International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 3 Issue 12, December-2014

Experimental Analysis on Single-Mode Fiber Based on LP_{01}-LP_{11} ModesCoupling for WDM Systems

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***Abstract*—Wavelength Division Multiplexing (WDM) has attracted considerable attention in recent years.Thistechnique can be implemented to not only design ultra-high capacity fiber-optic communicationsystems, but also upgrade the existing systems. In the present work is a general description of the theory that originates the linearly polarized in a fiber optic modes. Also, experimental analysis of linearly polarized modes (LP_{01} and LP_{11}) is carried out in a single-mode step-index fiber.Which is obtained by performing a manual mechanical adjustment in the proposed arrangements and also the corresponding simulations were carried out.**

***Keywords*— Single-Mode Fiber, Mode Coupling, WDM.**

I. INTRODUCTION

In the case of the optical fiber is a wave equation cylindrical, with boundary conditions laid down by the core and cladding, which describes the propagation of electromagnetic field within the dielectric fiber material, showing the different characteristic and linearly polarized propagation modes, indicating the proportion of the contribution of the electrical and magnetic in the disruption of the optics inside fields of fiber [1].

A WDM system requires a device to combine the optical channels at the transmitting end andanother to separate different channels from each other at the receiving end. The WDM device atthe transmitting side is essentially a power combiner and is referred to as a multiplexer. Thedevice at the receiver side is called a demultiplexer and should ideally separate out variouschannels with negligible insertion loss and signal distortion. Both multiplexer and demultiplexerare referred to as WDM device in the references [2-3].

Demultiplexers are usually more difficult to implement than multiplexers and thus require a morethorough design. Multiplexers in most applications are wideband directional couplers, whiledemultiplexers are essentially spectral filters that separate out different optical channels. They canbe designed and implemented utilizing a variety of devices including wavelength selectivewaveguide couplers, Bragggratings, Fabry-Perot interferometers, Mach Zehnder interferometers, and Acousto-optical filters [4-5]. By another hand, at the time of applying an electromagnetic theory to the phenomenon of the light guide, the approaches become a mess. Although the fact of introducing light with one angle less than the numerical aperture does not guarantee that such light is guide, is also required to form on the inside of waveguide field whose transverse component to be stationary distribution. The different ways in which such a condition can be achieved (in last term, a series of mathematical solutions of a wave equation) are called modes. We can say that a mode is a unique “electromagnetic field pattern” that travels through the fiber [6].Any field within the fiber distribution can be expressed as a combination of modes. The two modes guided of low order in a guide wave cylindrical such as fiber optics are LP_{01} and LP_{11} modes.

The transverse refractive index profiles of many optical fibers are radially symmetric, i.e., the refractive index depends only on the radial coordinate *r* and not on the azimuthal coordinate φ. Also, the index profiles of nearly all fibers exhibit only a small index contrast, so that the fiber can be assumed to be only weakly guiding. In this situation, the calculation of the fiber modes is greatly simplified. One obtains the linearly polarized LP modes [7].Most fiber based spectral filters investigated in the past consist of two single-mode fibers. As a wavelength filter, the LP_{01}-LP_{11} mode coupler offers much narrower spectral width than a coupler whose constituent fibers are single-mode. The proposed coupler may also be used as a mode converter for dispersion compensation application. Using the coupled-mode theory of parallel waveguides, a set of differential equations governing the coupling of LP_{01} and LP_{11} modes in the proposed coupler is derived in [8].

Various dispersion phenomena in optical fibers and the application of the proposed coupler as a mode converter for compensation of chromatic dispersion are discussed in [9]. Numerical results for transmission characteristics of the coupler and for the dispersion of the $LP_{01}$and LP_{11} mode are presented in [10].

II. THEORETICAL BACKGROUND

All guided modes have β values which lie between the plane-wave wavenumbers of the core and the cladding. Modes with β values close to the lower limit (the cladding wavenumber) have a small ω parameter, leading to a slow decay of the radial function in the cladding.One may calculate the effective refractive index of a fiber as its β value divided bythe vacuum wavenumber. For guided modes, that effective index lies between the refractive indices of core and cladding.

The lowest-order mode LP_{01} has an intensity profile which is similar to that of a Gaussian beam, particularly in cases with not too high *V*-number. Particularly for the higher m values, the resulting radial functions can oscillate in the fiber core, whereas it decays more or less rapidly in the cladding.Figure 1 shows the radial functions for an experimental analysis case. Here, we have two modes with l = 0 (LP_{01}, LP_{02}) to SM600single-mode fiber and one mode each for *l* = 1 and *l* = 2.

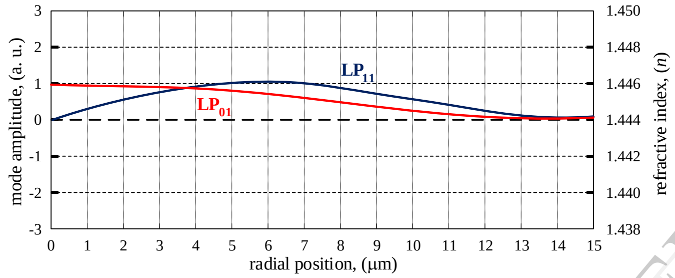

Figure 1. Radial functions of the fiber modes for a SM600 step-index fiber [11].

