Experimental and FE Modal Analysis for Elevated Steel Water Tanks

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Abstract-Liquid containing elevated tanks are critical and strategic structures. They are considered one of those sensitive structures affected by dynamic loads. In order to study the behavior of elevated cylindrical water tanks caused by dynamic loading, experimental modal analysis (EMA) and finite element method (FEM) were undertaken. The dynamic parameters (frequencies and mode shapes) of scaled models were experimentally determined. Using the FEM, the structural properties of the shell wall with their supporting columns and the liquid properties can be included. The main goal of the current research is the verification of previous analytical approaches that were used to obtain modal parameters of water tanks as a basic step to study the behavior of these structures under seismic loads. Due to the complex nature of theoretical approaches especially when considering the dynamic nature of structure, ANSYS finite element software was used. Housner method was adopted to represent the dynamic behavior of water elevated tank subjected to horizontal base excitation. Two cases of tanks were studied, and their validated 3D models showed quite good agreement with the experimental modal results. The analytical approach efficiently simulated the dynamic behavior of all tanks in the current study.

Index Terms-Circular tanks, Elevated tanks, Earthquakes, Finite element, Operational modal analysis, Output – only response, Dynamic testing.

1. Introduction

Elevated water tanks are subjected to several kinds of loading such as their own weight, liquid weight, wind loads and earthquake loads. Of all of these, earthquake load can be considered as the most dangerous one. It causes sloshing of a liquid in the tank. This sloshing causes high loads on the walls. This leads to high compressive stresses on tank structural components. Earthquakes can also damage tanks by causing potential buckling to the upper course of the tank [1].

Various numerical models have been developed on water tanks to express their seismic behavior. These models considered the effect of sloshing of the contained fluid. Housner [1] studied the behavior of a ground-supported tank under horizontal acceleration. In his model, the whole tank is considered as one frame where water is simulated by two masses. The first mass is called impulsive mass which is directly attached to the tank wall. The second mass is called convective mass which is attached to the tank wall by a linear spring.Housner [2] concluded 2D model to represent elevated liquid tank under horizontal acceleration. In his model, the contained liquid is simulated as Housner [1]. The tank wall, base and supporting frame are represented by frame element with their equivalent stiffness and masses.

Veletsos and Younan [3 and 4] studied the effect of the wall flexibility and rigidity; respectively for solid containing cylindrical tanks under horizontal base shaking. Haroun and Housner [5] developed a model of a ground-supported tank that takes into account the flexibility of the tank walls. Haroun and Housner [6] developed a numerical model for ground-supported cylindrical tanks. In their model, the liquid is treated analytically and only the tank wall is modeled by finite elements. The reliability of the analysis is illustrated by computing modes and natural frequencies of full-scale tanks and comparing them with the results of vibration tests [7].

Haroun and Tayel [8 and 9] studied the effect of vertical acceleration on cylindrical tank. They concluded that, liquid exerts axisymmetric hydrodynamic pressure on tank wall due to the influence of vertical excitation. After that Rizk [10] developed a FORTRANprogram to study the liquid cylindrical ground supported tank under the vertical component of the earthquake. The computer program was based on the finite element method formulation.

El-Hosiny et al. [11] developed a mathematical model to simulate ground-supported tanks using ANSYS program. In his model, the fluid element and the interface bottom element were used to detect the sloshing effect and the uplift of the base. The results obtained by the finite element model were compared with those obtained by Housner [1] model. His results showed that their finite element model gives accurate results compared with Housner [1] model.El-Rakabawyet al. [12] andAboul-Ella et al [13] studied elevated water tanks under the earthquake loads by the finite element method and these tanks were solved by ANSYS program. Moslemia et al [14] modeled the fluid domain using displacement-based fluid elements. Both time history and modal analyses are performed on an elevated tank.

Several techniques to obtain dynamic parameters of structures from experimental measurements have been developed in the last decades. In recent years there has been a considerable development at the level of the equipment for measurement of the dynamic response of structures. Sensors are having a higher sensitivity and data acquisition systems a better resolution, both allowing to measure adequately structural responses with very small amplitudes. Accurate estimations of modal parameters (natural frequencies, mode shapes and damping) can be obtained from the measured response of structures to their service loads (wind, traffic, etc.). Operational Modal Analysis (OMA) and some advanced identification techniques have been used to obtain dynamic properties of different civil structures [15 and 16]. Wide range of applications is based on experimental modal analysis such as modal updating, structure modification and structural health monitoring [17].

