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EXACT SOLUTION OF TRIPLE DIFFUSIVE MARANGONICONVECTION IN A COMPOSITE LAYER

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Abstract

The Triple-Diffusive Marangoni-convection problem is investigated in a two layer system comprising an incompressible three component fluid saturated porous layer over which lies a layer of the same fluid. The lower surface of the porous layer is rigid and the upper free surface are considered to be insulating to temperature and solutes concentration perturbations. At the upper free surface, the surface tension effects depending on temperature and both the solute concentrations are considered. At the interface, the normal and tangential components of velocity, heat and solute concentrations and their fluxes are assumed to be continuous. The resulting eigenvalue problem is solved Exactly and an analytical expression for the Thermal Marangoni Number is obtained. The effect of variation of different physical parameters on the same is investigated in detail.

1.Introduction

Hydrothermal growth is a crystal growth from aqueous solution at high temperature and pressure. Even under hydrothermal conditions most of the materials grown have very low solubilities in pure water. Thus to achieve reasonable solubilities large quantities of other materials called mineralizers are added which do not react with the material being grown but affect the density gradients. The convection involved is multi component convection There are many fluid systems in which more than two components are present. The problem under investigation also has many applications like solidification of alloys, the materials processing, the moisture migration in thermal insulation and stored grain, underground spreading of chemical pollutants, waste and fertilizer migration in saturated soil and petroleum reservoirs.

. For example, Degens *et al* [3] have reported that the saline waters of geothermally heated Lake Kivu are strongly stratified by temperature and salinity which is the sum of comparable concentrations of many salts, while the oceans contain many salts in concentrations less than a few percent of the sodium chloride concentration i. e. one can expect a multicomponent system. Even in laboratory experiments on double diffusive convection, dyes or small temperature anamolies introduce a third property which affects the density of the fluid. In these cases the study of double diffusive convection becomes very restrictive. Therefore, one has to consider the stability of multi component systems. Turner *et al* [17] and Griffiths [4] have initiated the work in this direction by conducting laboratory experiments in which the fluxes of several components across diffusive interfaces are measured. Shivakumara [13] has investigated the onset of triple diffusive convection, where the effect of third diffusing component upon the onset of marginal, oscillatory convection and bifurcation from the static solution are discussed.
The problems of triple diffusive convection in clear fluids are also studied by Pearlstein *et al* [8] and Lopez *et al* [5]. Rudraiah and Vortmeyer [11] have studied the linear stability of three- component system in a porous medium in the presence of a gravitationally stable density gradient. Poulikakos [9] has in his brief communication established the presence of a third diffusing component with small diffusivity can seriously alter the nature of the convective instabilities in the system. Triple diffusive convection in composite layers is not given much importance. Where as Single component convection in composite layers is investigated by Many of the researchers started by Nield [7] , Rudraiah [12], Taslim and Narusawa [16], McKay [6], Chen [2] . Recently I. S. Shivakumara *et. al* [14] have investigated the onset of surface tension driven convection in a two layer system comprising an incompressible fluid saturated porous layer over which lies a layer of the same fluid. The critical Marangoni number is obtained for insulating

boundaries both by Regular Perturbation technique and also by exact method. They also have compared the results obtained by both the methods and found in agreement.

Double diffusive convection in composite layers has wide applications in crystal growth and solidification of alloys. Inspite of its wide applications not much work has been done in this area. Chen and Chen [1] have considered the problem of onset of finger convection using BJ-slip condition at the interface. The problem of double diffusive convection for a thermohaline system consisting of a horizontal fluid layer above a saturated porous bed has been investigated experimentally by Poulikakos and Kazmierczak [10]. Venkatachalappa *et al* [17] have investigated the double diffusive convection in composite layer conducive for hydrothermal growth of crystals with the lower boundary rigid and the upper boundary free with deformation. The double diffusive magneto convection in a composite layer bounded by rigid walls is investigated in Sumithra [15] .

2.Formulation of the problem

We consider a horizontal three - component fluid saturated isotropic sparsely packed porous layer of thickness d_m underlying a three component fluid layer of thickness d. The lower surface of the porous layer is considered to rigid and the upper surface of the fluid layer is free at which the surface tension effects depending on temperature and both the species concentrations. Both the boundaries are kept at different constant temperatures and salinities. A Cartesian coordinate system is chosen with the origin at the interface between porous and fluid layers and the z – axis, vertically upwards as shown in Fig.1.

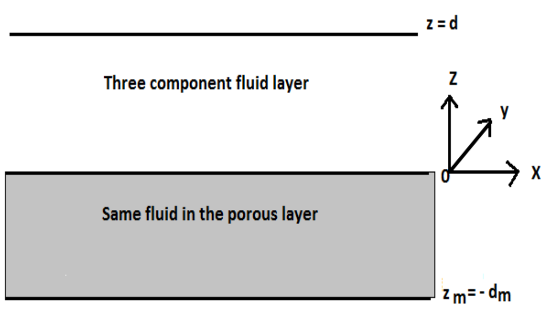

Fig1. Physical Configuration

. The continuity, momentum, energy, species concentration1 and species concentration2 equations are,

$$\nabla \cdot \vec{q} = 0 \quad (1)$$

$$\rho_0 \left[\frac{\partial \vec{q}}{\partial t} + \left(\vec{q} \cdot \nabla \right) \vec{q} \right] = -\nabla P + \mu \nabla^2 \vec{q} \quad (2)$$

$$\frac{\partial T}{\partial t} + \left(\vec{q} \cdot \nabla \right) T = \kappa \nabla^2 T \quad (3)$$

$$\frac{\partial C_1}{\partial t} + \left(\vec{q} \cdot \nabla \right) C_1 = \kappa_1 \nabla^2 C_1 \quad (4)$$

$$\frac{\partial C_2}{\partial t} + \left(\vec{q} \cdot \nabla \right) C_2 = \kappa_2 \nabla^2 C_2 \quad (5)$$

For the porous layer,

$$\nabla_m \cdot \vec{q}_m = 0 \quad (6)$$

$$\rho_0 \left[\frac{1}{\varepsilon} \frac{\partial \vec{q}_m}{\partial t} + \frac{1}{\varepsilon^2} \left(\vec{q}_m \cdot \nabla_m \right) \vec{q}_m \right] = -\nabla_m P_m + \mu_m \nabla^2 \vec{q}_m - \frac{\mu}{K} \vec{q}_m \quad (7)$$

$$A \frac{\partial T_m}{\partial t} + \left(\vec{q}_m \cdot \nabla_m \right) T_m = \kappa_m \nabla_m^2 T_m \quad (8)$$

$$\varepsilon \frac{\partial C_{m1}}{\partial t} + \left(\vec{q}_m \cdot \nabla_m \right) C_{m1} = \kappa_{m1} \nabla_m^2 C_{m1} \quad (9)$$

$$\varepsilon \frac{\partial C_{m2}}{\partial t} + \left(\vec{q}_m \cdot \nabla_m \right) C_{m2} = \kappa_{m2} \nabla_m^2 C_{m2} \quad (10)$$

Where the symbols in the above equations have the following meaning. $\vec{q} = (u, v, w)$ is the velocity vector, t is the time, μ is the fluid viscosity, P is the pressure, ρ_0 is the fluid density, T is the temperature, κ is the thermal diffusivit C_1 is the species concentration1 or the salinity field 1, κ_1 is the solute1 diffusivity of the fluid, C_2 is the species concentration2 or the salinity field2, κ_2 is the solute1 diffusivity of the fluid, $A = \frac{(\rho_0 C_p)_m}{(\rho C_p)_f}$ is the ratio of heat capacities, C_p is the specific heat, K is the permeability of the porous medium. The subscripts m and f refer to the porous medium and the fluid respectively.

