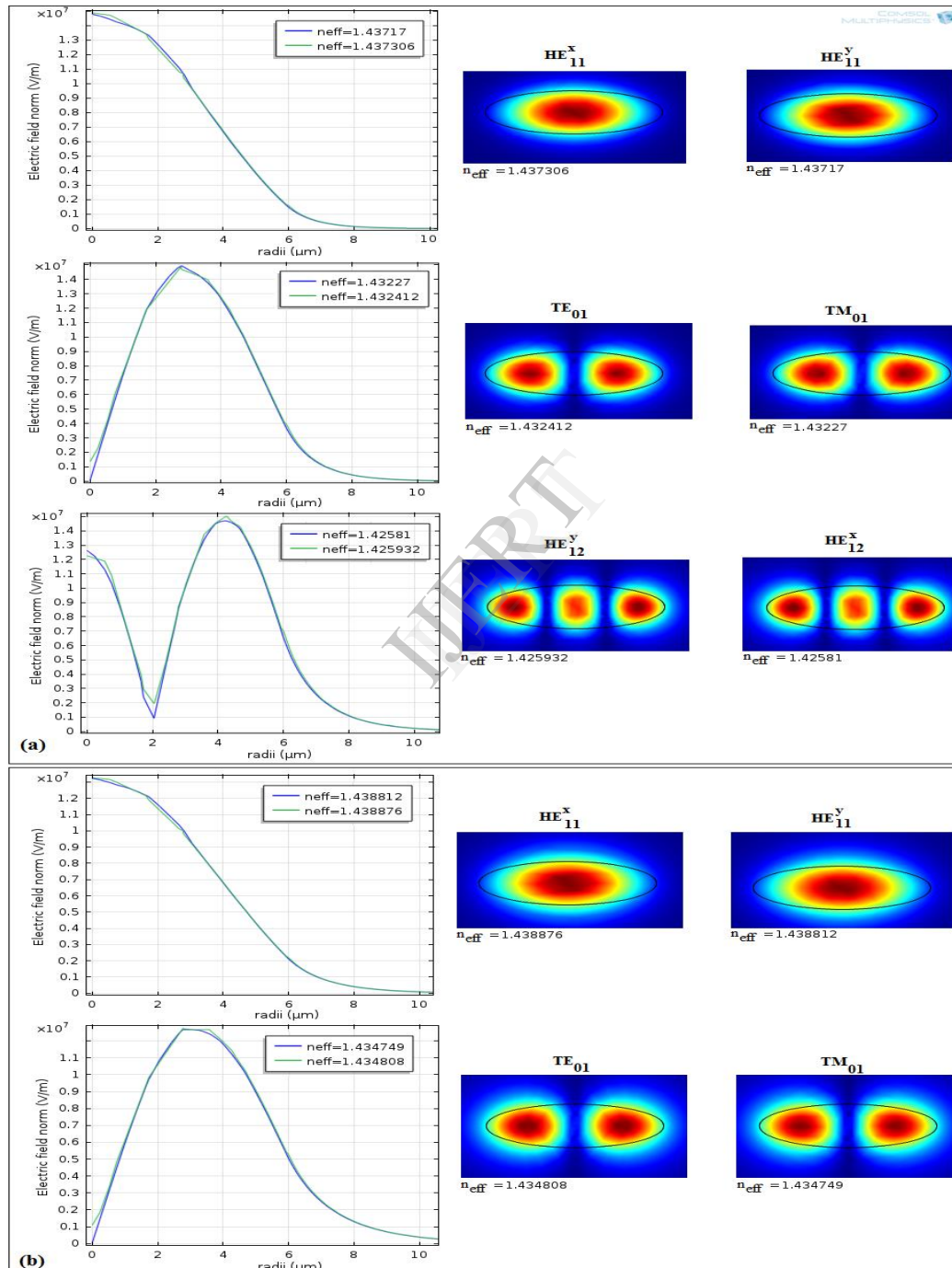
Table 2: Summary of results of the study

$\Delta n$	NA	$V$	$\Delta n_{eff}$	Normalized Birefringence
0.0307	0.29635	2.402611	0.000136	0.144299
0.0297	0.291535	2.36357	0.000132	0.149645
0.0287	0.286635	2.323844	0.000127	0.154184
0.0277	0.281646	2.283399	0.000122	0.159001
0.0267	0.276564	2.242195	0.000117	0.164121
0.0257	0.271383	2.20019	0.000111	0.168057
0.0247	0.266097	2.157336	0.000107	0.175384
0.0237	0.2607	2.113582	0.000101	0.179814
0.0227	0.255185	2.068872	9.6E-05	0.186303
0.0217	0.249545	2.023142	9.1E-05	0.193251
0.0207	0.243769	1.97632	8.5E-05	0.198371
0.0197	0.23785	1.928328	8E-05	0.206138
0.0187	0.231775	1.879075	7.5E-05	0.214476
0.0177	0.225532	1.82846	6.9E-05	0.220243
0.0167	0.219106	1.776367	6.4E-05	0.229481

### 3.2 Analysis of Modes

Figure 6(a) shows the spatial guidingmode fields for  $n_{core} = 1.4457$  and  $n_{clad} = 1.415$  (i.e.  $\Delta n = 0.0307$ ). As shown in Figure 5, there are three pairs of degenerate modes namely: ( $H_{11}^x, H_{11}^y$ ); ( $TE_{01}, TM_{01}$ ) and ( $HE_{12}^x, HE_{12}^y$ ). Figure 6(b) also shows the spatial guiding modes for  $n_{core} = 1.4457$  and  $n_{clad} = 1.429$  (i.e.  $\Delta n = 0.0167$ ). The results in

Figure 6 show that for the value of  $\Delta n = 0.0167$ , there exists only two pairs of degenerate modes: ( $H_{11}^x, H_{11}^y$ ) and ( $TE_{01}, TM_{01}$ ). Comparing Figure 6 (a) and Figure 6 (b), it is apparent that the elliptical core fibre can support more modes at higher values of  $\Delta n$ .



**Figure 6: Calculated mode profiles and effective indices of the elliptical fibre for (a)  $\Delta n = 0.0307$  (b)  $\Delta n = 0.0167$ . The associated E-field plots for the modes are shown to the left**

It has been clearly shown that the guided modes are well confined to the core region of the fibre and presents an obvious ellipse. Figure 6 has also

shown that it is possible to excite the fundamental mode with maximum electric field intensity in the middle of the core. However, two higher order



modes including the second order mode ( $TE_{01}$ ,  $TM_{01}$ ) and third order mode ( $HE_{12}$ ), with a minimum in the middle of the core, evolve as the fundamental mode ( $H_{11}$ ) splits [17,18]. Because of the twofold symmetrical shape of the core, the presence of a form asymmetry in the fibre is supported by the weak birefringence caused by the refractive index difference of the propagating modes [19,20].

#### 4. Conclusion

An elliptical core fibre with elliptical ratio  $a/b = 3$  was successfully analysed using the FEM in COMSOL Multiphysics®. The results confirm that it is possible to give weak phase birefringence due to twofold symmetry in the elliptical fibre structure. This type of specialty fibre is likely to find applications in fibre mode converters as well as mode selective couplers. Compared to traditional analytical methods of analyzing optical waveguides, it has been shown that the FEM computational package, COMSOL Multiphysics® has the ability to model homogeneous elliptical core optical fiber regions with a high resolution and allows the analysis of other parameters such as the electric intensity and mode field distribution across the fiber structure.

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