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Even-Odd Cordial Labeling on Zero Divisor Graphs

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Abstract - Graph labeling is a well – establish are of graph theory that has attracted considerable attention due to its wide range of applications. Among the various labeling schemes, even-odd cordial labeling is a recently studied concept. Let G = (V,E) be a graph with vertex set V and edge set E. A bijective function f ; V(G) → $\{1, 2, \ldots\ldots |V(G)|\}$ is called an even – odd cordial labeling if it generates an edge labeling f* : E(G) → $\{0, 1\}$ according to the parity of the labels assigned to the end vertices. Specifically, an edge uv ∈ E(G) receives the label 0 whenever f(u) and f(v) have opposite parity. While it is assigned the label 1 when both vertex labels are either even or odd. The labeling is said to be an even-odd cordial labeling if the absolute difference between the number of edges labeled 0 and those labeled 1 does not exceed one. A graph that admits such a labeling is referred to as an even-odd cordial graph. In this work, we investigate the existence of even – odd cordial labelings for several classes of zero – divisor graphs, including $\Gamma(Z_{np})$, $\Gamma(Z_{pq})$ and $\Gamma(Z_{p^2})$, and establish the conditions under which these graphs satisfy the even-odd cordial property.

Note: Two integeres have the same parity when they are both even or both odd.

Keywords and Phrases: Cordial labeling, Even-odd cordial labeling, Zero divisors, Zero - divisor graphs.

1. INTRODUCTION

Let G = (V(G),E(G)) be a finite, simple, and undirected graph. Graph labeling is a well-established and widely studied branch of graph theory in which integers are assigned to the vertices, edges, or both, subject to specified rules or conditions. Over the years, several types of graph labeling such as graceful, harmonious, cordial, mean, and difference cordial labeling have been extensively studied because of their theoretical significance and wide range of applications in coding theory, communication networks, circuit design, and combinatorial optimization.

Among these labeling methods, cordial labeling and its variants have attracted considerable attention due to their balanced labeling properties. In recent years, several new variations of cordial labeling have been developed by researchers, among which even cordial labeling, odd cordial labeling, and even-odd cordial labeling have gained particular attention. In an even-odd cordial labeling, the vertices of a graph are labeled so that the resulting edge labels are distributed as evenly as possible between 0 and 1 according to the parity of the vertex labels. This concept provides a new direction for studying parity-based labeling structures in graphs.

On the other hand, zero-divisor graphs provide an important link between algebra and graph theory. They were introduced as a graphical tool for studying the algebraic structure of commutative rings through their zero divisors. The notion of the zero divisor graph was first proposed by Istvan Beck. Later, David F.Anderson and Philip S. Livingston [6] further developed and modified this notion. Subsequently, S.P Redmond [5] investigated graph labeing techniques associated with zero-divisors in commutative rings and established several labeling results for particular classes of zero-divisor graphs arising from finite rings. These graphs possess rich structural and combinatorial properties, making them suitable for investigating various graph labeling techniques.

In recent years, various forms of cordial labeling have been extensively investigated on zero-divisor graphs as well as on graphs associated with them. However, the study of even-odd cordial labeling on zero divisor graphs remains limited. Motivated by the growing interest in parity-based graph labeling and the structural properties of

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zero divisor graphs, this paper investigates the existence of even-odd cordial labeling for certain classes of zero divisor graphs

2. PRELIMINARIES

Definition 2.1 Cordial Labeling

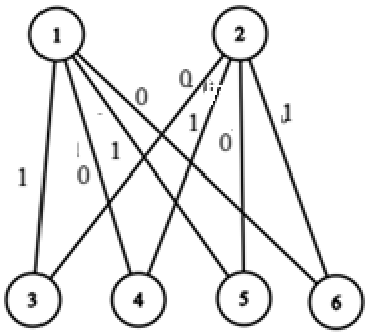
Let G = (V,E) be a graph. A cordial labeling is a binary vertex labeling f: V(G) → {0,1 } such that the number of vertices labeled 0 and 1 differ by at most one. This vertex labeling induces an edge labeling f*: E(G) → {0,1 } defined by f*(uv) = $|f(u) - f(v)|$ for every edge uv ∈ E(G). This labeling is called a cordial labeling if the number of edges labeled 0 and 1 differ by at most one.

Definition 2.2 Even –Odd Cordial Labeling

Let G (V,E) be a graph. Let f be a map from V(G) to {1,2,…,$|V(G)|$ } is bijective such that the induced edge mapping f* : E(G) → {0,1 } an edge uv is assigned the label 0 if f(u) and f(v) have different parities and label 1 if f(u) and f(v) have same parities. If the number of edges labeled with 0 and the number of edges labeled with 1 differ by atmost 1 then it is called even-odd cordial labeling(EOCL) and a graph admits even-odd cordial labeling(EOCL) is called even-odd Cordial Graph(EOCG).

Note: Two integers have the same parity when they are both even or both odd.

Example2.3 The following is a simple example of an even-odd Cordial labeling(EOCL).



EOCL Graph $k_{2,4}$ Figure 2.1

Definition 2.4 Divisor graph

A divisor graph is a graph where vertices represent elements of a set of positive integers, and two vertices are connected by an edge if one integer divides the other.

Definition 2.5 zero Divisors

Let R be a ring , an element a ∈ R,a ≠ 0 is called a zero divisor if there exists b ∈ R,b ≠ 0 such that a . b = 0 or b . a = 0

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Definition 2.6 Zero divisor graph $\Gamma(Z_n)$

The non zero zero - divisor of a commutative ring Z_n with unity forms an undirected graph is called zero divisor graph, if the vertices are the non zero zero divisors of Z_n, and two vertices are adjacent if their product is zero.

Main Section

In this section, we identity the zero-divisor graphs that admit even-odd cordial labeling.

Theorem 3.1. For any prime number (k>2), the zero-divisor graph $\Gamma(Z_{2k})$ is an even-odd cordial graph.

Proof: Let G = $\Gamma(Z_{2k})$, where (k) is a prime number greater than 2. We show that G admits an even-odd cordial labeling. The vertex set of $\Gamma(Z_{2k})$ consists of all nonzero zero divisors of Z_{2k}.