The basic steady state is assumed to the quiescent and we consider the solution of the form,

$$\left[u,v,w,P,T,C_1,C_2\right] = \left[0,0,0,P_b(z),T_b(z),C_{1b}(z),C_{2b}(z)\right] \quad (11)$$

in the fluid layer and in the porous layer

$$\left[u_m,v_m,w_m,P_m,T_m,C_{1m},C_{2m}\right] = \left[0,0,0,P_{mb}(z_m),T_{mb}(z_m),C_{1mb}(z_m),C_{2mb}(z_m)\right] \quad (12)$$

Where the subscript 'b' denotes the basic state. The temperature and species concentration distributions $T_b(z)$, $T_{mb}(z_m)$, $C_b(z)$, $C_m(z_m)$ and $C_{2b}(z)$, $C_{2mb}(z_m)$, respectively are found to be

$$T_b(z) = T_0 - \frac{(T_0 - T_u)z}{d} \text{ in } 0 \le z \le d \quad (13)$$

$$T_{mb}(z_m) = T_0 - \frac{(T_l - T_0)z_m}{d_m} \text{ in } 0 \le z_m \le d_m \quad (14)$$

$$C_{1b}(z) = C_{10} - \frac{(C_{10} - C_{1u})z}{d} \text{ in } 0 \le z \le d \quad (15)$$

$$C_{1mb}(z_m) = C_{10} - \frac{(C_{1l} - C_{10})z_m}{d_m} \text{ in } 0 \le z_m \le d_m \quad (16)$$

$$C_{2b}(z) = C_{20} - \frac{(C_{20} - C_{2u})z}{d} \text{ in } 0 \le z \le d \quad (17)$$

$$C_{2mb}(z_m) = C_{20} - \frac{(C_{2l} - C_{20})z_m}{d_m} \text{ in } 0 \le z_m \le d_m \quad (18)$$

Where

$$T_0 = \frac{\kappa d_m T_u + \kappa_m d T_l}{\kappa d_m + \kappa_m d},$$

$$C_{10} = \frac{\kappa_1 d_m C_{1u} + \kappa_{1m} d C_{1l}}{\kappa_1 d_m + \kappa_{1m} d},$$

$$C_{20} = \frac{\kappa_2 d_m C_{2u} + \kappa_{2m} d C_{2l}}{\kappa_2 d_m + \kappa_{2m} d}$$

are the interface temperature and concentrations.

In order to investigate the stability of the basic solution, infinitesimal disturbances are introduced in the form,

$$\left[\vec{q},P,T,C_1,C_2\right] = \left[0,P_b(z),T_b(z),C_1(z),C_2(z)\right] + \left[\vec{q}',P',\theta,S_1,S_2\right] \quad (19)$$

And

$$\left[\vec{q}_m,P_m,T_m,C_{m1},C_{m2}\right] = \left[0,P_{mb}(z_m),T_{mb}(z_m),C_{m1b}(z_m),C_{m2b}(z_m)\right] + \left[\vec{q}_m',P_m',\theta_m,S_{m1},S_{m2}\right] \quad (20)$$

Where the primed quantities are the perturbed ones over their equilibrium counterparts. Now Eqs. (19) and (20) are substituted into the Eqs. (1) to (10) and are linearised in the usual manner. Next, the pressure term is eliminated from (2) and (7) by taking curl twice on these two equations and only the vertical component is retained. The variables are then nondimensionalised using d, $\frac{d^2}{\kappa}$, $\frac{\kappa}{d}$, $T_0 - T_u$, $C_{10} - C_{1u}$ and $C_{20} - C_{2u}$ as the units of length, time, velocity, temperature, species concentrations in the fluid layer and d_m, $\frac{d_m^2}{\kappa_m}$, $\frac{\kappa_m}{d_m}$, $T_l - T_0$, $C_{1l} - C_{10}$ and $C_{2l} - C_{20}$ as the corresponding characteristic quantities in the porous layer. Note that the separate length scales are chosen for the two layers so that each layer is of unit depth.

In this way the detailed flow fields in both the fluid and porous layers can be clearly obtained for all the depth ratios $\zeta = \frac{d}{d_m}$. The dimensionless equations for the perturbed variables are given by, in $0 \le z \le 1$

$$\frac{1}{\Pr}\frac{\partial \nabla^2 w}{\partial t} = \nabla^4 w \quad (21)$$

$$\frac{\partial \theta}{\partial t} = w + \nabla^2 \theta \quad (22)$$

$$\frac{\partial S_1}{\partial t} = w + \tau_1 \nabla^2 S_1 \quad (23)$$

$$\frac{\partial S_2}{\partial t} = w + \tau_2 \nabla^2 S_2 \quad (24)$$

In $-1 \le z_m \le 0$

$$\frac{\beta^2}{\Pr_m}\frac{\partial \nabla_m^2 w_m}{\partial t} = \hat{\mu}\beta^2 \nabla_m^4 w_m - \nabla_m^2 w_m + R_m \nabla_{2m}^2 \theta_m \quad (25)$$

$$A\frac{\partial \theta_m}{\partial t} = w_m + \nabla_m^2 \theta_m \quad (26)$$

$$\varepsilon \frac{\partial S_{m1}}{\partial t} = w_m + \tau_{m1} \nabla_m^2 S_{m1} \quad (27)$$

$$\varepsilon \frac{\partial S_{m2}}{\partial t} = w_m + \tau_{m2} \nabla_m^2 S_{m2} \quad (28)$$

For the fluid layer $\Pr = \frac{\nu}{\kappa}$ is the Prandtl number, $\tau_1 = \frac{\kappa_1}{\kappa}$ is the ratio salinity1 diffusivity to thermal diffusivity, $\tau_2 = \frac{\kappa_2}{\kappa}$ is the ratio salinity2 diffusivity to thermal diffusivity. For the porous layer, $\Pr_m = \frac{\varepsilon \nu_m}{\kappa_m}$ is the Prandtl number, $\beta^2 = \frac{K}{d_m^2} = Da$ is the Darcy number, $\hat{\mu} = \frac{\mu_m}{\mu}$ is the viscosity ratio, $\tau_{m1} = \frac{\kappa_{m1}}{\kappa}$ is the ratio salinity1 diffusivity to thermal diffusivity, $\tau_{m2} = \frac{\kappa_{m2}}{\kappa}$ is the ratio salinity2 diffusivity to thermal diffusivity.