The refractive-index differences Δ of practical fiber is of the order of 1%. Moreover index differences of the most important single-mode fibers for optical communication are about Δ≅ 0.3-0.8%. Then we can approximate as $n_1/n_0 \cong 1$. This approximation allows us to simplify the analysis of fiber based on the approximation with $n_1/n_0 \cong 1$. Afterwards such mode groups with this approximation were designated LP modes. Since an LP mode is derived by the approximation of $n_1/n_0 \cong 1$, it means that the light confinement in the core is not so tight. Therefore this approximation is called weakly guiding approximation [12].

Table 1. Compares the relation between LP modes and conventional modes. Three kinds of mode corresponding to m=1, TE_{0l}, TM_{0l} and HE_{2l} satisfy approximately the same dispersion equation and can be expressed in the unified form:

$$\frac{J_m(u)}{uJ_{m-1}(u)} = -\frac{K_m(w)}{wK_{m-1}(w)} \quad (1)$$

Table 1. Comparison of LP modes with conventional modes [13].

LP mode (l ≥ 1)	Conventional mode (l ≥1)	Dispersion equation
LP0*l* *mode (m=0)*	HE_{1l} *mode* TE_{0l} *mode*	$\frac{J_0(u)}{uJ_1(u)} = \frac{K_0(w)}{wK_1(w)}$
LP_{1l} *mode* $(m = 1)$	TM_{0l} *mode* HE_{2l} *mode*	$\frac{J_1(u)}{uJ_0(u)} = -\frac{K_1(w)}{wK_0(w)}$
LP_{ml} *mode* $(m \geq 2)$	$EH_{m-1,l}$ *mode* $EH_{m+1,l}$ *mode*	$\frac{J_m(u)}{uJ_{m-1}(u)} = -\frac{K_m(w)}{wK_{m-1}(w)}$

The light propagation condition for an optical fiber is determined by the *V*-number or normalized frequency. The fiber that is designed to allow only the fundamental mode (LP_{01}) to propagate at the required wavelength is called the single mode fiber. When the *V*-number is less than 2.405, only the LP_{01} mode, can propagate through the fiber core. If the wavelength λ of the source is reduced sufficiently, the fiber can accommodate more optical modes as *V* exceeds 2.405. Therefore, the fiber has become multimode. Hence the cut-off wavelength $\lambda_c$above which the fiber becomes single mode is given by the normalized frequency $V_c$as:

$$Vc = \frac{2\pi a}{\lambda c} NA = 2.405 \qquad (2)$$

Where:

$$NA = \text{numerical aperture} = \sqrt{n_1^2 - n_2^2} \qquad (3)$$

a is a radius of the fiber core,λc is the free space cut-offwavelength, n_1 and n_2 are the refractive indices of the coreand cladding, respectively.The radiation losses caused by periodic microbendingcan be calculated by computing the power-couplingcoefficient of the lowest-order mode to higher mode and byassuming that the higher-order mode does not carry anypower [14].The radiation loss from the SM600 fiber bymicrobending is considered by assuming that two modescan coexist in the fiber, but that only the lowest-order modecarries power. This mode is coupled to the higher-orderleaky mode as well as to the radiation modes.

Within the mode groups such as $LP_{01}^{(p)}$, $LP_{11}^{(p)}$, and $LP_{21}^{(p)}$, two types of degenerate modes sharing the same propagation constant can be recognized as even and odd modes as defined by their parities in angular dependence, see Figure 2.

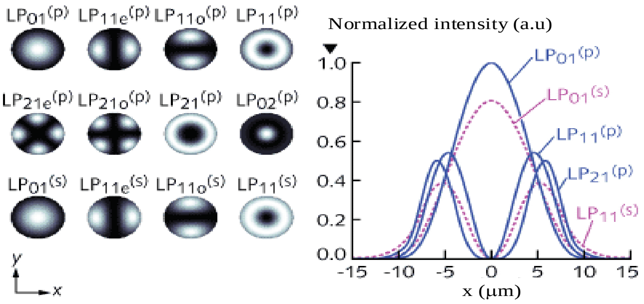

Figure 2. The intensity profile of pump and signal modes are shown for a few-mode fiber amplifier (left), along with their normalized intensity profiles as viewed along the x-axis (right) [15].

III. METHODOLOGY AND TARGETS

The methodology proposed is of theoretical and experimental class where is carried out a review on the State of the art and continues to perform the necessary settings, it also makes use of the equipment needed to perform experiments, this methodology is described in the diagram of Figure 3.

Is pertinent to clarify that the methodology does not describe the simulation process since it is not part of the experimental analysis of linearly polarized modes LP or not interfere with the results of such analysis, described step by step simply practical events in the laboratory using the materials and equipment described in the next section.

