

Evaluation of Some Raindrop Size Distribution Models for Different Rain Rates

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Abstract— Results from the performance of the Negative exponential and Gamma function distribution model in a tropical location is presented in this work. The inputs to the drop size distribution (DSD) models have been obtained from Heipang, Nigeria (9.04°N, 7.5°E) using the pointing Micro Rain Radar (MRR). The effect of the raindrop diameter on the drop size distribution models (DSD) is investigated over the operating frequency range of 4 GHz to 40 GHz. The estimated rain rate for 99.99% availability for Heipang was used as input to the DSD models. From the results obtained, we observe that the highest drop size distribution for the diameter range $3.5 \text{ mm} \leq D \leq 5.0 \text{ mm}$ was 24.44 for a rainfall rate of 120 mm/h considering the Gamma function distribution. While that of the negative exponential distribution model was 88.87 for the range $3.0 \text{ mm} \leq D \leq 5.0 \text{ mm}$. Results further suggest that a raindrop size of 0.7 mm can cause link outage for communication links operating with a frequency of 40 GHz. This is because of a wavelength large factor that ranged between 10.714 to 1.875. This study also shows that increasing rain rates of the convective rain types arise from the combined increase of the raindrop diameters across all frequencies. The results obtained from this study are useful for adequate network design and planning for achieving a higher quality of service across radio communication links.

Keywords—Raindrop size, Wireless communication, Gamma function, Radio communication, Micro rain radar

I. INTRODUCTION

Modeling raindrop size distributions (DSD) is fundamental for connecting remote sensing observations with reliable precipitation products for hydrological applications [1]-[3]. To date, various standard probability distributions have been proposed to build DSD models [4], [5]. Microstructural parameters necessary for the prediction of attenuation due to rainfall includes the rainfall rate and raindrop size distribution (DSD). The DSD directly contributes to the determination of rainfall attenuation. The determination of attenuation due to rain is carried out by short-term or long-term studies of rainfall rates for the location of interest which depends on the availability of rainfall data. Rainfall being a complex phenomenon is mostly random, especially when considering the active natural variables that induce it. Studies from [6]-[9] show the practicability of using rainfall rates for the prediction of attenuation in different parts of the world. In this study, we test the performance of the negative exponential distribution model proposed by [10] and the Gamma function distribution model proposed by [11]. A generalized function of which the

negative exponential is a special case, the gamma function was first proposed as a raindrop size distribution by [12] in the form:

$$N(D) = N_0 D^p \exp(-\Lambda D^q) \quad (1)$$

The methods from [10], [11] were used to estimate DSD values for the tropical location with frequency range 4 GHz to 40 GHz using the following rainfall rates as input: 40 mm/h, 60 mm/h, 80 mm/h and 120 mm/h. Since rainfall rate has been exhaustively used to predict attenuation in different parts of the world, the DSD models are used to determine their suitability in a tropical location. Internet service providers in a bid to improve the quality of service to customers, try to ensure the availability of services by compensating for errors due to attenuation resulting from rainfall through several corrective schemes such as base station power control algorithms. However, to adequately determine the amount of power to be added, proper estimations of attenuation values and fade margins are needed. To this end, the effect of raindrop diameters on DSD models becomes necessary. The effect of rain attenuation in microwave and satellite communication becomes increasingly disturbing at frequencies above 10 GHz [8], [13]-[15]. Since higher frequencies are characterized with smaller wavelengths, the effects from the rain droplets in the rainy medium often lead to signal absorption and scattering, and therefore, signal outage and deterioration [16]. Several statistical models have been used for the drop size distribution procedure. Popular among these statistical models include Marshall-Palmer negative exponential model, modified gamma model, and Weibull raindrop size distribution model [17]. The availability of rainfall DSD measurements provides a much more qualitative insight for radio engineers since it takes into reckoning the mechanics of rainfall microstructure [16]. Efforts have been made in the past to identify appropriate statistical models to represent rainfall DSD [18]. This is noticeable in the array and robustness of such models currently in use. Signal outages reduce bandwidth efficiency and spectrum utilization, which comes at a great cost to providers of network services [19], [20]. According to [21]-[24] the most deleterious factor to signal propagation in satellite/terrestrial link design is rainfall. Over the years, demand for utilization of higher frequencies has become paramount hence studies on drop size distribution models have become necessary.

II. RESEARCH METHOD

The equipment used for the collection of data is the Micro Rain Radar equipment located at Heipang, Nigeria (9.04°N, 7.5°E). The radar equipment operates at a frequency of 26GHz and is a frequency modulated continuous wave. The determination of the rain cell statistics is carried out by a conversion of measured Doppler spectra into drop diameters which enables the realization of the radar reflectivity (Z), liquid water content (W), and rain rates. The spectral volume reflectivity with depth as realized by [22] is given as follows:

$$\eta(r, f)df = p(r, f)df c \frac{r^2}{\delta r} t^{-1}(r) \quad (2)$$

Where: $p(r, f)$ = spectral power, f = doppler frequency in Hz, C = radar constant.