In 2005, El-Damatty et al [18] reported the first experimental study conducted on a small-scale combined liquid-filled conical shell model. Shake table testing (input-output test) was conducted to determine their dynamic parameters. Results of the experiments were used to validate the assumptions employed in a previously developed analytical model for the free surface sloshing motion. Also in 2010, Wu et al [19] and Curadelli et al [20] determined the dynamic parameters of fluid tanks using classic modal analysis. Furthermore Bayraktar et al [21] tested steel storage tanks under natural excitations to obtain their dynamic characteristics (natural frequencies, mode shapes and damping ratios), experimentally**.** Also they developed a finite element model by ANSYS software. In their model, Fluid element was selected to present the contained fluid. A good agreement was found between their experimental and theoretical results.

In 2011, Amiri and Sabbagh-Yazdi [22] determined the dynamic parameters for full scale fixed roof ground supporting tank from ambient vibration test. In this test, output only response data were experimentally measured, where the tank was naturally excited by micro-seismic and surrounding wind load. It is quite difficult to excite such large tank with artificial excitation methods.

In the current paper, output-only response dynamic test using operational modal analysis was carried out to estimate the experimental modal parameters (frequencies and mode shapes) of two different steel tanks. For each tank, the partially filled cases were studied. FEM results using ANSYS program were presented.In these models, the contained water was represented as Housner model [1]. Comparisons between the results of both the experimental work and FEM analysis showed good agreement. This emphasizes the efficiency of the analytical approach to simulate the dynamic behavior of partially filled elevated water tanks.

2. Structural Properties of Tanks

Two different scaled models of elevated cylindrical steel water tanks were chosen in this study to identify their modal parameters. These dynamic properties were estimated for each tank in the partially filled case. The tank tower consisted of four columns. Each column is formed from steel pipe with height 800 mm. Their outer diameter is 26.7 mm and their thickness is 2.87 mm. The four columns were welded at their base by steel plate 400 mm x 400 mm with thickness 2.00 mm. This plate was fixed on concrete footing through thirteen screw anchors. The footing dimension is 100 cm x 100 cm and its thickness 20 cm. The heights of the tank vessels are 30 cm and 20 cm, their dimensions are shown in table 1. The material properties of the tanks vessels and their columns are illustrated in table 2. It should be noted that bracing systems was constructed to join the columns at the beginning of the test and the test was carried out. The measurements displacement showed that the tested tank is constrained so that the bracing was removed and the tests were repeated.

3. Dynamic Test

3.1. Measurement Procedure

Ambient vibration tests were conducted on tanks using the Brüel&Kjær PULSE Multi-analyzer system (Type 3560). The mounted sensors (Fig.1) are 5 uni-axial piezoelectric accelerometers Brüel&Kjær Type 4507 B04 with a sensitivity of 100 mV/ms-2. Eight testing setups allowed measuring the response of different degrees of freedom of the steel vessel and the supporting columns. Fig. 2 shows one setup where two accelerometers as reference channels were placed in two main orthogonal directions at the top circumference of vessel's wall, while roving the other sensors on both the lower and bottom circumferences of the cylindrical vessel. Measurements on the columns were taken in the same directions of the reference channels (X & Y). For each test, the ambient acceleration-time histories were recorded for 320 sec, so that the time windows acquired is larger than 1000-2000 times the period of the structure's fundamental mode [23] for all tank cases as indicated from pretest model. The sample rate was 1.6 KHz as adjusted in PULSE providing good waveform definition. The height of water for the partially filled case was 200 mm and 150 mm for tanks 1 and 2; respectively. The tanks were excited by random impact hammering at the base in horizontal directions (X &Y) alternately.