V(G) = $\{2,4,\ldots\ldots.2(k-1),k\}$

Let V(G) = $\{v_1,v_2,\ldots\ldots v_{k-1},v_k\}$

From this, we have the E(G) = $\{v_iv_k \cong 0(mod\ 2k)\backslash 1 \le i \le k-1\}$

$|V(G)|$ = k and $|E(G)|$ = k-1

Define f : V(G) $\rightarrow \{1,2,\ldots\ldots k\}$

f(v_i) = i for $1 \le i \le k$

the induced edge labeling f*: E(G) $\rightarrow \{0,1\}$ is defined as follows

f*$(v_\delta v_k)$ = $\begin{cases} 1 & if\ \delta\ is\ odd\ for\ 1 \le \delta \le k-1 \\ 0 & if\ \ \delta\ is\ even\ for\ 1 \le \delta \le k-1 \end{cases}$

Here $e_{f^*}(1) = \frac{p-1}{2}$ and $e_{f^*}(0) = \frac{p-1}{2}$

Therefore, the labeling fulfills the even – odd cordial condition, namely $|e_{f^*}(1) - e_{f^*}(0)| \le 1$, then $\Gamma(Z_{2k})$ is an even – odd cordial graph for every prime k > 2.

Example 3.2 :

Here is a example of a even-odd cordial labeling of zero divisor graph $\Gamma(Z_{14})$

For k =7, G = $\Gamma(Z_{14})$

$|V(G)|$ = 7 and $|E(G)|$ = 6

V(G) = $\{2,4,6,8,10,12,7\}$ = $\{l_\theta\ /1\ \le \theta \le 7\}$

E(G) = $\{l_\theta l_7 \cong 0(mod\ 14)\backslash 1 \le \theta \le 6\}$

Here f: V(G) $\rightarrow \{1,2,3,4,5,6,7\}$ and is given by

f(l_1) = 1, f(l_2) = 2, f(l_3) = 3, f(l_4) = 4,

f(l_5) = 5, f(l_6) = 6, f(l_7) = 7.

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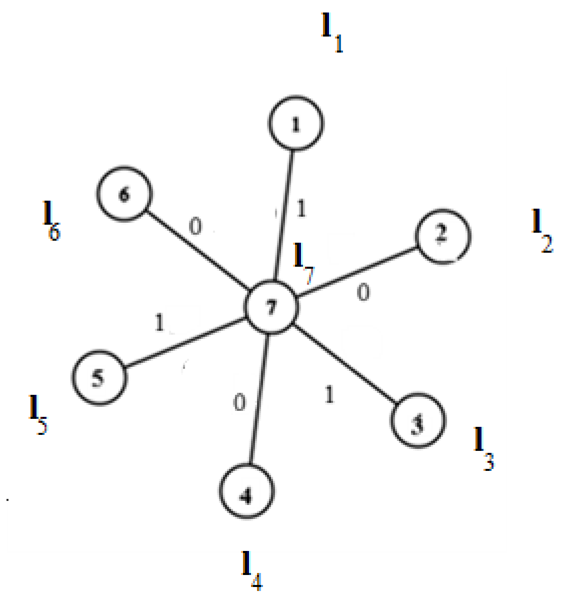

Figure 3.1 E.O.C.L of G = $\Gamma(Z_{14})$

Here $e_{f^*}(1)$ = 3 and $e_{f^*}(0)$ = 3

Consequently, the labeling satisfies $|e_{f^*}(1) - e_{f^*}(0)| \leq 1$, which establishes that the zero – divisor graph $\Gamma(Z_{14})$ is an even – odd cordial graph.

Theorem 3.3. Whenever m is a prime greater than 3, the graph $\Gamma(Z_{3m})$ possesses an even-odd cordial labeling.

Proof. Let G = $\Gamma(Z_{3m})$

Since the nonzero divisors in Z_{3m} are precisely the multiples of 3 and the multiples of m, the vertext set of G is V(G) = $\{3,6, \dots \dots 3(m-1), m, 2m\}$.

Let V(G) = $\{v_1, v_2, \dots \dots v_{m-1}, a_1, a_2\}$

From this, we have the E(G) = $\{v_\delta a_1, v_\delta a_2 \cong 0(mod\ 3m) / \ 1 \leq \delta \leq m-1\}$

$|V(G)|$ = m+1 and $|E(G)|$ = 2m-2

Define f : V(G) → $\{1,2, \dots \dots m+1\}$

$f(v_\delta) = \delta$ for $1 \leq \delta \leq m-1$ and f(a_1) =m and f(a_2) = m+1

The labeling f on the vertices gives rise to an edge labeling f* : E(G) → {0,1}, in which each edge uv ∈ E(G) is labeled according to the following criterion.

$$f^*(v_\tau a_1) = \begin{cases} 1 & if\ \tau\ is\ odd\ for\ 1 \leq \tau \leq m-1 \\ 0 & if\ \tau\ is\ even\ for\ 1 \leq \tau \leq m-1 \end{cases}$$

$$f^*(v_\tau a_2) = \begin{cases} 0 & if\ \tau\ is\ odd\ for\ 1 \leq \tau \leq m-1 \\ 1 & if\ \tau\ is\ even\ for\ 1 \leq \tau \leq m-1 \end{cases}$$

Here $e_{f^*}(1) = m-1$ and $e_{f^*}(0)$ = m-1

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Since $\left|e_{f^*}(1) - e_{f^*}(0)\right| \leq 1$, the graph G fulfills the condition for even-odd cordial labeling. Therefore, $\Gamma(Z_{3m})$ is an EOCL graph whenever m > 3 is prime.

Example 3.4.

Here is a example of a even-odd cordial labeling of zero - divisor graph $\Gamma(Z_{15})$.

For m = 5, G becomes $\Gamma(Z_{15})$

$|V(G)|$ = 6 and $|E(G)|$ = 8

V(G) = $\{3,6,9,12,5,10\} = \{l_1, l_2, l_3, l_4, a_1, a_2\}$

E(G) = $\{l_\delta a_1, l_\delta a_2 \cong 0(mod\ 3m)/\ 1 \leq \delta \leq 4\}$

Here f: V(G) → $\{1,2,3,4,5,6\}$ and is given by

f(l_1) = 1, f(l_2) = 2, f(l_3) = 3, f(l_4) = 4, f(a_1) = 5, f(a_2) = 6.