We make the normal mode expansion and seek solutions for the dependent variables in the fluid and porous layers according to

$$\begin{bmatrix} w \\ \theta \\ S_1 \\ S_2 \end{bmatrix} = \begin{bmatrix} W(z) \\ \Theta(z) \\ \Sigma_1(z) \\ \Sigma_2(z) \end{bmatrix} f(x,y)e^{nt} \quad (29)$$

And

$$\begin{bmatrix} w_m \\ \theta_m \\ S_{m1} \\ S_{m2} \end{bmatrix} = \begin{bmatrix} W_m(z_m) \\ \Theta_m(z_m) \\ \Sigma_{m1}(z_m) \\ \Sigma_{m2}(z_m) \end{bmatrix} f_m(x_m, y_m)e^{n_m t} \quad (30)$$

With $\nabla_2^2 f + a^2 f = 0$ and $\nabla_{2m}^2 f_m + a_m^2 f_m = 0$, where a and a_m are the nondimensional horizontal wavenumbers, n and n_m are the frequencies. Since the dimensional horizontal wavenumbers must be the same for the fluid and porous layers, we must have $\frac{a}{d} = \frac{a_m}{d_m}$ and hence $a_m = \hat{d}a$.

Substituting Eqs. (29) and (30) into the Eqs.(21) to (28) and denoting the differential operator $\frac{\partial}{\partial z}$ and $\frac{\partial}{\partial z_m}$ by D and D_m respectively, an eigenvalue problem consisting of the following ordinary differential equations is obtained,

In $0 \le z \le 1$,

$$\left(D^2 - a^2 + \frac{n}{\Pr}\right)\left(D^2 - a^2\right)W = 0 \quad (31)$$

$$\left(D^2 - a^2 + n\right)\Theta + W = 0 \quad (32)$$

$$\left[\tau_1\left(D^2 - a^2\right) + n\right]\Sigma_1 + W = 0 \quad (33)$$

$$\left[\tau_2\left(D^2 - a^2\right) + n\right]\Sigma_2 + W = 0 \quad (34)$$

In $-1 \le z_m \le 0$

$$\left[\hat{\mu}\beta^2\left(D_m^2 - a_m^2\right) + \frac{n_m \beta^2}{\Pr_m} - 1\right]\left(D_m^2 - a_m^2\right)W_m = 0 \quad (35)$$

$$\left(D_m^2 - a_m^2 + An_m\right)\Theta_m + W_m = 0 \tag{36}$$

$$\left[\tau_{m1}\left(D_m^2 - a_m^2\right) + n_m\varepsilon\right]\Sigma_{m1} + W_m = 0 \tag{37}$$

$$\left[\tau_{m2}\left(D_m^2 - a_m^2\right) + n_m\varepsilon\right]\Sigma_{m2} + W_m = 0 \tag{38}$$

It is known that the principle of exchange of instabilities holds for triple diffusive convection in both fluid and porous layers separately for certain choice of parameters. Therefore, we assume that the principle of exchange of instabilities holds even for the composite layers. In otherwords, it is assumed that the onset of convection is in the form of steady convection and accordingly we take $n = n_m = 0$.

In $0 \le z \le 1$,

$$\left(D^2 - a^2\right)^2 W = 0 \tag{39}$$

$$\left(D^2 - a^2\right)\Theta + W = 0 \tag{40}$$

$$\tau_1\left(D^2 - a^2\right)\Sigma_1 + W = 0 \tag{41}$$

$$\tau_2\left(D^2 - a^2\right)\Sigma_2 + W = 0 \tag{42}$$

In $-1 \le z_m \le 0$

$$\left[\hat{\mu}\beta^2\left(D_m^2 - a_m^2\right) - 1\right]\left(D_m^2 - a_m^2\right)W_m = 0 \tag{43}$$

$$\left(D_m^2 - a_m^2\right)\Theta_m + W_m = 0 \tag{44}$$

$$\left[\tau_{m1}\left(D_m^2 - a_m^2\right) + n_m\varepsilon\right]\Sigma_{m1} + W_m = 0 \tag{45}$$

$$\left[\tau_{m2}\left(D_m^2 - a_m^2\right) + n_m\varepsilon\right]\Sigma_{m2} + W_m = 0 \tag{46}$$

Thus we note that, in total we have a twentyth order ordinary differential equation and we need twenty boundary conditions to solve them.

3. Boundary conditions

The bottom boundary is assumed to be rigid and insulating to both temperature and species concentrations, so that at $z_m = -d_m$,

$$w_m = 0,\ \frac{\partial w_m}{\partial z_m} = 0,\ \frac{\partial T_m}{\partial z_m} = 0,\ \frac{\partial C_{m1}}{\partial z_m} = 0,\ \frac{\partial C_{m2}}{\partial z_m} = 0 \tag{47}$$

The upper boundary is assumed to be free insulating both temperature and species concentrations so, the appropriate boundary conditions at $z = d$,

$$w = 0,\ \frac{\partial T}{\partial z} = 0, \frac{\partial C_1}{\partial z} = 0, \frac{\partial C_2}{\partial z} = 0 \tag{48}$$

One more velocity condition at the free surface is the continuity of the tangential stress given by

$$\mu\frac{\partial^2 w}{\partial z^2} = -\frac{\partial \sigma_t}{\partial T}\nabla_2^2 T - \frac{\partial \sigma_t}{\partial C_1}\nabla_2^2 C_1 - \frac{\partial \sigma_t}{\partial C_2}\nabla_2^2 C_2 \tag{49}$$

Where σ_t is the surface tension and is given by $\sigma_t = \sigma_0 - \sigma_T T - \sigma_{C_1} C_1 - \sigma_{C_2} C_2$

$$\sigma_T = -\left(\frac{\partial \sigma_t}{\partial T}\right)_{T=T_0}, \sigma_{C_1} = -\left(\frac{\partial \sigma_t}{\partial C_1}\right)_{C_1=C_{10}},$$

$$\sigma_{C_2} = -\left(\frac{\partial \sigma_t}{\partial C_2}\right)_{C_2=C_{20}}$$

At the interface (i.e., at $z = 0, z_m = 0$), the normal component of velocity, tangential velocity, temperature, heat flux, species concentration and mass flux are continuous and respectively yield following Nield (1977),

$$w = w_m, \quad \frac{\partial w}{\partial z} = \frac{\partial w_m}{\partial z_m},$$

$$T = T_m, \quad \kappa\frac{\partial T}{\partial z} = \kappa_m\frac{\partial T_m}{\partial z_m},$$

$$C_1 = C_{m1}, \quad \kappa_1\frac{\partial C_1}{\partial z} = \kappa_{m1}\frac{\partial C_{m1}}{\partial z_m},$$

$$C_2 = C_{m2}, \quad \kappa_2\frac{\partial C_2}{\partial z} = \kappa_{m2}\frac{\partial C_{m2}}{\partial z_m} \tag{50}$$