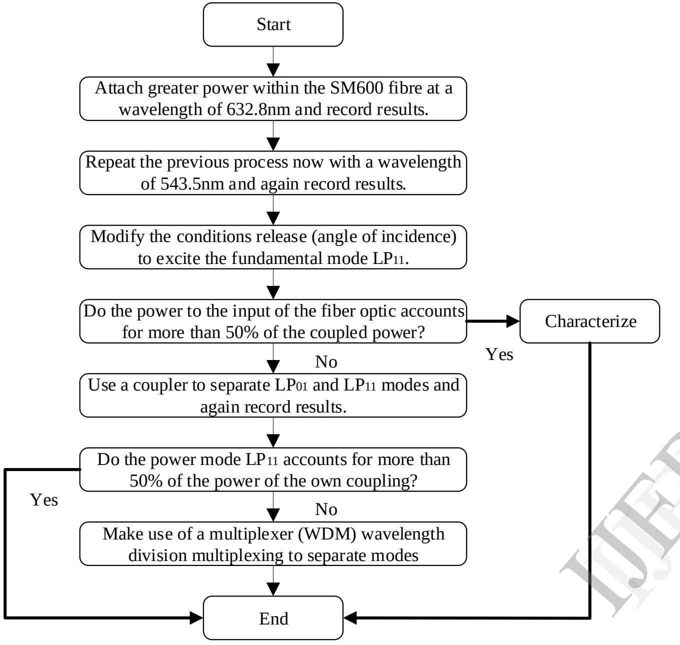

Figure 3. Methodology proposed for the experimental analysis of linearly polarized LP modes.

The targets of this work are basically four and are described below:

1. Separate a fiber mode LP$_{11}$ SM600 single mode at a wavelength of 543nm.
2. Confine in single-mode fiber nearly70% of the light from the laser.
3. Filter fiber mode LP$_{01}$.
4. See how LP$_{11}$mode out of fiber with more than 40% of the total power confined.

IV. MATERIALS AND EQUIPMENT

Melles Griot™ to 632.8nm (red) Helium-Neon (HeNe) Laser with powers from 0.8 mW to 22.5 Mw, (1 Laser).
Melles Griot™ to 543.58nm (green) Helium-Neon (HeNe) Laserwith powers from 0.3 mW to 3 mW, (1 Laser).
SM600 - Single Mode Optical Fiber, 600nm, Ø125 µm Cladding, (3 meters).
ThorLabs™ CMTSMA-SMA Fiber Adapter Plate with C-Mount (1.00"-32) Thread, (1 pieces).
The 3-axis (simplex and duplex) fiber optic mount and fiber optic holders are useful in many applications for alignment of fibers, (2 pieces).
Polymicro Technologies™ Fiber Optic Assemblies, (2 pieces).
Newport™ Microscope Objective Lens, 20x, 0.40NA, 9.0mm Focal Length, (1 piece).
Anritsu™ Optical Power Meter Mod. ML9001A, (1 piece)
ThorLabs™ Mod. VOA50, variable optical attenuators, single mode, 50dB attenuation to 600-1500nm, (1 piece).
ThorLabs™ Linear Film Polarizers 400-1630nm windows, (1 piece).
ThorLabs™ Quarz-Wedge Glass, (2 pieces).
JDSU™ WaveReady MDC Coupler600nm, 1310nm, and 1550nm transmission systems.

V. EXPERIMENTAL PROCEDURE

Before making use of fiber optics is necessary to implement three important steps to prepare it, which consist of: remove using a clip of 3 or 4cm polymer that covers to the finish on the end of the optical fiber, then item is cleaned with ethyl alcohol discovered fiber, and finally; with the help of the cutter is a small portion of the fiber without coating of plastic about 2 cm.

Care must be taken not to touch the fiber after wiping with alcohol since this would affect the transmitted signal. On the other hand, it is necessary to be very careful to cut it because of the importance that has the plastic film covering of the fiber, so that it becomes very fragile and retrieved cross-section can create any rift with any blow to the tip of the fiber caused any careless.

Finally place the end of the fiber optic in one of the capillaries, leaving only a few millimeters (2mm) to the end of the capillary fibre.

With optical fiber prepared, is set up the arrangement shown in Figure 4, the laser output is placed the microscope lens to focus the light from the source to the input of fiber held by a capillary. At the other end of the fiber is placed an optical power sensor for measuring output.

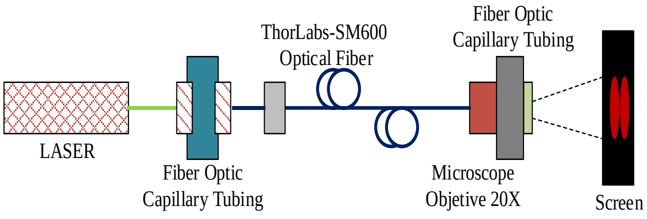

Figure 4. Setup used for imaging the low-order modes supported by SM600 optical fiber.

The first test was a He-Ne Laser of 632.8nm with a maximum power of 4mW, a fiber optic singlemode SM600 of which it is only possible to observe the fundamental mode LP01 to wavelength emitted by the laser. We sought input from the fiber end is aligned on the focal point of the objective of the microscope, which observed a number of light more confined in the fiber at a distance of 3.5mm about the objective of the microscope to the fiber, in such a way is to obtain the highest measurement by the detector with small measurement approaches, the radiation pattern of the fiber was observed on a screen situated front of the final end of the fiber optic capillary tubing.