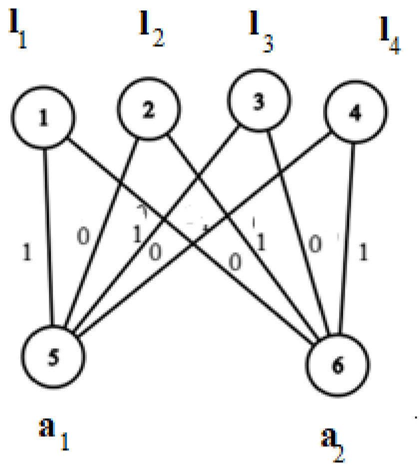

Figure 3.2 E.O.C.L of G = $\Gamma(Z_{15})$

Here $e_{f^*}(1)$ = 4 and $e_{f^*}(0)$ = 4

Thus, the required balance condition, $\left|e_{f^*}(1) - e_{f^*}(0)\right| \leq 1$, is satisfied. Hence, $\Gamma(Z_{15})$ possesses an even-odd cordial labeling.

Theorem 3.5. Let G = $\Gamma(Z_{4s})$, where s is a prime number with s ≥ 2. Then G satisfies the conditions of even-odd cordial labeling, and hence $\Gamma(Z_{4s})$ is an EOCL graph.

Proof. Let G = $\Gamma(Z_{4s})$, for any prime number s, the vertex set of $\Gamma(Z_{4s})$ is

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V(G) = $\{s, 2s, 3s\} \cup \{2,4, \dots \dots .2(s-1),\ 2(s+1), \dots \dots 2(2s-1)\}$

Let V(G) = $\{l_1, l_2, l_3\} \cup \{b_1, b_2, \dots \dots b_{s-1}, b_{s+1}, \dots \dots b_{2s-1}\}$

From this,

we have the E(G) = $\{b_\alpha l_j \cong 0\ (mod\ 4s)\ /\ 1 \leq j \leq 3,\ 1 \leq \alpha \leq s-1,\ s+1 \leq \alpha \leq 2s-1\}$

$|V(G)|$ = 2s+1 and $|E(G)|$ = 4s-4

Define f : V(G) $implies\ to\ \{1, 2, \dots ..2s+1\}$

f(l_1) = 1, f(l_2) = 2, f(l_3) = 3

$$f(b_\alpha) = \begin{cases} 3+\alpha & for\ 1 \leq \alpha \leq s-1 \\ 2+\alpha & for\ s+1 \leq \alpha \leq 2s-1 \end{cases}$$

Based on the vertex labeling, the induced edge function f*: E(G) → {0,1}, is defined according to the following rule.

for r = 1 and 3

$$f^*(u_r v_\alpha) = \begin{cases} 0 & if \alpha\ is\ odd\ in\ 1 \leq \alpha \leq s-1 \\ 1 & if \alpha\ is\ even\ in\ 1 \leq \alpha \leq s-1 \\ 1 & if \alpha\ is\ odd\ in\ s+1 \leq \alpha \leq 2s-1 \\ 0 & if \alpha\ is\ even\ in\ s+1 \leq \alpha \leq 2s-1 \end{cases}$$

$$f^*(u_3 v_\alpha) = \begin{cases} 1 & if \alpha\ is\ odd\ in\ 1 \leq \alpha \leq s-1 \\ 0 & if \alpha\ is\ even\ in\ 1 \leq \alpha \leq s-1 \\ 0 & if \alpha\ is\ odd\ in\ s+1 \leq \alpha \leq 2s-1 \\ 1 & if \alpha\ is\ even\ in\ s+1 \leq \alpha \leq 2s-1 \end{cases}$$

Here $e_{f^*}(1) = 2s-2$ and $e_{f^*}(0)$ = 2s-2

Hence, the induced edge labeling satisfies the condition $|e_{f^*}(1) - e_{f^*}(0)| \leq 1$, Therefore, the zero – divisor graph $\Gamma(Z_{4s})$ admits an even – odd cordial labeling and is consequently an EOCL graph for every prime numbers s > 3.

Example 3.6 :

Here is a example of a even-odd cordial labeling of zero divisor graph $\Gamma(Z_{20})$.

For p = 5, G = $\Gamma(Z_{20})$

$|V(G)|$ = 11 and $|E(G)|$ = 16

V(G) = $\{b_\alpha / 1 \leq \alpha \leq 4,6 \leq \alpha \leq 9\} \cup \{l_1, l_2, l_3\}$

we have the E(G) = $\{b_\alpha b_j \cong 0(mod\ 20) / 1 \leq j \leq 3,1 \leq \alpha \leq 4,6 \leq \alpha \leq 9\}$

Here f: V(G) → {1,2,3,4,5,6,7,8,9,10,11} and is given by

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$f(l_1) = 1$, $f(l_2) = 2$, $f(l_3) = 3$, $f(b_1) = 4$, $f(b_2) = 5$,

$f(b_3) = 6$, $f(b_4) = 7$, $f(b_6) = 8$, $f(b_7) = 9$, $f(b_8) = 10$, $f(b_9) = 11$.