Table 1. Geometric properties of tested tanks

Geometric properties	Tank (1)	Tank (2)
Diameter of the vessel, mm	300	300
Height of the vessel, mm	300	200
Height of the tower, mm	800	800
Wall thickness, mm	1.5	1.5
Base thickness, mm	3	3

Table 2. Material properties of tested tanks

Material properties	Value
Modulus of elasticity	210 GPa
Unit weight	78.1 KPa
Poisson's ratio	0.3

3.2.Operational Modal Analysis (OMA)

Also known as Output-Only Modal Analysis, has in the recent years, been used for determination of modal parameters of several civil engineering structures. The advanced signal processing tools used in OMA technique allow to determine the inherent properties of a structure (Resonance Frequencies, Damping, Mode Shapes), by measuring only the response of the structure, without using an artificial excitation. In this paper, the non-parametric technique based Frequency Domain Decomposition (FDD) was the modal identification method as applied in ARTeMISExtarctor V5.3 [24].

Assuming that the unmeasured input is white noise, and that the system is lightly damped with limited modal coupling, the power spectral density of the response signals **y** can be expressed as follows:

$$G_{yy}(j\omega)=\sum_{k=1}^{N}\frac{d_k\phi_k\phi_k^T}{j\omega-\lambda_k}+\frac{\bar{d}_k\bar{\phi}_k\bar{\phi}_k^T}{j\omega-\bar{\lambda}_k} \quad ....(1)$$

Where dk is a scalar constant, ωis the angular frequency,λ_k is the pole location (i.e. resonance frequency and damping) for the k_{th} mode, and ϕ_k is the corresponding mode shape [25].The identification algorithm first estimates the PSD $\hat{G}_{yy}(j\omega)$, known at discrete frequencies $\omega=\omega_i$, then applies the singular value decomposition (SVD) of the PSD matrix:

$$\hat{G}_{yy}(j\omega_i)=U_iS_iU_i^H \quad ...... (2)$$

where,

$U_i=[u_{i1},u_{i2},....,u_{im}]$is a unitary matrix, of the singular vectors u_{ij}

S_i is a diagonal matrix of the scalar singular values s_{ij}

Near a peak corresponding to a mode, the first singular vector u_{ij} is an estimate of the mode shape: $\hat{\phi}=u_{ij}$, and the corresponding singular value is the auto power spectral function of the corresponding single degree of freedom system. By peak-picking, applied to the first singular values, the significant modes for both cases were identified in the frequency range from 12 Hz to 174 Hz. The following sections describe the modal identification results for the studied cases.

As expected, due to the relative flexibility of steel tanks, the system exhibits many mode shapes, some are of very close frequencies and of similar pattern of vibration. To help distinguish these mode shapes, initial FE model was used to determine the target modes to be considered in the current study. Seven specific mode shapes were chosen as illustrated in the following sections.

The identification algorithm FDD produced many frequency peaks as seen in Fig. 3. This is expected as the steel tank structure is flexible and prone to vibration under its base excitation. The dynamic behavior is characterized by very close modes for the first sway modes; however the identification technique efficiently allowed the detection of all targeted global modes of the tank. The first modes are characterized as swaying modes of the whole tank in two orthogonal directions at 12.0 Hz and 12.75 Hz. Torsion mode was identified at 43 Hz. Higher modes are more related to the circumferential modes along the walls of the tank vessel as shown at 58 Hz, 96 Hz, and 110.5 Hz. Oval shaped modes are obtained at higher frequencies as shown for mode No 7. Very low frequency peaks were captured by AVT at the range 1-1.9 Hz, this can be explained as representation of the sloshing effect of water measured by the high sensitive sensors mounted on the vessel wall. Seven estimated global modes of interest that are chosen for correlation of FEM are shown in Fig. 4.

Figure 1. The mounted reference accelerometers, roving sensors and random hammer impact at the base