We take two more boundary conditions at the interface. Since we have used the Darcy-Brinkman equations of motion for the flow through the porous medium, the physically feasible boundary conditions on velocity are the following, at $z = 0$ and $z_m = 0$

$$P_m - 2\mu_m \frac{\partial w_m}{\partial z_m} = P - 2\mu \frac{\partial w}{\partial z}$$

which will reduce to

$$\mu\left(3\nabla_2^2 + \frac{\partial^2}{\partial z^2}\right)\frac{\partial w}{\partial z} = -\frac{\mu_m}{K}\frac{\partial w_m}{\partial z_m} + \mu_m \beta^2\left(3\nabla_{2m}^2 + \frac{\partial^2}{\partial z_m^2}\right)\frac{\partial w_m}{\partial z_m} \quad (51)$$

The other appropriate velocity boundary condition at the interface $z = 0, z_m = 0$ can be ,

$$\mu\left(-\frac{\partial^2 w}{\partial z^2} + \nabla_2^2 w\right) = \mu_m\left(-\frac{\partial^2 w_m}{\partial z_m^2} + \nabla_{2m}^2 w_m\right) \quad (52)$$

All the twenty boundary conditions (47) to (52) are nondimenstionalised by using the same scale factors that of equations and are subjected to normal mode analysis and they are given .

$$W(1) = 0,\ D^2W(1) + a^2 M\Theta(1) + a^2 M_{s1}\Sigma_1(1) + a^2 M_{s2}\Sigma_2(1) = 0,$$

$$D\Theta(1) = 0,\ D\Sigma_1(1) = 0,\quad D\Sigma_2(1) = 0$$

$$W(0) = \frac{\zeta}{\varepsilon_t} W_m(0),\quad DW(0) = \frac{\zeta^2}{\varepsilon_t} D_m W_m(0),$$

$$\left(D^2 + a^2\right)W(0) = \frac{\hat{\mu}\zeta^3}{\varepsilon_t}\left(D_m^2 + a_m^2\right)W_m(0)$$

$$\left(D^3 W(0) - 3a^2 DW(0)\right) = -\frac{\zeta^2}{\varepsilon_t Da} D_m W_m(0) + \frac{\hat{\mu}\zeta^4}{\varepsilon_t}\left(D_m^3 W_m(0) - 3a_m^2 D_m W_m(0)\right)$$

$$\Theta(0) = \frac{\varepsilon_t}{\zeta}\Theta_m(0),\quad D\Theta(0) = D_m\Theta_m(0),$$

$$\Sigma_1(0) = \frac{\varepsilon_{s1}}{\zeta}\Sigma_{m1}(0),\quad D\Sigma_1(0) = D_m\Sigma_{m1}(0),$$

$$\Sigma_2(0) = \frac{\varepsilon_{s2}}{\zeta}\Sigma_{m2}(0),\quad D\Sigma_2(0) = D_m\Sigma_{m2}(0),$$

$$W_m(-1) = 0,\quad D_m W_m(-1) = 0,\quad D_m\Theta_m(-1) = 0,$$

$$D_m\Sigma_{m1}(-1) = 0,\quad D_m\Sigma_{m2}(-1) = 0 \quad (53)$$

Where $M = -\frac{\partial \sigma_t}{\partial T}\frac{(T_0 - T_u)d}{\mu\kappa}$ is the thermal Marangoni number, $M_{s1} = -\frac{\partial \sigma_t}{\partial C_1}\frac{(C_{10} - C_{1u})d}{\mu\kappa}$ is the solute1 Marangoni number, $M_{s2} = -\frac{\partial \sigma_t}{\partial C_2}\frac{(C_{20} - C_{2u})d}{\mu\kappa}$ is the solute2 Marangoni number, $\zeta = \frac{d}{d_m}$ is the depth ratio, $\varepsilon_t = \frac{\kappa}{\kappa_m}$ is the ratio of thermal diffusivities of fluid to porous layer , $\varepsilon_{s1} = \frac{\kappa_{s1}}{\kappa_{s1m}}$ is the ratio of solute1 diffusivities of fluid to porous layer , $\varepsilon_{s2} = \frac{\kappa_{s2}}{\kappa_{s2m}}$ is the ratio of solute2 diffusivities of fluid to porous layer.

The Eqs.(41) to (46) are to be solved with respect to the boundary conditions (53).

4. Exact Solution

The equations (39) and (43) are independent of $\Theta, \Sigma_1, \Sigma_2$ and $\Theta_m, \Sigma_{m1}, \Sigma_{m2}$ respectively and they can be solved independently to get the general solutions in the form,

$$W(z) = A_1 Cosh(az) + A_2 z Cosh(az) + A_3 Sinh(az) + A_4 z Sinh(az) \quad (54)$$

$$W_m(z) = A_5 Cosh(a_m z_m) + A_6 Sinh(a_m z_m) + A_7 Cosh(\delta z_m) + A_8 Sinh(\delta z_m) \quad (55)$$

Where A_1 to A_4 and A_5 to A_8 constants to be determined using the velocity boundary conditions of $(53)^1, (53)^6, (53)^7, (53)^8, (53)^9, (53)^{10}, (53)^{11}$ and obtain

$$W(z) = A_1[Cosh(az) + a_1 z Cosh(az) + a_2 Sinh(az) + a_3 z Sinh(az) \quad (56)$$

$$W_m\left(z\right)=A_1[a_4Cosh\left(a_mz_m\right)+a_5Sinh\left(a_mz_m\right)$$
$$+a_6Cosh\left(\delta z_m\right)+a_7Sinh\left(\delta z_m\right)] \quad (57)$$

The heat equations (40) and (44) are then solved using thermal boundary conditions of (53), the expressions for Θ, Θ_m are obtained as,

$$\Theta\left(z\right)=A_1\left[a_8Cosh\left(az\right)+a_9Sinh\left(az\right)+f\left(z\right)\right] \quad (58)$$

$$\Theta_m\left(z\right)=A_1[a_{10}Cosh\left(a_mz_m\right)$$
$$+a_{11}Sinh\left(a_mz_m\right)+f_m(z_m)] \quad (59)$$

The Species concentration1equations (41) and (45) are then solved using species1 boundary conditions of (53), the expressions for $\sum_1, \sum_{m1}$ are obtained as,

$$\Sigma_1\left(z\right)=A_1\left[a_{12}Cosh\left(az\right)+a_{13}Sinh\left(az\right)+\frac{f(z)}{\tau_1}\right] \quad (60)$$

$$\Sigma_{m1}\left(z_m\right)=A_1[a_{14}Cosh\left(a_mz_m\right)$$
$$+a_{15}Sinh\left(a_mz_m\right)+\frac{f_m(z_m)}{\tau_{m1}}] \quad (61)$$