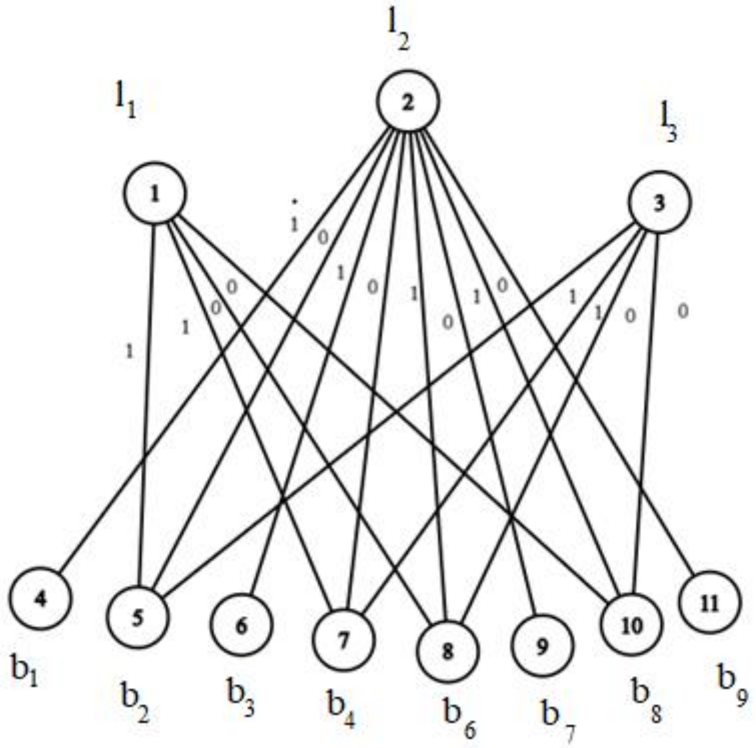

Figure 3.3 E.O.C.L of G = $\Gamma(Z_{20})$

Here $e_{f^*}(1) = 8$ and $e_{f^*}(0) = 8$

Hence, the induced edge labeling satisfies the condition $\left|e_{f^*}(1) - e_{f^*}(0)\right| \leq 1$, Therefore, the zero – divisor graph $\Gamma(Z_{20})$ admits an even-odd cordial labeling and is consequently and EOCL graph.

Theorem 3.7. Whenever t is a prime with t > 5, the graph $\Gamma(Z_{5t})$ possesses an even-odd cordial labeling

Proof. Let G denote the zero-divisor graph $\Gamma(Z_{5t})$. For any prime t, its vertex set is given by

For t > 5, the V(G) = $\{5,10, \dots \dots 5(t-1)\} \cup \{t, 2t, 3t, 4t\}$

Let V(G) = $\{v_1, v_2, \dots \dots v_{t-1}\} \cup \{a_1, a_2, a_3, a_4\}$

From this, we have the E(G) = $\left\{v_\alpha a_\beta \cong 0(mod 5t) \backslash\ 1 \leq \alpha \leq t-1; 1 \leq \beta \leq 4\right\}$

$|V(G)|$ = t+3 and $|E(G)|$ = 4t-4

Define f : V(G) → $\{1,2, \dots \dots t+3\}$

$f(v_\alpha) = \alpha$ for $1 \leq \alpha \leq t-1$ and

$f(a_\beta)$ = t+ β -1 for $1 \leq \beta \leq 4$

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Based on the vertex labeling f, the corresponding induced edge labeling is given by f*: E(G) → {0,1}, with edge labels determined as follows:

$$f^*(v_\alpha\, a_\beta) = \begin{cases} 0 \quad if\ \alpha\ \ is\ odd\ \&\ \beta\ is\ even\ for\ 1 \le \alpha\ \le t-1; 1 \le \beta \le 4 \\ 1 \quad if\ \alpha\ \ is\ odd\&\ \beta\ is\ odd\ for\ 1 \le \alpha\ \le t-1; 1 \le \beta \le 4 \\ 1\ if\ \alpha\ \ is\ even\ \&\ \beta\ is\ even\ for\ 1 \le \alpha\ \le t-1; 1 \le \beta \le 4 \\ 0 \quad if\ \alpha\ \ is\ even\ \&\ \beta\ is\ odd\ for\ 1 \le \alpha\ \le t-1; 1 \le j \le 4 \end{cases}$$

Here $e_{f^*}(1) = 2t - 2$ and $e_{f^*}(0)$ = 2t-2

Thus, the induced edge labeling satisfies the inequality $|e_{f^*}(1) - e_{f^*}(0)| \le 1$, Consequently, the graph G admits an even-odd cordial labeling and is therefore an EOCL graph for every prime number t > 5.

Example 3.8 :

Here is a example of a even-odd cordial labeling of zero divisor graph $\Gamma(Z_{35})$.

For t = 7, G = $\Gamma(Z_{35})$

$|V(G)|$ = 10 and $|E(G)|$ = 24

V(G) = {5,10,15,20,25,30} ∪ {7,14,21,28}

Let V(G) = $\{k_1, k_2, k_3, k_4, k_5, k_6\} \cup \{a_1, a_2, a_3, a_4\}$

E(G) = $\{k_i a_j \cong 0(mod35) \backslash\ 1 \le i \le 6; 1 \le j \le 4\}$

Here f: V(G) → $\{1,2, \dots \dots .10\}$ and is given by

f(k_1) = 1, f(k_2) = 2, f(k_3) = 3, f(k_4) = 4, f(k_5) = 5, f(k_6) = 6,

f(a_1) = 7, f(a_2) = 8, (a_3) = 9, f(a_4) = 10.

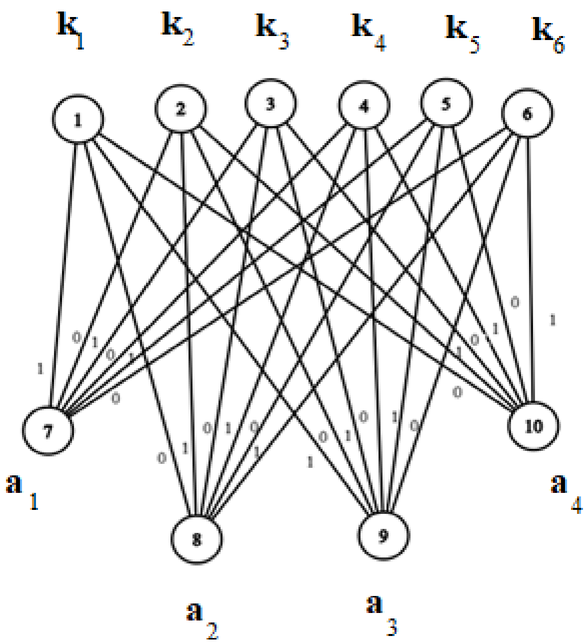

Figure 3.4 E.O.C.L of G = $\Gamma(Z_{35})$

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Here $e_{f^*}(1)$ = 12 and $e_{f^*}(0)$ = 12

Accordingly, the induced edge labeling satisfies the fundamental condition $|e_{f^*}(1) - e_{f^*}(0)| \leq 1$. This establishes that the zero-divisor graph $\Gamma(Z_{35})$ admits an even-odd cordial labeling; hence, $\Gamma(Z_{35})$ is an EOCL graph.