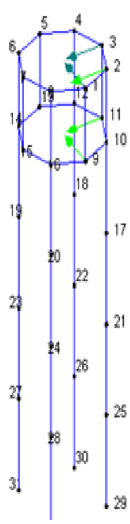
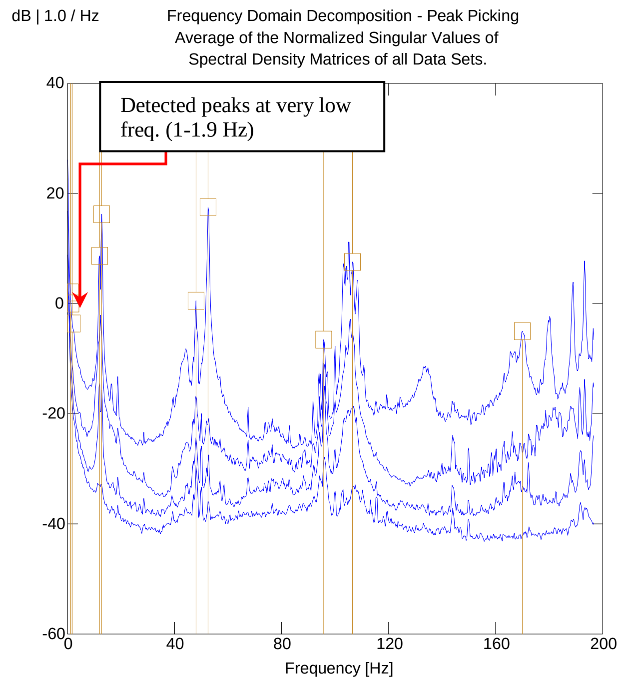
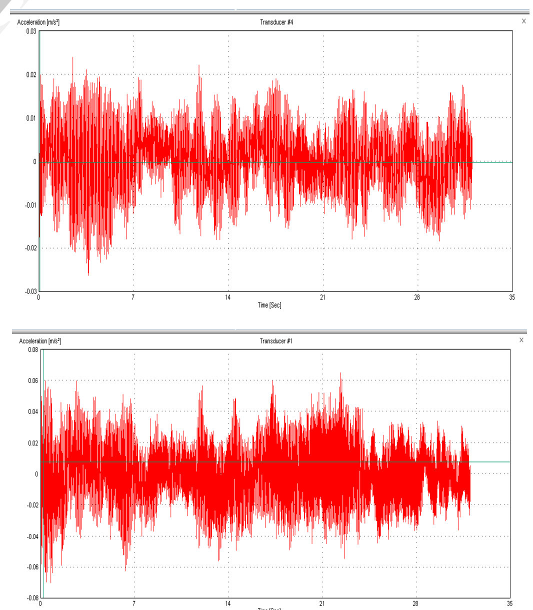


Figure 2. Measurement points on the tank



a) Average of the normalized singular values of data sets for partially filled tank (1)



b) Typical measured time histories in both direction X & Y

Figure 3. Experimental ambient vibration results of partially filled tank (1)

The modal results for partially filled case of tank (2) produced similar dynamic characteristics as for tank (1). Although many frequency peaks are produced form the measurements.Very low frequency peaks were captured by AVT at the range

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1-1.8 Hz, this represents the effect of water sloshing. Only seven global modes were chosen for correlation with FEM results. The first close sway modes of the whole tanks are detected at 15 Hz and 15.8 Hz. The third mode shape is torsion mode at 45 Hz. The circumferential (radial modes of the vesselwall) appeared at frequency peaks 75.5 Hz, 115 Hz, and 155 Hz. The seventh mode was identified at 174 Hz. It is noticed that the value of frequencies for tank (2)increased comparable to the modal frequencies of tank (1). This clearly illustrated that the mass influential of water in the taller tanks is larger than it stiffness participation. This influence was less in the shorter vessel due to the lesser height of fluid in the partially filled case. All modal shapes for tank (2) are identified in the range 15 Hz-174 Hz as shown in Fig. 6.