The Species concentration2 equations (42) and (46) are then solved using species2 boundary conditions of (53), the expressions for $\sum_2, \sum_{m2}$ are obtained as,

$$\Sigma_2\left(z\right)=A_1\left[a_{16}Cosh\left(az\right)+a_{17}Sinh\left(az\right)+\frac{f(z)}{\tau_2}\right] \quad (62)$$

$$\Sigma_{m2}\left(z_m\right)=A_1[a_{18}Cosh\left(a_mz_m\right)$$
$$+a_{19}Sinh\left(a_mz_m\right)+\frac{f_m(z_m)}{\tau_{m2}}] \quad (63)$$

where

$$\delta=\sqrt{a_m^2+\frac{1}{\hat{\mu}\beta^2}}$$
$$\delta_1=\frac{-\zeta^2}{Da\ \varepsilon_t}a_m-\frac{3a_m^2\zeta^4}{\varepsilon_t}+\frac{a_m^3\zeta^4}{\varepsilon_t}$$
$$\delta_2=\frac{-\zeta^2\delta}{Da\ \varepsilon_t}-\frac{3a_m^2\zeta^4\delta}{\varepsilon_t}+\frac{\delta^3\zeta^4}{\varepsilon_t}$$
$$\delta_3=\frac{2a^2\varepsilon_t\delta}{\hat{\mu}\zeta^3}$$
$$\delta_4=\left(\frac{\varepsilon_t\left(\delta^2+a_m^2\right)}{\zeta}-\delta_3\right)\frac{1}{a_m^2-\delta^2}$$
$$\delta_5=\frac{\delta_4}{a_m^2-\delta^2}$$
$$\delta_6=-\delta_5\left(Cosha_m+Cosh\delta\right)$$
$$\delta_7=\delta_4Cosha_m+\left(\frac{\varepsilon_t}{\zeta}-\delta_4\right)Cosh\delta$$
$$\delta_8=\delta_5\left(\delta Sinh\delta-a_mSinha_m\right)$$
$$\delta_9=\delta_4a_mSinha_m+\left(\frac{\varepsilon_t}{\zeta}-\delta_4\right)\delta Sinh\delta$$
$$\delta_{10}=-\frac{\delta_1}{2a^3},\quad \delta_{11}=-\frac{\delta_2}{2a^3}$$
$$\delta_{12}=-a\delta_{10}+\frac{\zeta^2a_m}{\varepsilon_t},\quad \delta_{13}=-a\delta_{11}+\frac{\zeta^2\delta}{\varepsilon_t}$$
$$\delta_{14}=Sinha,\quad \delta_{15}=\delta_{12}Cosha+\delta_{10}Sinha$$
$$\delta_{17}=Cosha,\quad \delta_{16}=\delta_{13}Cosha+\delta_{11}Sinha$$
$$\delta_{18}=-\frac{\delta_{15}}{\delta_{14}},\quad \delta_{19}=-\frac{\delta_{16}}{\delta_{14}},\quad \delta_{20}=-\frac{\delta_{17}}{\delta_{14}}$$
$$\delta_{21}=\delta_6\delta_{18}-Sinha_m,\quad \delta_{22}=\delta_6\delta_{19}-Sinh\delta,$$
$$\delta_{23}=\delta_6\delta_{20}+\delta_7,\quad \delta_{26}=\delta_8\delta_{20}+\delta_9$$
$$\delta_{24}=\delta_8\delta_{18}-a_mCosha_m,\quad \delta_{25}=\delta_8\delta_{19}-\delta Cosh\delta,$$
$$\delta_{27}=-\frac{\delta_{22}}{\delta_{21}},\quad \delta_{28}=-\frac{\delta_{23}}{\delta_{21}},$$

$$a_7 = \frac{-\left(\delta_{28}+\delta_{26}\right)}{\delta_{24}\delta_{27}+\delta_{25}}, \quad a_5 = \delta_{27}a_7+\delta_{28}, \quad a_3 = \delta_{18}a_5+\delta_{19}a_6+\delta_{20}$$

$$a_1 = \delta_{12}a_5+\delta_{13}a_7, \quad a_2 = \delta_{10}a_5+\delta_{11}a_7,$$

$$a_4 = \delta_4-\delta_5 a_3, \quad a_6 = \frac{\varepsilon_t}{\zeta}-a_4,$$

$$\delta_{29} = -\frac{aCosha+Sinha}{2a}$$

$$-\frac{a_1}{4a}\left(aCosha+Sinha-\frac{aSinha+Cosha}{a}\right)$$

$$-\frac{a_2}{2a}\left(aSinha+Cosha\right)$$

$$-\frac{a_3}{4a}\left(aSinha+2Cosha-\frac{aCosha+Sinha}{a}\right)$$

$$\delta_{30} = -\frac{\varepsilon_t}{\zeta}\frac{a_6}{\delta^2-a_m^2}-\frac{\varepsilon_t}{\zeta}\frac{a_7}{\delta^2-a_m^2}$$

$$\delta_{31} = \frac{a_1}{4a^2}+\frac{a_2}{2a}-\frac{a_5}{2a_m}$$

$$\delta_{32} = -\frac{a_4\left(a_m Cosha_m+Sinha_m\right)}{2a_m}$$

$$+\frac{a_5}{2a_m}\left(a_m Sinha_m+Cosha_m\right)$$

$$-\frac{a_6\delta Sinh\delta}{\delta^2-a_m^2}-\frac{a_7\delta Sinh\delta}{\delta^2-a_m^2}$$

$$\delta_{33} = \frac{\delta_{32}\varepsilon_t aSinha}{\zeta a_m Sinha_m}-\delta_{30}aSinha-\delta_{31}Cosha-\delta_{29}$$

$$\delta_{34} = \frac{\varepsilon_t a_m Cosha}{\zeta}+\frac{\varepsilon_t aCosha_m Sinha}{Sinha_m}$$

$$a_{11} = \frac{\delta_{33}}{\delta_{34}}, \quad a_{10} = \frac{a_{11}a_m Cosha_m-\delta_{32}}{a_m Sinha_m}$$

$$a_9 = \frac{a_m}{a}a_{11}+\delta_{31}, \quad a_8 = \frac{\varepsilon_t}{\zeta}a_{10}+\delta_{30},$$

$$\delta_{35} = \frac{1}{\tau_1}\left(\frac{a_1}{4a^2}+\frac{a_2}{2a}\right)-\frac{1}{\tau_{m1}}\left(\frac{a_5}{2a_m}\right)$$

$$\delta_{36} = \frac{1}{\tau_{m1}}\{\frac{-a_4\left(a_m Cosha_m+Sinha_m\right)}{2a_m}$$

$$+\frac{a_5}{2a_m}\left(a_m Sinha_m+Cosha_m\right)\}$$

$$\frac{1}{\tau_{m1}}\left(-\frac{a_6\delta Sinh\delta}{\delta^2-a_m^2}-\frac{a_7\delta Sinh\delta}{\delta^2-a_m^2}\right)$$