Then seeks to maximize the optical power coupled, moving positioners (*X*, *Y*, *Z* displacements) mounts until the brightness on the screen was the maximum. The corresponding measurements. A picture of the distribution of intensity over an opaque screen (glass grinding and attenuator), was made so that he could get from this image, a graphic three-dimensional mode LP_{01} full profile, as shown in the Figure 5. Reconstruction of thedisplayed profile was obtained with help of MATLAB programming software.

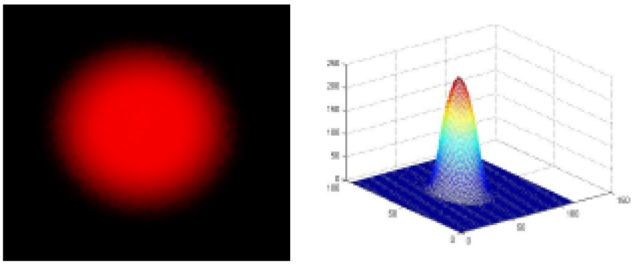
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Figure 5. a) Fundamental mode LP_{01}. b) Simulation of the fundamental mode in the conditions described above.

Modes excitation is performed manually, using a mount with *X*,*Y*,*Z* travel, which allows you to vary the angle of incidence of the light inside the kernel to the interior of the fiber. For this wavelength, fiber behaves as a single mode. It is important to mention that it was attached to fiber nearly 50% (1.9mW) of the total output power of the laser (4mW).

We repeat the previous process, using a laser He-Ne to 543.5nm of wavelength with a maximum power of 1.9mW, in this case we can observe the same fiber behaves like a once multimode fiber that spreads radiation with a wavelength below which was manufactured, see Figure 6.In certain moments the modes are superimposed and the radiation pattern is distorted.

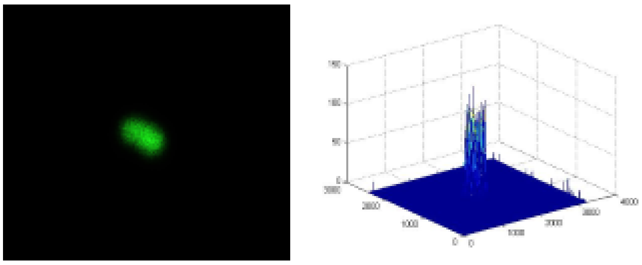
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Figure 6. a) Fundamental modes LP_{01} and LP_{11} superimposed. b) Simulation of the fundamental modes in the conditions described above.

The found maximum was 1.125mW where there are overlapping modes. It is difficult to separate both modes, especially these that are of low order, given that there are more confined to the core. Now, manipulating the optical fiber, i.e. without moving the Positioners of the mounts, only making circles in the fiber is manages to filter LP_{11} mode as shown in Figure 7. Finally, in theFigure 8 shows the result of the filtering that is done again and gets the LP_{11} mode. Obtaining a power of 650mW with loops of 3 cm in radius. Is then manages to filter mode LP_{01} reconfiguring the positioners in *X*, *Y*, *Z*; this form is recorded a capacity of 630mW and again this image is simulated and presented.Despite the fact that the expected results are obtained, the system is unstable and sensitive.

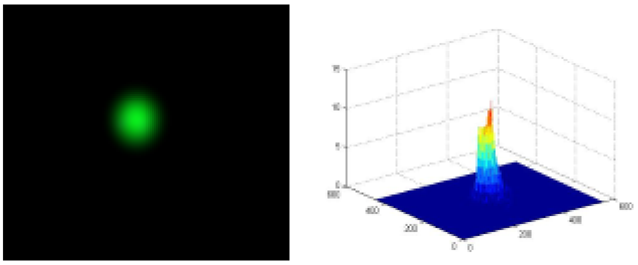
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Figure 7. a) Fundamental modes LP_{01}. b) Simulation of the fundamental modein the conditions described above.

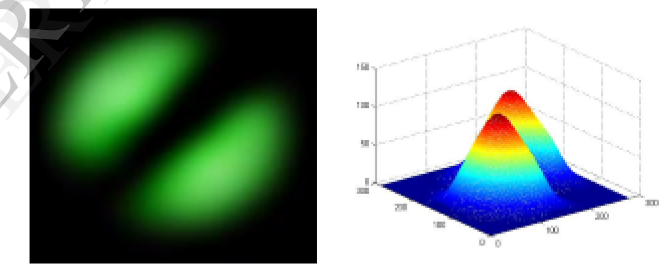
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Figure 8. a) Result of the filtering: LP_{11} mode. b) Simulation of the fundamental mode LP_{11} in the conditions described above.

We repeat the experiment by modifying the array of input, “angle of incidence of the laser to fiber”the obtained results are shown in Figure9.