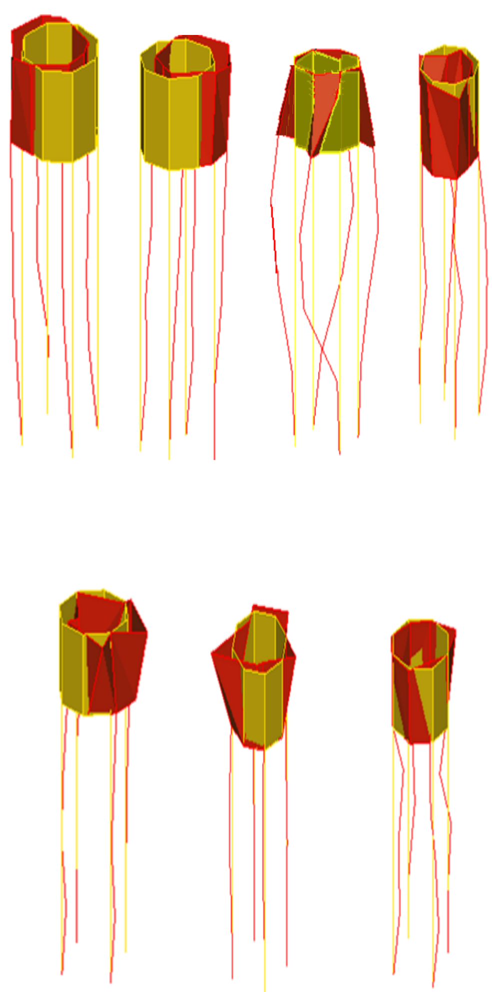
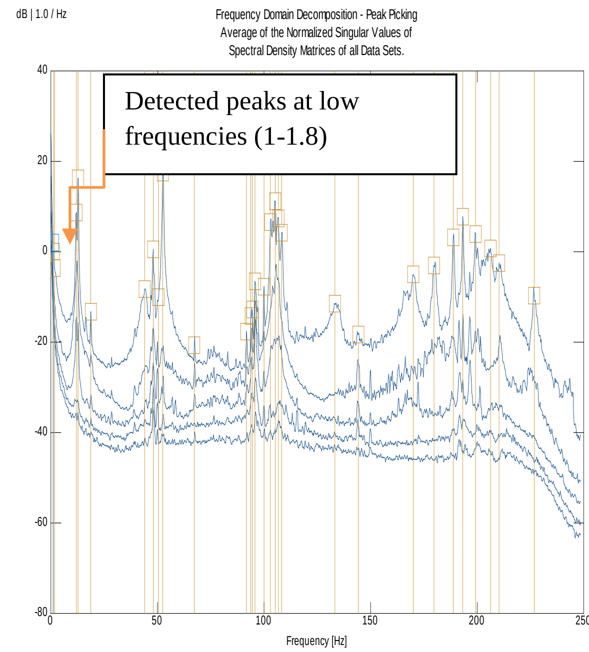


Figure 4. Experimentally identified mode shapes of partially filled tank (1)



a) Average of the normalized singular values of data sets for partially filled tank (2)

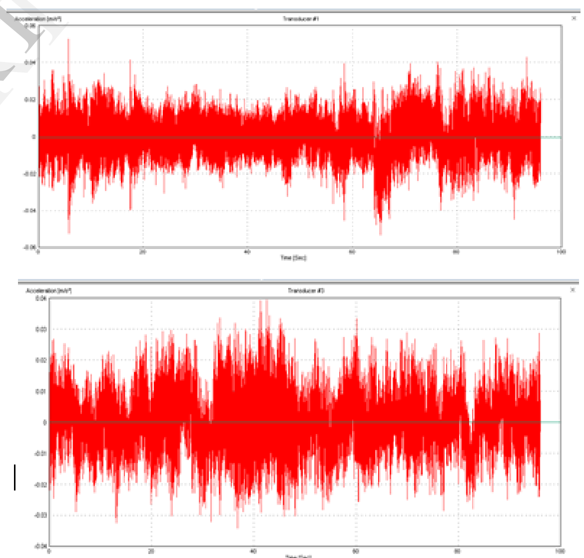


Figure 5. Experimental ambient vibration results of partially filled tank (2)

4. Theoretical Model

FEM was done using ANSYS program [26] for the two tanks. In these models, the vessel of the tank was formulated by Shell63 element. This element is a Four node quadrilateral element which have both bending and membrane capabilities. The tank columns were defined by Beam4 element. This element has six degree of freedom at each node. It is a unit-axial element with tension, compression, torsion, and bending capabilities.