$$\delta_{37} = \frac{\delta_{36}\varepsilon_{s1}aSinha}{\zeta a_m Sinha_m}-\frac{\delta_{30}aSinha}{\tau_{m1}}-\delta_{35}Cosha-\frac{\delta_{29}}{\tau_1}$$

$$\delta_{38} = a_m Cosha+\frac{\varepsilon_{s1}aCosha_m Sinha}{\zeta Sinha_m}$$

$$\delta_{39} = \frac{1}{\tau_2}\left(\frac{a_1}{4a^2}+\frac{a_2}{2a}\right)-\frac{1}{\tau_{m2}}\left(\frac{a_5}{2a_m}\right)$$

$$\delta_{40} = \frac{1}{\tau_{m2}}\{\frac{-a_4\left(a_m Cosha_m+Sinha_m\right)}{2a_m}$$

$$+\frac{a_5}{2a_m}\left(a_m Sinha_m+Cosha_m\right)\}$$

$$\frac{1}{\tau_{m2}}\left(-\frac{a_6\delta Sinh\delta}{\delta^2-a_m^2}-\frac{a_7\delta Sinh\delta}{\delta^2-a_m^2}\right)$$

$$\delta_{41} = \frac{\delta_{40}\varepsilon_{s2}aSinha}{\zeta a_m Sinha_m}-\frac{\delta_{30}aSinha}{\tau_{m2}}-\delta_{39}Cosha-\frac{\delta_{29}}{\tau_2}$$

$$\delta_{42} = a_m Cosha+\frac{\varepsilon_{s2}aCosha_m Sinha}{\zeta Sinha_m}$$

$$a_{19} = \frac{\delta_{41}}{\delta_{42}}, \quad a_{18} = \frac{a_{19}a_m Cosha_m-\delta_{40}}{a_m Sinha_m}$$

$$a_{17} = \frac{a_m}{a}a_{19}+\delta_{39}, \quad a_{16} = \frac{\varepsilon_{s2}}{\zeta}a_{18}+\frac{\delta_{30}}{\tau_{m2}},$$

$$a_{15} = \frac{\delta_{37}}{\delta_{38}}, \quad a_{14} = \frac{a_{15}a_m Cosha_m-\delta_{36}}{a_m Sinha_m}$$

$$a_{13} = \frac{a_m}{a}a_{15}+\delta_{35}, \quad a_{12} = \frac{\varepsilon_{s1}}{\zeta}a_{14}+\frac{\delta_{30}}{\tau_{m1}},$$

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5.The Thermal Marangoni number

Now the thermal Marangoni number is obtained by the boundary condition $(53)^2$ as

$$M = \frac{-\left[D^2W(1)+a^2M_{s1}\Sigma_1(1)+a^2M_{s2}\Sigma_2(1)\right]}{a^2\Theta(1)} \tag{64}$$

Simplifying we get

$$M = \frac{-\begin{bmatrix} \Delta + a^2 M_{s1} \\ \left\{ a_{12}Cosh(a)+a_{13}Sinh(a)+\dfrac{f(1)}{\tau_1} \right\} \\ +a^2M_{s2} \\ \left\{ a_{16}Cosh(a)+a_{17}Sinh(a)+\dfrac{f(1)}{\tau_2} \right\} \end{bmatrix}}{a^2\left[a_8Cosh(a)+a_9Sinh(a)+f(1)\right]} \tag{65}$$

Where

$$f(1) = \frac{Sinha}{2a}+\frac{a_1}{4a}\left(Sinha-\frac{Cosha}{a}\right) + \frac{a_2}{2a}(Cosha)+\frac{a_3}{4a}\left(Cosha-\frac{Sinha}{a}\right)$$

And

$$\Delta = a^2Cosha + a_1\left(a^2Cosha+2aSinha\right) + a_2a^2Sinha + a_3\left(a^2Sinha+2aCosha\right)$$

6. Results and discussion

The Thermal Marangoni number M obtained as a function of the parameters is drawn versus the depth ratio ζ and the results are represented graphically showing the effects of the variation of one physical quantity, fixing the other parameters. The fixed values of the parameters are $\varepsilon_t = 0.25,\ \varepsilon_{s1} = 0.25,\ \varepsilon_{s2} = 0.25,\ \tau_1 = 0.25,$, $\tau_2 = 0.25,\ \tau_{m1} = 0.25,\ \tau_{m2} = 0.25$, $\hat{\mu} = 1,\quad a = 3.0,$. $M_{s1} = 10, M_{s2} = 100,\ Da = 10.0,\quad \varepsilon = 1.0$.

The effects of the parameters $a, Da,\ \varepsilon_{s1},$ $M_{s1}, M_{s2},\ \tau_2,\ \tau_{m2}$ and $\hat{\mu}$ on the thermal Marangoni number are obtained and portrayed in the Figures 2 to 9 respectively.

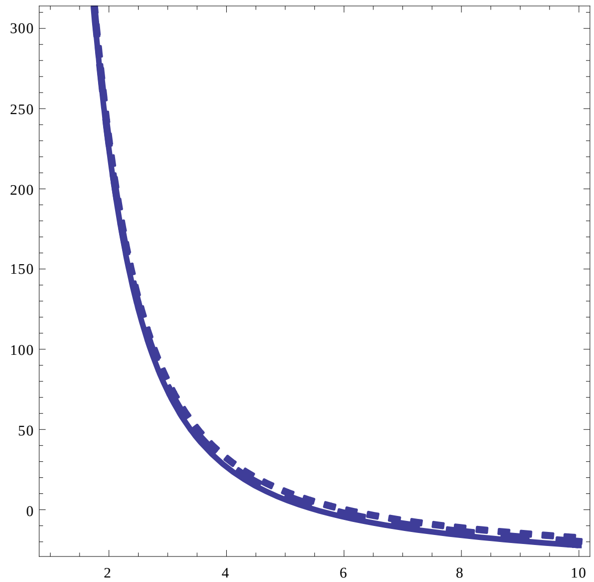

Fig.2. The effects of a on Thermal Marangoni number M

The effects of the horizontal wave number a, on the thermal Marangoni number M are shown in Fig.2. The graph has three diverging curves. The line curve is for $a = 3.0$, the big dotted curve is for 3.1 and the small dotted line curve is for 3.2. Since the curves are diverging, it indicates that the increasing values of τ will have effect only for larger values of the depth ratio $\zeta = \dfrac{d}{d_m}$, that is for fluid layer dominant composite systems. From the curves one can see that for a fixed value of ζ, increase in the value of a is to increase the value of the thermal Marangoni number i.e., to stabilize the system by delaying the onset of surface tension driven convection.