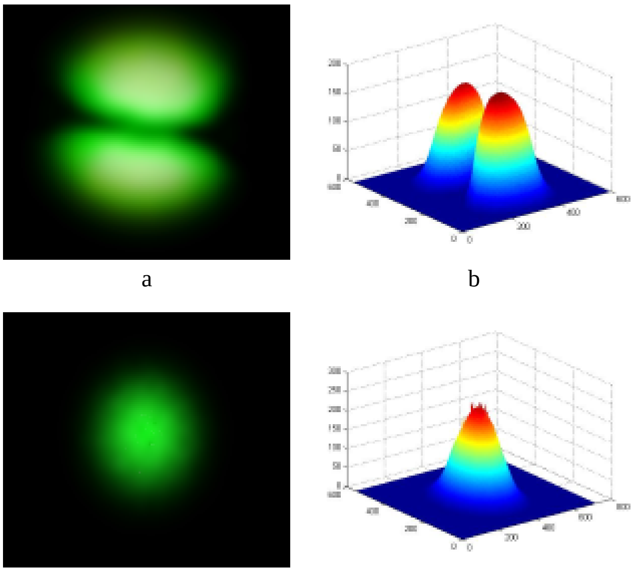
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Figure 9. a) For this mode is achieved a power of 0.58mW. b)Simulation of the fundamental mode LP_{11}.c) making loops is achieved an output of 1.12mW with a radius of 15mm. d) Simulation of the fundamental mode LP_{01}.

An efficient way of separating LP_{01} and a fiber LP_{11} modes is through the use of a coupler, reason by which it opts for try this scheme, see Figure 10.

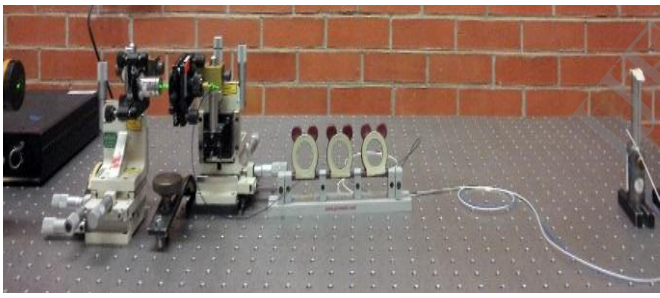

Figure 10. Proposed outline: Laser, mounts, fiber optic, polarizer and coupling capillary.

It proposes the use of a driver of polarization and a coupling of 650nm 50-50, see Figure 11. However it appears that the fiber coupler behaves like monomodal, then proceed to splicing a fiber SM600 is bimodal to a wavelengthof 543.5nm, and exit again observe that only supports a mode.

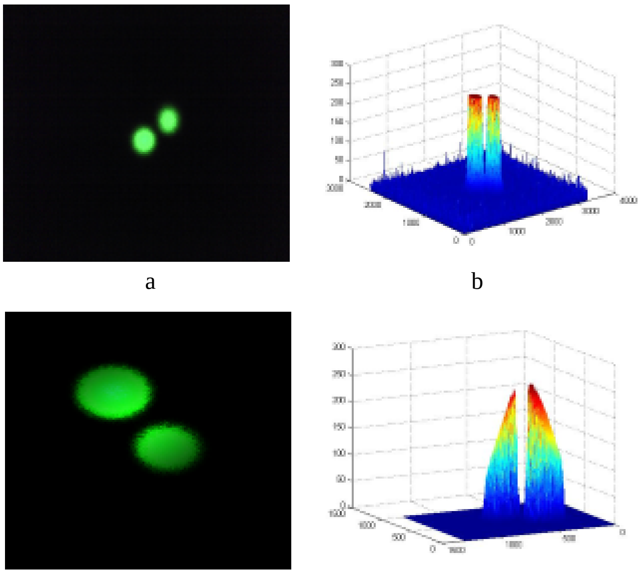
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Figure 11. a) Two LP_{11} fundamental modes out of the coupling before the joint. b) Simulation of the fundamental mode LP_{11}. c) Two LP_{11} fundamental modes out of the coupling after joint. d) Simulation of the fundamental mode LP_{11}.

We repeating the experiment using other broadband coupler, this 1x1Fixed Fiber-to-Fiber Coupler, 1550 nm, SM600, FC/APC has a 10-90 (3 dB) coupling ratio. It has bare fiber ends, see Figure 12.

Figure 12. Broadband fiber coupler proposed array

It is to change the coupler 1550nm, 10-90. We observed the combination of four different modes LP_{01}, LP_{02}, LP_{11}, and LP_{21}. See Figure 13. It is important to highlight the figure c, in which we observe the initially desired modes LP_{01}=1.5mW, LP_{11}=3.6mW. Although they separate LP_{01} and LP_{11} modes power is not expected.