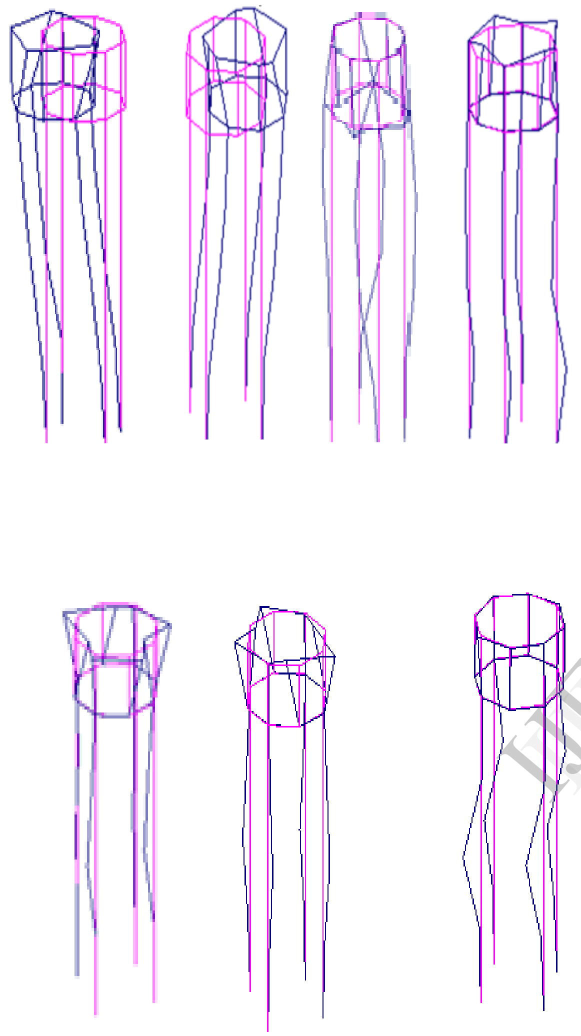


Fig. 6: Experimentally Identified mode shapes of partially filled tank (2)

4.1. Fluid model.

A satisfactory spring mass model to characterize basic dynamics for two mass model of elevated tank was proposed by Housner [1]. This model is commonly used in most of the international codes after the Chilean earthquake of 1960. During lateral mode of shaking of the water tank, an upper part of the water moves in a long period sloshing motion, while the rest part moves rigidly with the tank wall. The former one is recognized as convective mass (m_c) of water which exerts convective hydrodynamic pressure on tank wall and base, while the latter part is known as impulsive mass (m_o) of water which accelerates along with the wall and induces impulsive hydrodynamic pressure on tank wall. The impulsive mass (m_o) of water experiences the same acceleration as the tank container and contributes predominantly to the base shear and overturning moment. The convective mass m_c is attached to the tank wall by a linear spring of stiffness k_c. The impulsive and convective masses are located at distances h_o and $h_{c;}$ respectively from the bottom of the tank.

Housner [1] assumed that the tank walls are rigid, the liquid is incompressible and its displacement is small and the tank is open, with vertical side walls and horizontal bottom that is symmetrical with respect to the vertical X-Z and Y-Z plans to develop the following equations. These equations describe the derivation of the fluid masses, convective spring stiffness k_c and their corresponding heights for a cylindrical tank. Also he concluded equation (8) to calculate the period that represents the water movement (water sloshing). Their calculated values for partially filled tanks are illustrated in table 3. In the current work, the contained water in the two tanks was represented by two-mass model as shown in Fig. 7. Mass21 element was used to represent the two masses; the impulsive and convective masses and Link8 element was represent the linear spring. Link8 element has three degrees of freedom at each node (translations in the nodal x, y, and z directions).The theoretical models of tank (1) and tank (2) are shown in Fig. 8 and Fig. 9; respectively.

$$m_o = m \frac{tanh\left(\sqrt{3}\frac{R}{h}\right)}{\sqrt{3}\frac{R}{h}} \ldots\ldots (3)$$

$$m_c = m\left[0.318\frac{R}{h} tanh\left(1.84\frac{h}{R}\right)\right] \ldots\ldots (4)$$

$$h_o = \frac{h}{8}\left[\frac{4}{\frac{tan\,h\left(\sqrt{3}\frac{R}{h}\right)}{\sqrt{3}\frac{R}{h}}} - 1\right] \ldots\ldots (5)$$

$$h_c = h\left[1 - \frac{cosh\left(1.84\frac{h}{R}\right) - 2.01}{1.84\frac{h}{R} sinh\left(1.84\frac{h}{R}\right)}\right] \ldots\ldots (6)$$

$$K_c = mg\left\{\frac{0.58512}{h}\left[tanh\left(1.84\frac{h}{R}\right)\right]^2\right\} \ldots..(7)$$

$$T_w = 2\pi\sqrt{\frac{m_c}{K_c}} \ldots..(8)$$

Where:-g = acceleration due to gravity, m= the total masses of water, R= radius of the tank vessel and h= contained water height.