The effects of the Darcy number Da, on the thermal Marangoni number M are shown in Fig.3. The graph has three converging curves. The line curve is for $Da = 10$, the big dotted curve is for 20 and the small dotted line curve is for 30. Since the curves are converging, it indicates that the increasing values of Da will have effect only for smaller values of the depth ratio $\zeta = \frac{d}{d_m}$, that is for porous layer dominant composite systems. From the curves one can see that for a fixed value of ζ, increase in the value of Da is to increase the value of the thermal Marangoni number i.e., to stabilize the system by delaying the onset of surface tension driven convection.

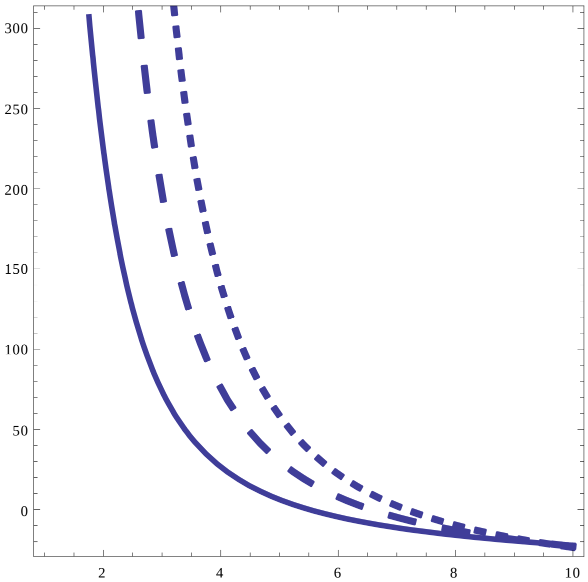

Fig.3 . The effects of Da on the Thermal Marangoni number M

The effects of the ratio of solute1 diffusivity of the fluid in the fluid layer to that of porous layer $\varepsilon_{s1} = \frac{\kappa_{s1}}{\kappa_{sm1}}$, on the thermal Marangoni number M are shown in Fig.4. The curves are converging at both the ends. The line curve is for $\varepsilon_{s1} = 0.25$, the big dotted curve is for 0.5 and the small dotted line curve is for 0.75. It is evident that the effect of ε_{s1} is prominent in the region $2 \le \zeta \le 8$ and here for a fixed value of ζ, increase in the value of ε_{s1} is to increase the value of the thermal Marangoni number M i.e., to stabilize the system by delaying the onset of surface tension driven convection.

Figure 5 displays the effects of the solute1 Marangoni number M_{s1}, on the thermal Marangoni number M. The graph has three converging curves. The line curve is for $M_{s1} = 25$, the big dotted curve is for 50 and the small dotted line curve is for 75. This number has dual effect on the thermal Marangoni number. For values of $\zeta \le 5$ the curves are converging and here for a fixed depth ratio the

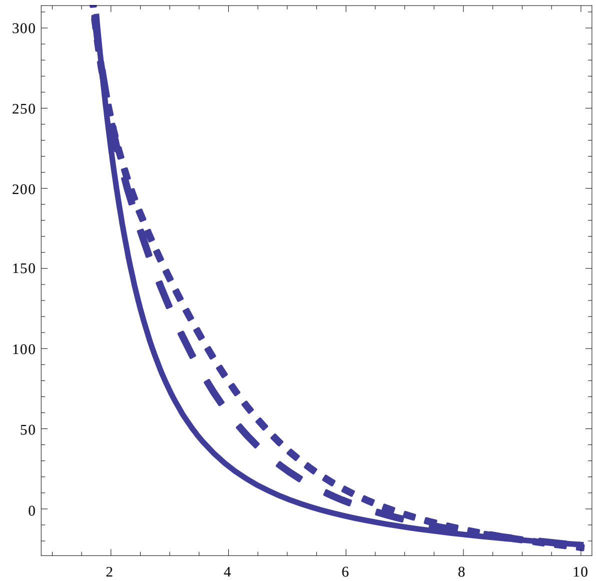

Fig.4. The effects of ε_{s1} on the Thermal Marangoni number M

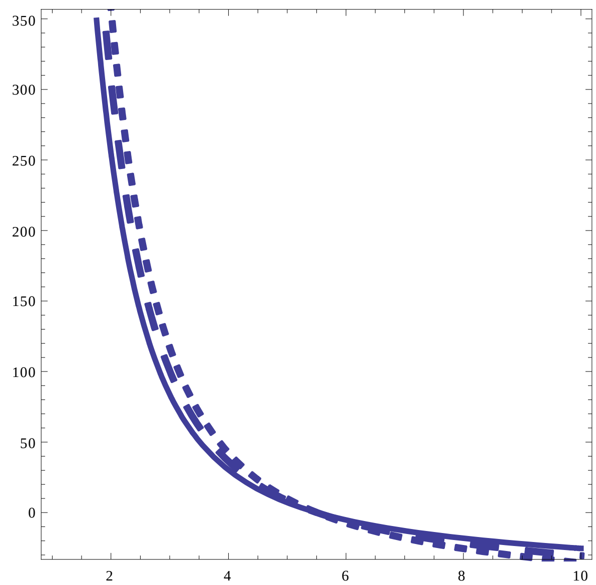

Fig.5. The effects of M_{s1} on the Thermal Marangoni number M

increase in value of M_{s1} increases the thermal marangoni number where as, for the values of depth ratio $\zeta \ge 5$ the curves are diverging, and here for fixed depth ratio the increase in value of M_{s1} decreases the thermal marangoni number.

Figure 6 displays the effects of the solute1 Marangoni number M_{s2}, on the thermal Marangoni number M. The graph has three converging curves. The line curve is for $M_{s2} = 100$, the big dotted curve is for 150 and the small dotted line curve is for 200. Its effect is similar to that of M_{s1}. This number has again dual effect on the thermal Marangoni number. For values of $\zeta \leq 5.5$ the curves are converging and here for a fixed depth ratio the increase in value of M_{s2} increases the thermal marangoni number where as, for the values of depth ratio $\zeta \geq 5.5$ the curves are diverging, and here for a fixed depth ratio the increase in value of M_{s2} decreases the thermal marangoni number.

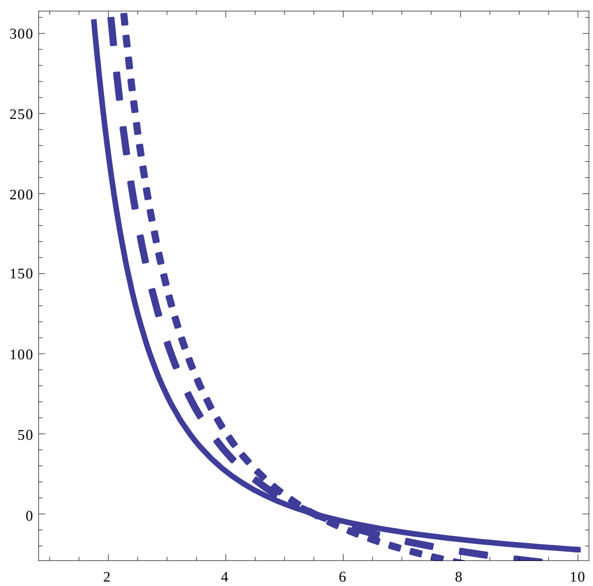

Fig.6. The effects of M_{s2} on the Thermal Marangoni number M

The effects of the ratio of solute 2 diffusivity to thermal diffusivity in the fluid layer , $\tau_2 = \dfrac{\kappa_2}{\kappa}$ on the thermal Marangoni number M are shown in Fig.7. The graph has three diverging curves. The line curve is for $\tau_2 = 0.25$, the big dotted curve is for 0.50 and the small dotted line curve is for 0.75. Since the curves are diverging, it indicates that the increasing values of τ_2 will have effect only for larger values of the depth ratio $\zeta = \dfrac{d}{d_m}$, that is for fluid layer dominant composite systems. From the curves one can see that for a fixed value of ζ , increase in the value of τ_2 is to increase the value of the thermal Marangoni number i.e., to stabilize the system by delaying the onset of surface tension driven convection.