Theorem 3.9. Let h be any prime number satisfying $h \geq 2$. Then the zero – divisor graph $\Gamma(Z_{6h})$ admits an even – odd cordial labeling; consequently, $\Gamma(Z_{6h})$ is an EOCL graph.

Proof.

Case(i): Let h = 2. Then the zero-divisor graph under consideration is G = $\Gamma(Z_{6h}) = \Gamma(Z_{12})$

In this case, G is an even-odd cordial labeling(EOCL) graph. The result follows directly from the previously established fact that $\Gamma(Z_{4h})$ admits an even – odd cordial labeling when h = 3.

Case(ii): Let h = 3. Then G = $\Gamma(Z_{6h}) = \Gamma(Z_{18})$.

An explicit even-odd cordial labeling for the zero – divisor graph G = $\Gamma(Z_{18})$ is given below.

$|V(G)|$ = 11 & $|E(G)|$ = 13

V(G) = $\{2,4,8,10,14,16\} \cup \{3,6,12,15\} \cup \{9\}$

Let V(G) = $V(Z_{18}) = \{t_1, t_2, t_3, t_4, t_5, t_6\} \cup \{m_1, m_2, m_3, m_4\} \cup \{w_1\}$

we have the E(Z_{18}) = $\{(c,d) : c,d \in V(Z_{18}),\ c \neq d, cd \equiv 0(mod 18)\}$

Here f: V(G) → $\{1,2, \dots \dots .11\}$ and is given by

$f(t_1) = 1$, $f(t_2) = 2$, $f(t_3) = 3$, $f(t_4) = 4$, $f(t_5) = 5$, $f(t_6) = 6$,

$f(m_1) = 7$, $f(m_2) = 9$, $f(m_3) = 8$, $f(m_4) = 10$, $f(w_1) = 11$.

Here $e_{f^*}(1)$ = 6 and $e_{f^*}(0)$ = 7

Hence, the induced edge labeling satisfies the condition $|e_{f^*}(1) - e_{f^*}(0)| \leq 1$. Therefore, the zero – divisor graph $\Gamma(Z_{18})$ admits an even – odd cordial labeling and is consequently an EOCL graph.

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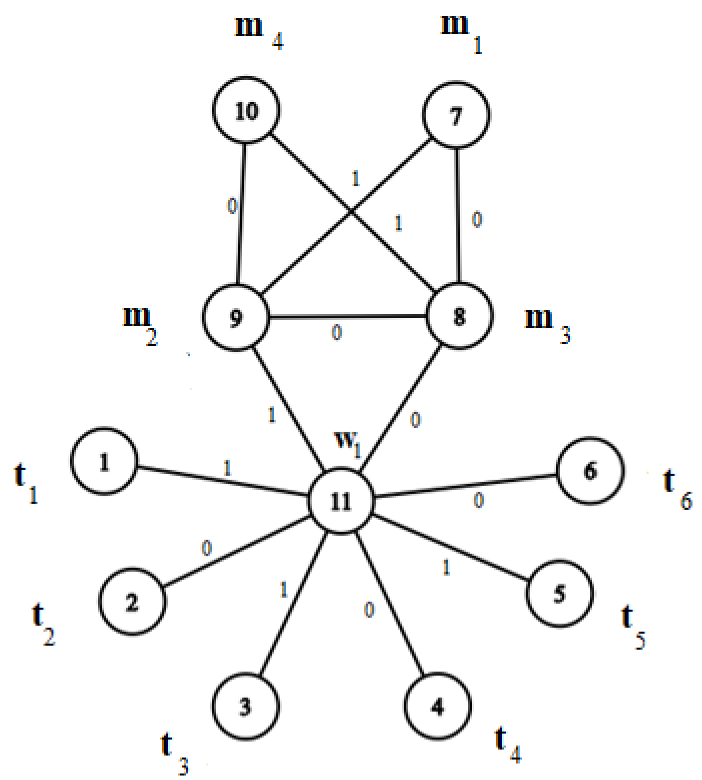

Figure 3.5 E.O.C.L of G = $\Gamma(Z_{18})$

Case(iii): For a prime number h > 3, Let G = $\Gamma(Z_{6h})$. The vertex set of G consists precisely of the nonzero zero – divisors of the ring Z_{6h}. Moreover, this set can be partitioned into six pairwise disjoint subsets, namely L, M, N, R, S, and T.

Therefore, V $\Gamma(Z_{6h})$ = L $\cup M \cup N \cup R \cup S \cup T$.

Where L = $\{x \in Z_{6h}: \gcd(x, 6h) = 2\}$, M = $\{x \in Z_{6h}: \gcd(x, 6h) = 3\}$,

N = $\{x \in Z_{6h}: \gcd(x, 6h) = 6\}$, R = $\{x \in h: \gcd(x, 6h) = h\}$ = $\{h, 5h\}$,

S = $\{x \in Z_{6h}: \gcd(x, 6h) = 2h\}$ = $\{2h, 4h\}$,T = $\{x \in Z_{6h}: \gcd(x, 6h) = 3h\}$ = $\{3h\}$

Let V(Z_{6h}) = $\{l_1, l_2, .. l_{2h-2}\} \cup \{m_1, m_2 \dots m_{h-1}\} \cup \{n_1, n_2 \dots n_{h-1}\} \cup \{a_h, a_{5h}\} \cup \{b_{2h}, b_{4h}\} \cup \{c_{3h}\}$

From this,

we have the E $\Gamma(Z_{6h})$= $\{(k, t) : k, t \in V(Z_{6h}),\ k \neq t, kt \equiv 0(mod 6h)\}$

$|V(G)|$ = 4h+1 & $|E(G)|$ = 9h-7

Define f : V(G) to $\{1,2, \dots .. 4h + 1\}$

f(c_{3h}) = 4h+1, f(b_{2h}) = 4h, f(b_{4h}) = 4h-1, f(a_h) = 4h-2, f(a_{5h}) = 4h-3

f(l_α) = α $\quad for\ 1 \leq \alpha \leq 2h - 2$,

f(m_β) = $\beta + 2h - 2$ $\quad for\ 1 \leq \beta \leq h - 1$,

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f(n_γ) = $\gamma + 3h - 3 \qquad for\ 1 \le \gamma \le h-1$,

The vertex labeling f uniquely induces an edge labeling f^*: E(G) → {0,1}, where the label assigned to each edge is determined by the following rule.