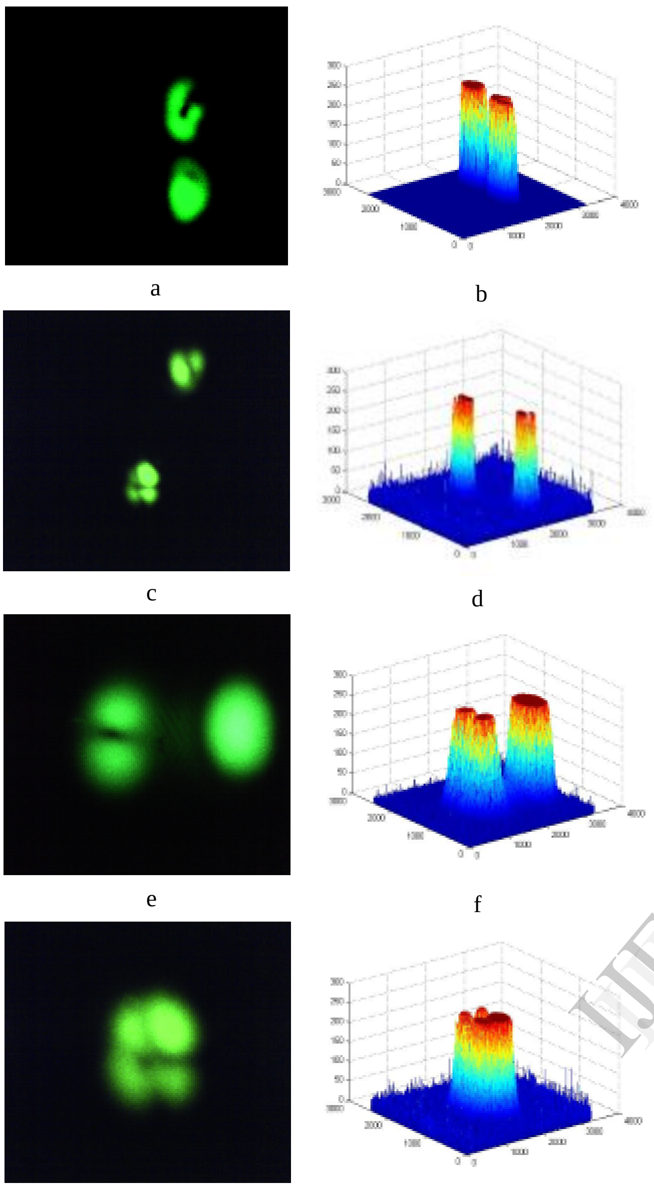

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Figure 13. a) Fundamental mode LP_{11}. b) Simulation of the LP_{11} fundamental mode. c) The initially desired modes LP_{01} and LP_{11}. d)Simulation of the LP_{11} fundamental mode. e) Both fundamental modes LP_{01} and LP_{11}. f) Simulation of the LP_{01} and LP_{11} modes. g) Fundamental mode LP_{21}, and h) Simulation of the LP_{21} fundamental mode.

The previous experiment allowed us to separate the fundamental modes LP_{01} and LP_{11}, however; the power in the system is low, and this is due to the relationship of coupling that is 10-90, for this reason is decides to change this coupling on the other operating at a wavelength of 1310nm with a ratio of 50-50, see Figure 14.

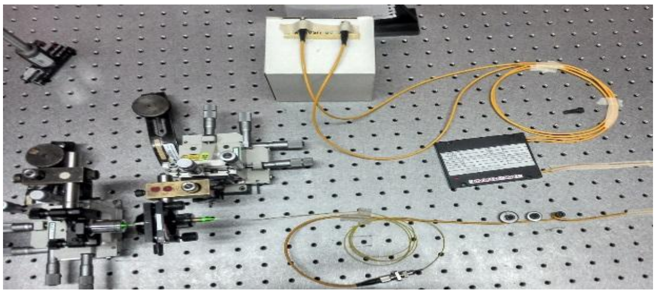

Figure 14. Arrangement proposed with the coupler at 1310nm

It is important to mention that the inclusion of light in this type of couplers is not easy due to the settings on both connectors, why use a pigtail, and a special coupling is made, see Figure 15.

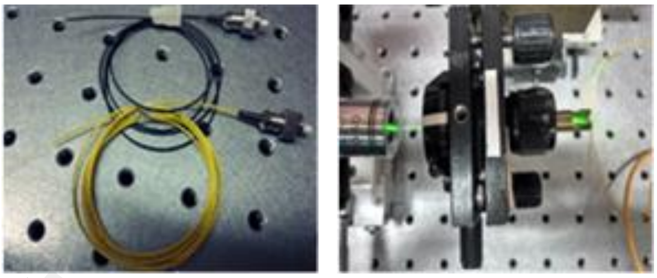

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Figure 15. a) The FC Fiber optic pigtails, and b) special coupling for FC fiber optic pigtail.

At the first exit of the coupler LP_{01} and LP_{11} modes are observed, at the second exit remaining LP modes. The transmitted power was 0.3mW for the first and 0.8mW for the second, which is displayed in Figure 16.