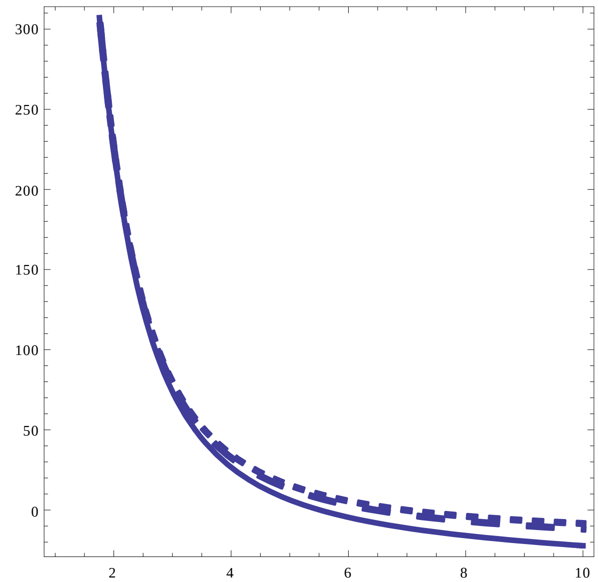

Fig.7. The effects of τ_2 on the Thermal Marangoni number M

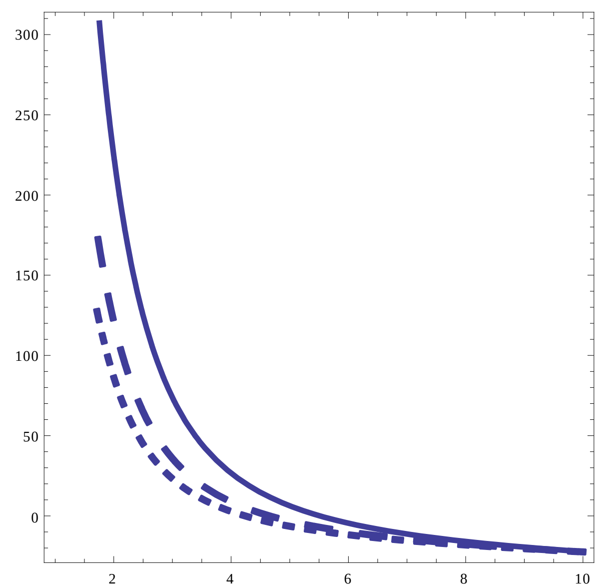

Fig.8. The effects of τ_{m2} on the Thermal Marangoni number M

The effects of the ratio of solute2 diffusivity to thermal diffusivity of the fluid in the porous layer $\tau_{m2} = \dfrac{\kappa_{m2}}{\kappa}$, on the thermal Marangoni number M are shown in Fig.8. The graph has three converging curves. The line curve is for $\tau_{m2} = 0.25$, the big dotted curve is for 0.50 and the small dotted line curve is for 0.75. Since the curves are converging, it indicates that the increasing values of τ_{m2} will have effect only for smaller values of the depth ratio $\zeta = \dfrac{d}{d_m}$, that is for porous layer dominant composite systems. From the curves one can see that for a fixed value of ζ, increase in the value of τ_{m2} is to decease the value of the thermal Marangoni number i.e., to destabilize the system so the onset of surface tension driven convection is faster.

The effects of the viscosity ratio $\hat{\mu} = \dfrac{\mu_m}{\mu}$, which is the ratio of the effective viscosity of the porous matrix to the fluid viscosity are displayed in Fig.9. The line curve is for $\hat{\mu} = 1$, the big dotted curve is for 2 and the small dotted line curve is for 3. Since the curves are converging, it indicates that the increasing values of $\hat{\mu}$ will affect the onset of convection only for the values of $\zeta \leq 10$. From the curves it is evident that for a fixed value of ζ, increase in the value of $\hat{\mu}$ is to increase the value of the thermal Marangoni number M i.e., to stabilize the system, so the onset of surface tention driven triple diffusive convection is delayed. In other words when the effective viscosity of the porous medium μ_m is made larger than the fluid viscosity μ, the onset of the convection in the fluid layer can be delayed.

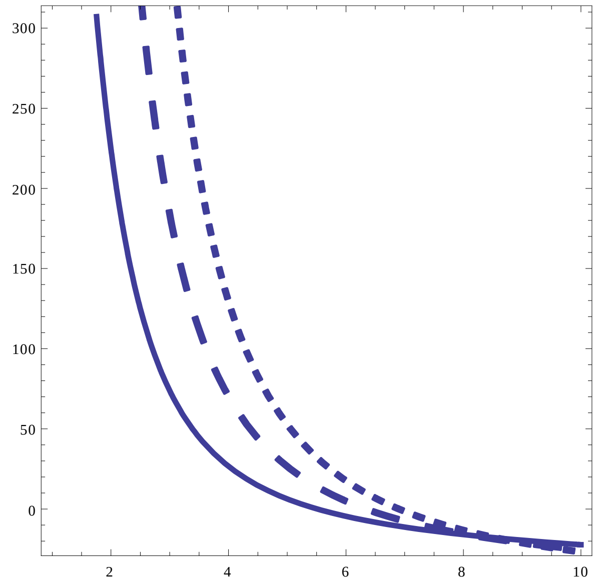

Fig.9. The effects of $\hat{\mu}$ on the Thermal Marangoni number M

6. Conclusions

1. For Fluid layer dominant composite systems, by increasing values of a τ_2 the surface tension driven triple diffusive convection can be delayed.

2. For Porous layer dominant composite systems, by increasing the values of Da, $\hat{\mu}$ and by decreasing the value of τ_{m2} the system can be stabilized.

3. Both the solute Marangoni numbers have similar effects on the convection. They exhibit opposite effects for the fluid layer dominant and porous layer dominant systems.

4. The effect of ratio of solute1 diffusivity of the fluid in fluid layer to porous layer is prominent for a range of values of depth ratio for certain choice of parameters. There is no effect of the ratio of solute2 diffusivity of the fluid in fluid layer to porous layer on the thermal marangoni number.

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