$f^*(c_{3h} l_\alpha) = \begin{cases} 1 & if\ \alpha\ is\ odd\ in\ 1 \le \alpha \le 2h-2 \\ 0 & if\ \alpha\ is\ even\ in\ 1 \le \alpha \le 2h-2 \end{cases}$

$f^*(c_{3h}\, n_\gamma) = \begin{cases} 1 & if\ \gamma\ is\ odd\ in\ 1 \le \gamma \le h-1 \\ 0 & if\ \gamma\ is\ even\ in\ 1 \le \gamma \le h-1 \end{cases}$

$f^*(b_{2h}\, n_\gamma)\ \&\ f^*(a_h\, n_\gamma) = \begin{cases} 0 & if\ \gamma\ is\ odd\ in\ 1 \le \gamma \le h-1 \\ 1 & if\ \gamma\ is\ even\ in\ 1 \le \gamma \le h-1 \end{cases}$

$f^*(b_{4h}\, n_\gamma)\ \&\ f^*(a_{5h}\, n_\gamma) = \begin{cases} 1 & if\ \gamma\ is\ odd\ in\ 1 \le \gamma \le h-1 \\ 0 & if\ \gamma\ is\ even\ in\ 1 \le \gamma \le h-1 \end{cases}$

$f^*(b_{2h}\, m_\beta) = \begin{cases} 0 & if\ \beta\ is\ odd\ in\ 1 \le \beta \le h-1 \\ 1 & if\ \beta\ is\ even\ in\ 1 \le \beta \le h-1 \end{cases}$

$f^*(b_{4h}\, m_\beta) = \begin{cases} 1 & if\ \beta\ is\ odd\ in\ 1 \le \beta \le h-1 \\ 0 & if\ \beta\ is\ even\ in\ 1 \le \beta \le h-1 \end{cases}$

$f^*(c_{3h} b_{4h}) = 1$ & $f^*(c_{3h} b_{2h}) = 0$

Here $e_{f^*}(1) = \frac{9h-7}{2}$ and $e_{f^*}(0) = \frac{9h-7}{2}$

Accordingly, the inequality $|e_{f^*}(1) - e_{f^*}(0)| \le 1$ holds for the induced edge labeling. This establishes that the zero – divisor graph $\Gamma(Z_{6h})$ possesses an even – odd cordial labeling and is therefore and EOCL graph whenever h is a prime number with h ≥ 2.

Example 3.10 :

When h = 5, the graph G = $\Gamma(Z_{6h}) = \Gamma(Z_{30})$ admits an even-odd cordial labeling. One such labeling is presented below.

For h = 5, G = $\Gamma(Z_{30})$

$|V(G)|$ = 21 and $|E(G)|$ = 38

V(G) = {2,4,8,14,16,22,26,28} ∪ {9,10,11,12} ∪ {6,12,18,24} ∪ {5,25} ∪ {10,20} ∪ {15}

Let V(G) = $V(Z_{30})$ =

$\{l_1, l_2, \dots . l_8\} \cup \{m_1, m_2, m_3, m_4\} \cup \{n_1, n_2, n_3, n_4\} \cup \{a_5, a_{25}\} \cup \{b_{10}, b_{20}\} \cup \{c_{15}\}$

we have the E(Z_{30}) = $\{(k,t) : k, t \in V(Z_{30}),\ k \ne t, kt \equiv 0 (mod 30)\}$

Here f: V(G) → {1,2, … ….21} and is given by

f(l_1) = 1, f(l_2) = 2, f(l_3) = 3, f(l_4) = 4, f(l_5) = 5, f(l_6) = 6, f(l_7) =7, f(l_8) = 8;

f(c_{15}) = 21, f(b_{10}) = 20, f(b_{20}) = 19, f(a_5) = 18, f(a_{25}) = 17

f(n_1) = 13, f(n_2) = 14, f(n_3) = 15, f(n_4) = 16,

f(m_1) = 9, f(m_2) = 10, f(m_3) = 11, f(m_4) = 12.