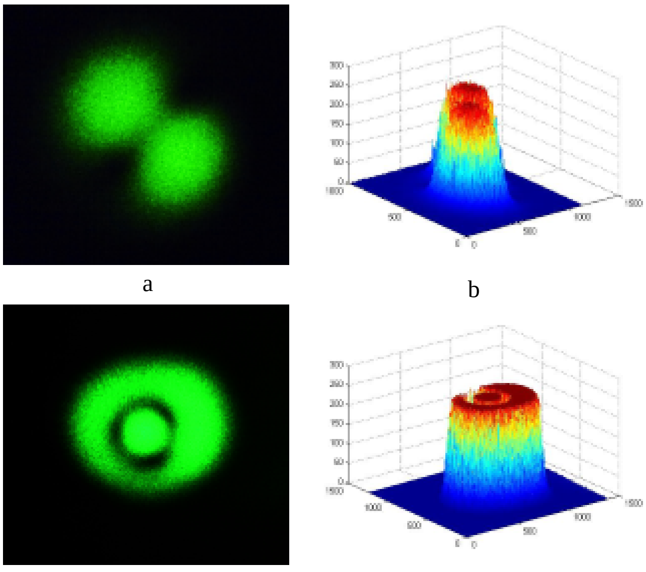

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Figure 16. a) and c) There are fundamental modes LP_{01} and LP_{11} fiber coupler output. b) and d) Simulation of the LP_{01} and LP_{11} fundamental modes.

Following with the technique and incorporating a coupler at 1310nm it is interesting to add to the array using fiber optic of low-birefringence LB1300 at the entrance of the coupling, since this type of fiber keeps the polarization and thus the stability of the system, see Figure 17.

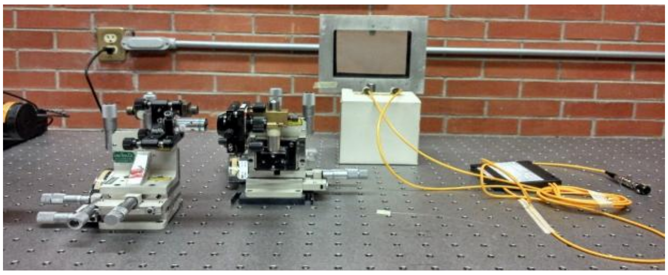

Figure 17. Low-birefringence single mode optical fiber coupling array.

The system is more stable, but there are more combinations of modes, which does not allow us to distinguish one mode other, the results of this configuration are shown in Figure 18.

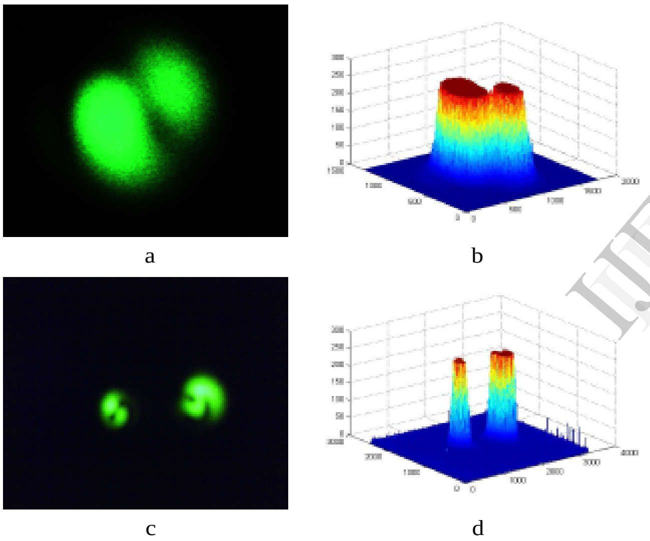

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Figure 18. Fundamental mode LP_{11} with using fiber optic of low-birefringence LB1300. b) Simulation of the LP_{11} fundamental mode using fiber optic of low-birefringence LB1300.

Although the above results are interesting do not meet the objectives posed, it is suggested the use of a multiplexer by wavelength density to separate LP_{01} and LP_{11} modes.1310/1550nm fused WDM can be used to combine or split 1310nm and 1550nm optical signals, to double the fibre transmission capability and ensure bi-direction communication in a single fiber, see Figure 19. It is widely used for fibre communication systems upgrade to expand the system capacity and is based on thin-film technology a thermal platform for optical device. This is used to combine or separate 1310 and 1550nm band signals. This device offers very flat and wide passband, low insertion loss, and high isolation, which make it ideal for optical amplification systems and WDM network applications.

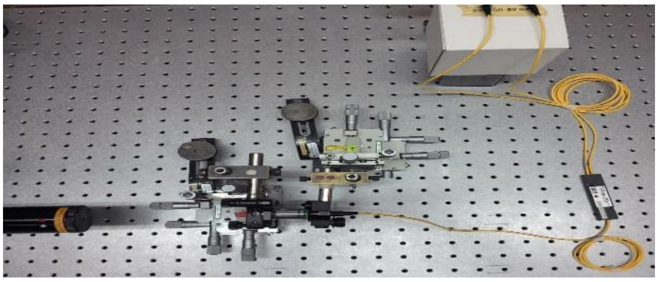

Figure 19. Multiplexer for wavelength, an output density operating at 1310nm and 1550nm one.

The first results that threw the experiment.