Table 3. Theoretical model parameters for the partially filled tanks

Parameter		Tank (1)	Tank (2)
Material Properties	m, $Kg.sec^2/m$	1.4	1.06
	m_o, $Kg.sec^2/m$	0.946	0.575
	m_c, $Kg.sec^2/m$	0.41	0.3205
Geometric Properties	h, mm	200	150
	h_o, mm	126	119.5
	h_c, mm	146	117.7
	K_c, Pa	570	374
Water mode (water sloshing)	T_w, Sec	0.533	1.876
	F_w, Hz	0.581	1.72

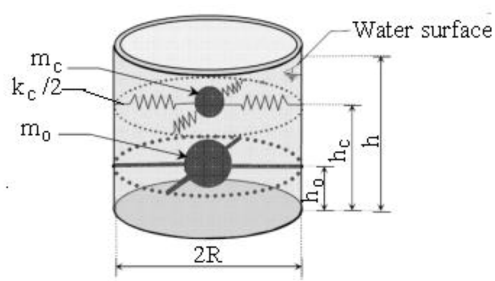


Figure 7. Equivalent spring –mass model of fluid inside the cylindrical tank

5. Validation of Theoretical FEM

The FEM for tank (1) gives first mode shape at frequency 1.8 Hz and tank (2) gives the first mode shape at frequency 1.73 Hz. It represents the tank water sloshing. The frequencyat the first mode shape for the two tanks is closed with the obtained frequencies from theexperimental test and Housner [1] simulation. Seven global modes are presented in Fig. 10 for tank (1) as obtained from the developed theoretical model. The comparison between the value of the frequencies from both of the theoretical model and the dynamic test are illustrated in table 4. Also seven global modes for tank (2) are presented in Fig. 11 and the comparison between the theoretical and experimental results are presented in table 5.

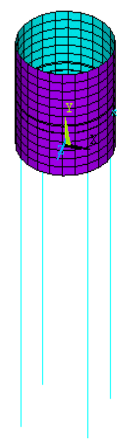


Figure 8. 3D FE model of partially filled tank (1)

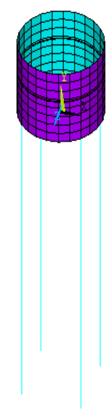


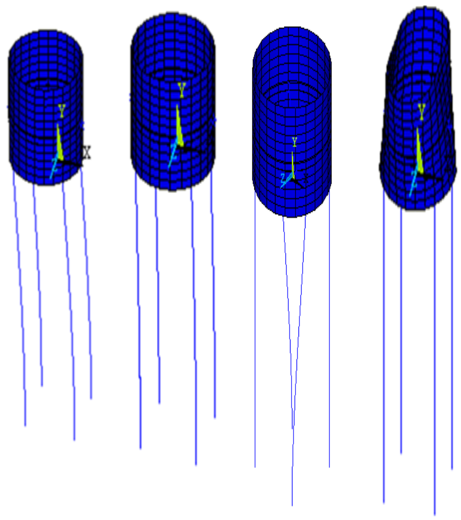
Figure 8. 3D FE model of partially filled tank (2)

Table 4: Comparison between experimental and theoretical frequencies for tank (1)

No. of mode	Experimental modal analysis Freq. (Hz)	Theoretical model Freq. (Hz)	Diff %
1	12.00	12.08	0.67
2	12.75	13.31	4.34
3	43.00	43.50	1.16
4	58.00	60.01	3.47
5	96.00	91.36	-4.83
6	110.5	112.61	1.91
7	167.0	169.1	1.26

