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Exploring Image Reconstruction with Orthogonal Matching Pursuit and Least Angle Regression

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***Abstract*—Image reconstruction is a process of recovering the required components that constitute an image. Over the last decade and a half, a technique to reconstruct images was developed. This technique, known as Compressed Sensing (CS), requires only sparse measurements while the Nyquist sampling theorem requires all necessary samples to recover a signal. This paper explores two algorithms which solve for a sparse solution, namely, OMP and LARS. It surveys the two procedures by measuring different metrics side-by-side and highlighting the essence of the algorithms.**

***Index Terms*—Image reconstruction, OMP, LARS, Signal Processing**

I. Introduction

The reconstruction of signals is a common goal in the field of signal processing. The Nyquist-Shannon sampling theorem suggests that in order to reconstruct a signal perfectly, it would have to be sampled at a rate that is at least twice its highest frequency component. For signals that are not naturally bandlimited, like images, the temporal resolution will determine the sampling rate. However, in data acquisition systems, due to aliasing, an antialiasing-low-pass filter is traditionally used to bandlimit the signal. Therefore, the Nyquist-Shannon theorem still plays an essential role.

Compressed sensing is a signal processing technique that can reconstruct a signal with fewer measurements than traditionally required. It fundamentally relies on two principles, namely, sparsity and incoherence. [1]

A. Sparsity

To demonstrate sparsity mathematically, consider an unknown vector $x \in \mathbf{R}^n$; such as the n-pixels of an image. Consider Ψ to be a standard basis; such as the fourier basis which can expanded as $\Psi = [\psi_1\ \psi_2 \,..\psi_n]\ x$ can be represented as follows :

$$x = \sum_{i=1}^{n} \Psi_i \Theta_i \tag{1}$$

Where Θ is understood as a sequence of coefficients of x. In order to compress x, the vector Θ_s will have to be sparse. It will majorly have zero value entries.

$$x_s = \sum_{i=1}^{n} \Psi_i \Theta \tag{2}$$

Given that Ψ is an $n \times n$ matrix, the compressible signal x can then be written as:

$$x_s = \Psi \Theta_s \tag{3}$$

B. Incoherence

The latter principle that compressed sensing relies on is incoherence. In the context of incoherent sampling, consider Φ, a sensing matrix $\in \mathbf{R}^{m \times n}$, where $m << n$. Let there exist a sparse matrix Ψ which is incoherent with respect to Φ, as their product will have new samples which aren't found in a standard basis [2]. Instead of recovering x, which will require n requirements, consider a measurement vector $y \in \mathbf{R}^m$.

$$y = \Phi \Psi x \tag{4}$$

From eq. (3) Θ_s is the sparsest possible sequence of coefficients that would make up the image x. The dictionary $C = \Phi\Psi$ substitutes the above equation to become:

$$y = Cx \tag{5}$$

Thus, the above equation is of the form $Ax = b$, wherein, with the information of y and C, one must solve for x. For the two algorithms that we are exploring in this paper, there are a few assumptions. It assumes $C \in \mathbf{R}^m$ is an input matrix and is l_2 normalized. The residual vector $r \in \mathbf{R}^m$ demonstrates the difference between the measurement vector y and the solution vector. A support set $S \in \mathbf{R}^k$ is a set that consists of the indices of the active columns of matrix C.

II. Algorithms for sparse approximation

In this paper we discuss the two algorithms for the reconstruction of a noisy image. Their individual features are highlighted below.

A. Least Angle Regression

Over a recent period of time, attention to the domain of l_1 normalization has increased drastically. It has offered techniques to solve for a sparse solution of underdetermined systems. Donoho and Tsaig [4] proposed that an algorithm named 'Homotopy algorithm' (a modified LARS algorithm), in addition to solving the l_1 minimization problem, can solve for the sparse solutions just as rapidly as OMP/LARS. The name "least angle" came from geometrical interpretation of the LARS process. It chooses the updated direction that makes the smallest and equal angle with all active columns [3].

Algorithm 1 Least Angle Regression

Input: (i) the measurement vector $y \in \mathbf{R}^m$ (ii) the matrix $C \in \mathbf{R}^{m\times n}$ (iii) The threshold for error ϵ

Output: The sparse vector $x \in \mathbf{R}^n$

Initialise :

1) The residual $r_0 = y$
2) The counter value $p = 0$
3) $x_0 = 0$
4) Support set $S = \{\}$

Compute:

1) $p = p + 1$
2) Calculate the correlation vector by $v_p = C^T r_{p-1}$
3) Calculate the absolute maximum value in the vector v_p, $\lambda_p = ||v_p||_\infty$
4) If the value of λ_p happens to be extremely small or even 0, then the algorithm terminates and the values of x_p is returned. Else, the algorithm continues and the following steps are continued.
5) The support set S will then be built using $\{j: v_p(j) = \lambda_p\}$
6) The least squares problem will then be solved, so that the active entries may be found. The updated direction should be considered.
   $C_S^T C_S d_p(S) = sign(v_p(S))$
   where $sign(v_p(S))$ returns the sign of the entries of the correlation vector v_k
7) The inactive entries of the updated direction become 0, $d_p(S^v) = 0$
8) Calulate the minimum step size for λ_p
9) The solution vector is updated to $x_p = x_{p-1} + \lambda_p d_p$
10) Find the new residual vector: $r_p = y - Cx_p$
11) If $\|r_p\|_2 < \epsilon$, the algorithm may be terminated and output $x = x_v$ as the solution vector. If not, increase the iteration k = k +1 and return to computing the correlation vector.