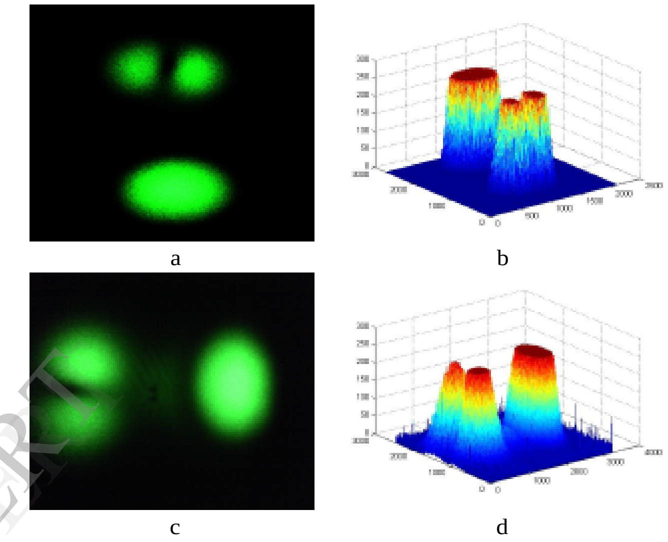

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Figure 20. a)and c) After searching the optimal launch conditions are achieved to separate modes LP_{01} and LP_{11}, resulting in a stable system to external disturbances. We have 1310nm output mode LP_{11} with a power of 0.5mW and 1550nm output have LP_{01} mode with 0.7mW, respectively. b) and d) Simulation of the LP01 and LP02 modes resulting in a stable system to external disturbances.

VI. RESULTS AND DISCUSSIONS

The first technique for the separation of LP_{01} and LP_{11} modes was to carry out micro curvatures with fiber SM600 and although he is achieved by attaching a percentage remarkably in each one of the ways the system is sensitive to any disturbance, which provokes a mixture of the two modes. It is important to highlight that mode LP_{01} never disappears.

In the second technique involves the use of a coupler with its different variants, the first is the use of a coupler 650nm with a ratio of 50-50, but the manufacturer does not specify the exact value of the normalized cut-off frequency, and therefore at the time of test it was identified that a length of 543.5nm the coupling supports single-mode fiber.

The second variant is to use a coupler to 1550nm to a relationship 10-90, this with the purpose of mitigating the power coupled mode LP_{01}. The coupler supports 4 modes: LP_{11}, LP_{21}, LP_{01}, LP_{02}, and although clearly they can only visualize LP_{01} and LP_{11} modes the remaining two appear distorted and mixed with each other. Important to note that contrary to what had been raised initially the fundamental mode manages to attach a power of 1.5mW and LP_{11}only mode at 36mW.

The third variant is to use a coupler to 1310nm 50-50 relationship. This type of coupling comes connectand protected with plastic coatings which hinders the task of inclusion of light and the same manipulation. Therefore the use of a pigtail to the inclusion of light are used. Now, in one of the outputs of the coupling are observed LP_{01} and LP_{11} modes and the other output modes LP_{21} and LP_{02} with powers ranging from the 0.3mW to the 0.8mW.

Given that the previous systems were unstable is the third variant which consists of the use of low birefringence LB1300 fiber and the same coupling, this in order to maintain the polarization and to avoid that the system is so sensitive to external shocks. And as predicted the system is more stable and less sensitive although coupler outputs we see the combination of the four modes therefore cannot visualize accurately any of them with the exception of the LP_{11} mode whose power is 80mW.

The latest variant is the use of the previous coupling more fiber SM600 (formerly used to observe only the first two linearly polarized modes) in one of the exits. This type of arrangement allows to keep LP_{01} mode while in the other output the three remaining modes are excited. It is important to note that the power in both outputs varies around the 130mW. The third technique consists in the use of a WDM, a departure is at 1310nm and 1550nm. Is enough to correctly align so that you only excite LP_{01} and LP_{11} modes with a power for the first of 0.7mW and 0.5mW for the second. Taking all this as reference, we can conclude that the use of wavelength division multiplexer is the most efficient technique for separating modes LP_{01} and LP_{11}, to be stable, not very sensitive to external shocks, provides images clear for each of the modes and most importantly considerable power in each.

VII. CONCLUSIONS

The theoretical and experimental behavior of light in conventional fibers is essential and important to the understanding of complex phenomena. A single-mode fiber designed to operate at certain wavelength can behave like a fiber multimodal if it spreads radiation with a wavelength below which was designed.

The lowest-order modeLP_{01} is more rooted to the nucleus and therefore to be able to filter it were not suitable techniques of polarizers or bending the fiber, on the other hand to bend it LP_{11} filtered. Make that the laser beam does not shine of perpendicular to the surface of the fiber input, causes the beam to travel parallel to the axis of the fiber and you will have a higher angle of incidence as a result of high order modes are excited.

Finally, fiber couplers are important components used in WDM systems to route and split signals, monitor the networks, or combine signal and pump wavelengths for feeding optical amplifiers.

VIII. ACKNOWLEDGEMENTS

The authors are grateful to Autonomous University ofHidalgo State (UAEH) and Optical Research Center(CIO) for the realization of this work.

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