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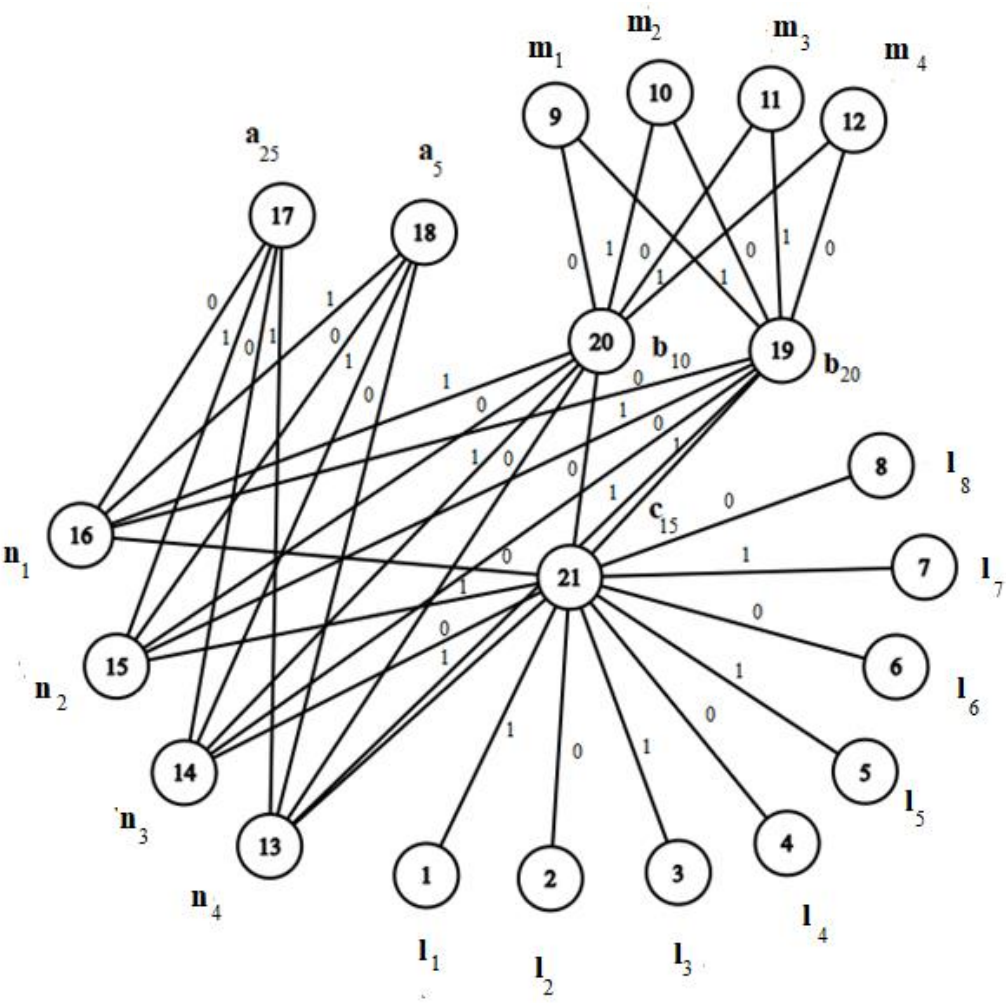

Figure 3.6 E.O.C.L of G = $\Gamma(Z_{30})$

Hence, the induced edge labeling satisfies the inequality $|e_{f^*}(1) - e_{f^*}(0)| \leq 1$. Consequently, the zero – divisor graph $\Gamma(Z_{30})$ admits an even – odd cordial labeling and is therefore an EOCL graph.

Theorem 3.11. Let g be a prime number satisfying g > 7. Then the zero – divisor graph $\Gamma(Z_{7g})$ admits an even – odd cordial labeling. Consequently, $\Gamma(Z_{7g})$ is an EOCL graph.

Proof. Let G = $\Gamma(Z_{7g})$, for any prime number p, the vertex set of $\Gamma(Z_{7g})$ is

V(G) = $\{7,14, \dots \dots 7(g-1)\} \cup \{g, 2g, 3g, 4g, 6p\}$

Let V(G) = $\{v_1, v_2, \dots \dots v_{g-1}\} \cup \{a_1, a_2, a_3, a_4, a_5, a_6\}$

From this, we have the E(G) = $\{v_\delta a_\theta \cong 0(mod 7g) \backslash\ 1 \leq \delta \leq g-1; 1 \leq \theta \leq 6\}$

$|V(G)|$ = g+5 and $|E(G)|$ = 6(g-1)

Define f : V(G) to $\{1,2, \dots \dots g+5\}$

f(v_δ) = δ for $1 \leq \delta \leq g-1$ and

f(a_θ) = g+ θ -1 $1 \leq \theta \leq 6$

the induced edge labeling is as follows f*: E(G) → {0,1} is given by

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$$f^*(v_\delta a_\theta) = \begin{cases} 0 \;\; if\ \delta\ is\ odd\ \ \&\ \theta\ is\ even\ for\ 1 \le \delta \le g-1; 1 \le \theta \le 6 \\ 1 \;\; if\ \delta\ is\ odd\&\ \theta\ is\ odd\ for\ 1 \le \delta \le g-1; 1 \le \theta \le 6 \\ 1\ if\ \delta\ is\ even\ \ \&\ \theta\ is\ even\ for\ 1 \le \delta \le g-1; 1 \le \theta \le 6 \\ 0 \;\; if\ \delta\ is\ even\ \ \&\ \ \theta\ is\ odd\ for\ 1 \le v \le g-1; 1 \le \theta \le 6 \end{cases}$$

Here $e_{f^*}(1) = 3g - 3$ and $e_{f^*}(0)$ = 3g-3

Hence, the induced edge labeling satisfies the inequality $\left|e_{f^*}(1) - e_{f^*}(0)\right| \le 1$. Consequently, G admits an even – odd cordial labeling and is therefore an EOCL graph for every prime number g.

Theorem 3.12. Let r and s be prime numbers. Then the zero – divisor graph $\Gamma(Z_{rs})$ admits an even – odd cordial labeling. Consequently, $\Gamma(Z_{rs})$ is an EOCL graph.

Proof.

For distinct prime number r and s , let G = $\Gamma(Z_{rs})$. The vertex set of the zero divisor graph consists of all non-zero multiples of r and s modula rs, and is therefore given by

V(G) = $\{r, 2r, \dots . r(s-1)\} \cup \{s, 2s, \dots \dots s(s-1)\}$

Let V = $\{v_1, v_2, \dots \dots v_{r-1}\} \cup \{a_1, a_2, \dots . . a_{s-1}\}$

From this, we have the E(G) = $\left\{v_\alpha a_\beta \cong 0(mod rs) \backslash\ 1 \le \alpha \le r-1; 1 \le\ \beta \le s-1\right\}$

$|V(G)|$ = r+s - 2 and $|E(G)|$ = (r-1)(s-1)

Define f : V(G) to $\{1,2, \dots . . r+s-2\}$

f(v_α) = α for $1 \le \alpha \le r-1$ and

f(a_β) = r + β -1 $1 \le\ \beta \le s-1$

Based on the vertex labeling f, the corresponding edge labeling is induced through the mapping f*: E(G) → {0,1}, defined as follows.

$$f^*(v_\alpha a_\beta) = \begin{cases} 0 \;\; if\ \alpha\ is\ odd\ \ \&\ \beta\ is\ even\ for\ 1 \le \alpha \le r-1; 1 \le\ \beta \le s-1 \\ 1 \;\; if\ \alpha\ is\ odd\ \&\ \beta\ is\ odd\ for\ 1 \le \alpha \le r-1;\ 1 \le\ \beta \le s-1 \\ 1\ if\ \alpha\ is\ even\ \&\ \ \beta\ is\ even\ for\ 1 \le \alpha \le r-1; 1 \le\ \beta \le s-1 \\ 0 \;\; if\ \alpha\ is\ even\ \&\ \ \beta\ is\ odd\ for\ 1 \le \alpha \le r-1; 1 \le\ \beta \le s-1 \end{cases}$$

Here $e_{f^*}(1) = \frac{(r-1)(s-1)}{2}$ $e_{f^*}(0) = \frac{(r-1)(s-1)}{2}$

Hence $\left|e_{f^*}(1) - e_{f^*}(0)\right| \le 1$, implying that $\Gamma(Z_{rs})$ is an EOCL graph for all distinct prime numbers r and s.