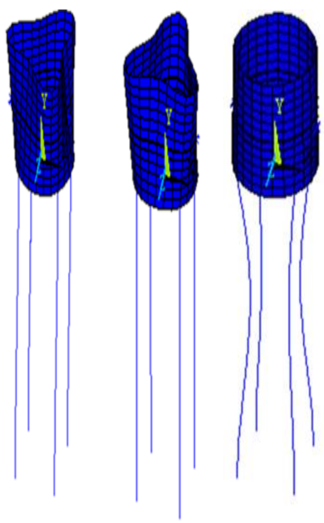
Table 5: Comparison between experimental and theoretical frequencies for tank (2)

No. of mode	Experimental modal analysis Freq. (Hz)	Theoretical model Freq. (Hz)	Diff %
1	15	14.50	-3.33
2	15.8	15.40	-2.53
3	47.6	45.00	-5.46
4	75.5	74.45	-1.39
5	115	110.0	-4.35
6	155	150.4	-2.96
7	174	170.5	-2.01



Mode#1 Mode#2 Mode # 3 Mode # 4

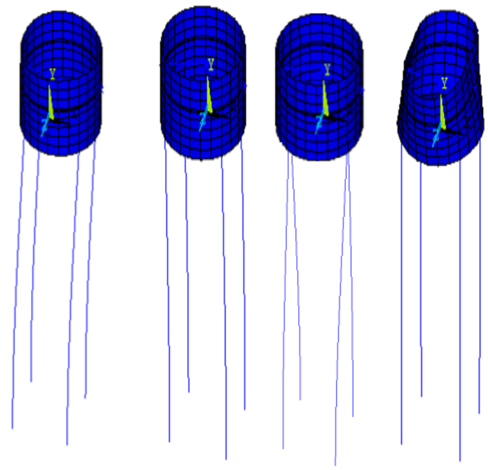
12.08 Hz 13.31 Hz 43.50 Hz 60.01 Hz



Mode#5 Mode#6 Mode#7

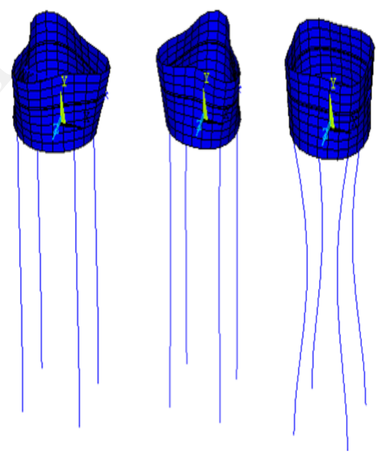
91.36 Hz 112.61 Hz 169.1 Hz

The FE modal analysis successfully produced all the target modes that are experimentally identified using AVT. The differences between mathematical model results and experimental data show quite good agreement, and the validity of the FE model is ascertained. As can be seen from tables 4 &5, the measured and simulated values differ by less than 5% for all modes except for the third mode of tank 2, is slightly higher giving difference of 5.46%. These variations are considered to be minor. Thus, the numerical model is representative of the actual structure in reliable way. For both cases the Housner model shown to be excellent approximation of the fluid behavior that allows simulating the dynamic behavior of the partially filled tanks.



Mode#1 Mode#2 Mode#3 Mode#4

14.50 Hz 15.40 Hz 45 Hz 74.45 Hz



Mode#5 Mode#6 Mode#7

110 Hz 150.42 Hz 170.5 Hz

Figure 11. Theoretical mode shape for tank (2)

6. Conclusion

The current research aims to verify the previous analytical approaches proposed by Housner that were used to obtain modal parameters of water tanks as a basic step to study the behavior of these structures under seismic loads.The dynamic behavior of two different cylindrical steel elevated tanks was reliably revealed using both AVT and FEM techniques. Ambient vibration test is shown to be convenient and sufficient method to identify the most significant modes of the considered liquid storage tanks. The sloshing modes of water at the very low frequency range were identified from AVT. Orientation of the acceleration transducers allowed the capture of both sway and circumferential modes in the range 12-174 Hz successfully. The Frequency Domain Decomposition, FDD technique, which is a non-parametric technique, provided very accurate results for natural frequencies and mode shapes. The developed three-dimensional finite element models using the Housner model reflect the real conditions of the tank-liquid systems and achieved good agreement with the measured modal parameters identified from dynamic measurements. Therefore, the constructed models can serve as a baseline model in structural dynamics for future analyses to be representative of the real structures.

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