B. Orthogonal Matching Pursuit

In 1993, Mallat and Zhang [5] proposed a sparse approximation algorithm that they named the Matching Pursuit (MP). This algorithm searches for a solution for an underdetermined linear system. The MP algorithm was later modified to the OMP, which uses a least squared formula instead and adds a step of othrogonalization, in order to prevent the algorithm from choosing a column of the matrix C repeatedly [6].

Algorithm 2 Orthogonal Matching Pursuit

Input: (i) the measurement vector $y \in \mathbf{R}^m$ (ii) the matrix $C \in \mathbf{R}^{m\times n}$ (iii)The threshold for error ϵ

Output: The sparse vector $x \in \mathbf{R}^n$ *Initialise* :

1) The residual $r_0 = y$
2) The counter value $p = 0$
3) $x_0 = 0$
4) Support set $S = \{\}$

Compute:

1) $p = p + 1$
2) Calculate the correlation vector $v_p = C^T r_{k-1}$
3) Find the next column of matrix C by using the index obtained from the largest absolute entry of v_p.
   $i = \arg\max_{j\in C^p} |v_p(j)|$
   Where C^p is a set that the excludes values that are in the support set S
4) Add i to $S_p = S_{p-1} \cup \{i\}$
5) Solve the least square problem: $C_S^T C_S x_p(S) = C_S^T y$
6) Calculate the residual vector $r_p = y - Cx_p$
7) If the residual vector $r \geq \epsilon$, the algorithm will go back to the first step. Else, the algorithm will terminate and return $x = x_p$

III. Experiment and simulation

We conducted an experiment by using a noisy image from a dataset and measure the performance of both algorithms on it.

A. Conditions

The Smartphone Image Denoising Dataset (SIDD) contains images representing 160 scenes. They are presented in pairs of noisy and ground truth. This dataset has pictures which were shot on the iPhone 7, LG G4, Google Pixel and Samsung Galaxy S6. For this experiment, patches of size (7,7) were extracted from a sample image. The dictionary for this algorithm was learnt in batches, and the comparison of the two algorithms is measured through two parameters. These parameters are Peak Signal-to-Noise Ratio (PSNR) and Structural Similarity Index (SSIM).

B. Results

The measured PSNR values for both methods are shown in Table I. For the LARS algorithm, the values do not increase drastically and centre around 28 dB. The OMP algorithm increases in PSNR with the increase of the non-zero coefficients. The OMP algorithm displayed higher values of PSNR. The SSIM is a perception-based metric that calculates image quality degradation.

As shown in Table II, the values of SSIM increase upwards with the increase in of non-zero coefficients. It is worth noting that OMP reaches a value that is extremely close to 1. From fig. 1, there are upward trends of both the algorithms. At no

TABLE I
PSNR VALUES FOR OMP AND LARS

PSNR (dB)		Non-zero coefficients (s)
OMP	LARS	
28.44	25.70	2
30.58	25.76	4
33.72	27.51	6
37.10	28.93	8
38.11	28.54	10
38.90	28.91	12
41.18	28.28	14

point do these values overlap, although a slight spike at 8 non-zero coefficients is noticeable.

TABLE II
SSIM VALUES FOR OMP AND LARS

SSIM		Non-zero coefficients (s)
OMP	LARS	
0.882	0.749	2
0.932	0.791	4
0.964	0.839	6
0.985	0.865	8
0.986	0.869	10
0.991	0.876	12
0.994	0.877	14

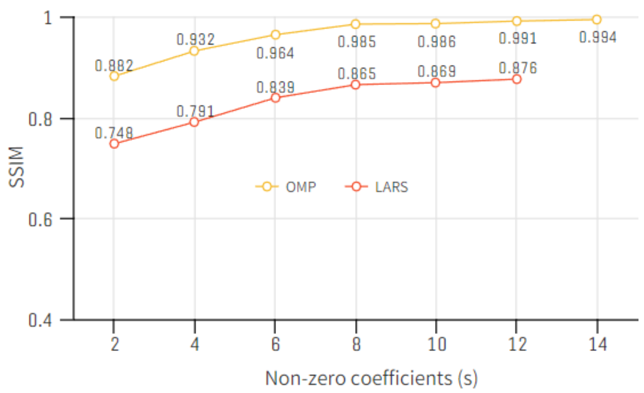

Fig. 1. Graph of SSIM vs. non-zero coefficients for LARS and OMP

IV. CONCLUSION

This paper explores two sparse approximation algorithms by testing them on an image reconstruction experiment. The gathered results as shown above were for a given noisy image, so that we could obtain a general insight into the metrics of OMP and LARS. The time taken to compute the calculation for OMP is far lesser than for LARS. This is because there are many additional steps that LARS executes, that OMP does not have. OMP updates the largest possible entries such that the values in the support set are orthogonal to residual r. As for LARS, the solution coefficients are updated to the smallest value, which will result in a column to join the support set or be dropped from it. For further study, it would be

Fig. 2. Original noisy image from SIDD

Fig. 3. Reconstruction with OMP and 4 non-zero coefficients

Fig. 4. Reconstruction with LARS and 4 non-zero coefficients

encouraged to analyse the performance of OMP and LARS in other circumstances where values of C are highly correlated.

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