Theorem 3.13. Whenever k is a prime integer, the zero – divisor graph $\Gamma(\ Z_{k^2})$ satisfies the condtions of an even – odd cordial labeling and is therefore an EOCL graph.

Proof. Let G = $\Gamma(\ Z_{k^2})$, where k is a prime. The vertex set of G, comprising all non-zero divisors of Z_{k^2}, is expressed as

V(G) = $\{k, 2k, \dots \dots (k-1)k\}$

Let V(G) = $\left\{m_1, m_2, \dots \dots m_{p-1}\right\}$

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From this, we have the E(G) = $\{m_i m_j \backslash\ 1 \le i, j \le k-1\}$

$|V(G)|$ = (k-1) & $|E(G)|$ = $\frac{(k-1)(k-2)}{2}$

Define f : V(G) to $\{1,2, \ldots\ldots (k-1)\}$

f(v_α) = α for $1 \le \alpha \le k-1$

The edge labels are obtained from the vertex labeling f through the function f*: E(G) → {0,1}, defined as follows.

$$f^*(m_\alpha m_\beta) = \begin{cases} 0 \;\; if\ \alpha\ is\ odd\ \&\ \beta\ is\ even\ for\ 1 \le \alpha, \beta \le k-1 \\ 1 \;\; if\ \alpha\ is\ odd\&\ \beta\ is\ odd\ for\ 1 \le \alpha, \beta \le k-1 \\ 1 \;\; if\ \alpha\ is\ even\ \&\ \beta\ is\ even\ for\ 1 \le \alpha, \beta \le k-1 \\ 0 \;\; if \alpha\ is\ even\ \&\ \beta\ is\ odd\ for\ 1 \le \alpha, \beta \le p-1 \end{cases}$$

Here $e_{f^*}(0) = \frac{\frac{(k-1)(k-2)}{2}+1}{2}$ $\;\; e_{f^*}(1) = \frac{\frac{(k-1)(k-2)}{2}-1}{2}$

The above construction ensures that $|\, e_{f^*}(1) - e_{f^*}(0) \,| \le 1$, thereby proving that G is an EOCL graph for every prime number k.

Example 3.14 :

The following illustrates a valid even – odd cordial labeling of the zero – divisor graph G = $\Gamma(Z_{49})$

The following illustrates a valid even-odd cordial labeling of the zero-divisor graph $G = \Gamma(\mathbb{Z}_{49})$.

For k = 7, G = $\Gamma(Z_{49})$

$|V(G)|$ = 6 and $|E(G)|$ = 15

V(G) = {7,14 ….42}

Let V(G) = $\{m_1, m_2, m_3, m_4, m_5, m_6\}$

From this, we have the E(G) = $\{m_i m_j \backslash\ 1 \le i \le j \le 6\}$

Here f: V(G) → {1,2, … … 6} and is given by

f(m_1) = 1, f(m_2) = 2, f(m_3) = 3,

f(m_4) = 4, f(m_5) = 5, f(m_6) = 6

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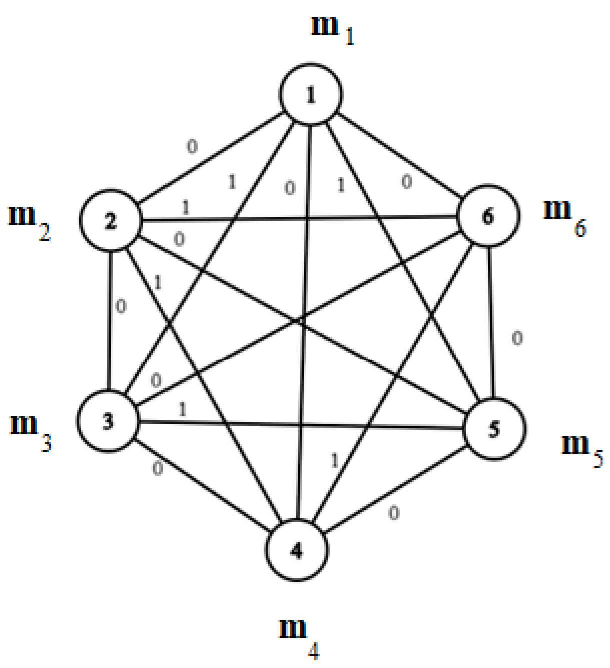

Figure 3.7 E.O.C.L of $G = \Gamma(Z_{49})$

Here $e_{f^*}(0) = 8$ and $e_{f^*}(1) = 7$

Accordingly, the established edge labeling meets the even – odd cordiality criterion, confirming that $\Gamma(Z_{49})$ is an EOCL graph.

CONCLUSION:

In closing, we investigate that cycle of zero divisor graphs

i) For every positive integer n and every prime number p for which $\Gamma(\mathrm{Z}_{\mathrm{np}})$ is defined under the prescribed conditions, the Zero – divisor graph $\Gamma(\mathrm{Z}_{\mathrm{np}})$ is even – odd cordial graph.

ii) Whenever p is prime, the zero – divisor graph associated with $\mathrm{Z}_{\mathrm{p}^2}$, denoted by $\Gamma(\mathrm{Z}_{\mathrm{p}^2})$, is an EOCL graph.

iii) For every pair of distinct prime numbers p and q, the zero – divisor graph $\Gamma(\mathrm{Z}_{\mathrm{pq}})$, is an EOCL graph.

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