The DSD can be estimated from the volume reflectivity $\eta(D)$ as follows:

$$N(D) = \frac{\eta(D)}{\sigma_D} \quad (3)$$

Where: $N(D)$ = number of drops with size D to $D+\Delta D$ in m/s. Mean fall velocity V_m is given as:

$$V_m = \frac{\lambda \int_0^\infty f.P(f)df}{2 \int_0^\infty P(f)df} \quad (4)$$

Where: $P(f)$ = spectral power, λ = wavelength.

The DSD model is given by [10] and has the general form:

$$N(D) = N_0 \exp(-\Lambda D) \quad (5)$$

Where: $N(D)$ = concentration of raindrops per diameter (mm), D = rain drop diameter, N_0 = intercept parameter given as $8 \times 10^3 \text{ mm}^{-1}\text{m}^{-3}$, Λ = slope parameter (mm)

$$\Lambda = 4.1R^{-0.21}\text{mm}^{-1} \quad (6)$$

Where: R = rainfall rate (mm/h)

The DSD model derived by [11] has the general form:

$$N(D) = N_0 D^p \exp(-\Lambda D^q) \quad (7)$$

Where: $N_0 = 1.76 \times 10^6 R^{-1.2}$, $p = 2$, $\Lambda = 10R^{-0.31}$, $q = 1$

To estimate the terminal velocity $v(D)$, we can apply the following equation:

$$v(D) = 9.65 - 10.30 \exp(-0.6D) + 0.65 \exp(-7D) \quad (8)$$

III. RESULTS AND DISCUSSION

Performance of the negative exponential and Gamma function distribution models are presented in Figures (1a)-(1d), (2a)-(2d). The figures show that as the mean diameter of raindrops increase, raindrop size distribution decreases.

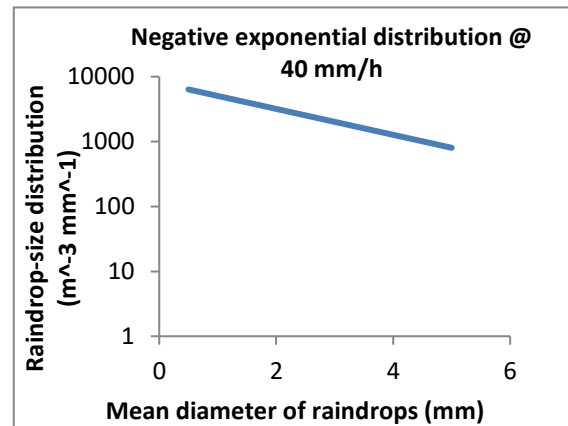


Fig. 1a. Drop size distribution @ 40 mm/h

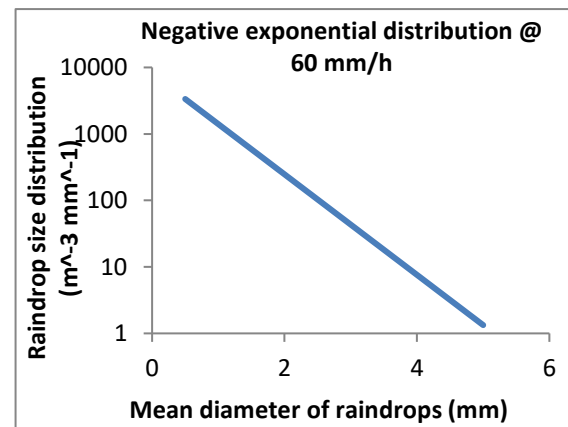


Fig. 1b. Drop size distribution @ 60 mm/h

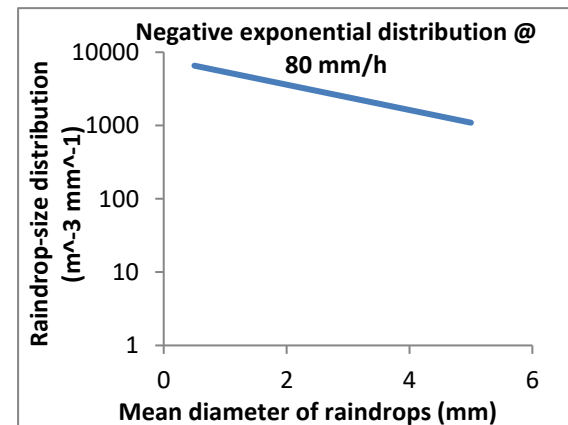


Fig. 1c. Drop size distribution @ 60 mm/h

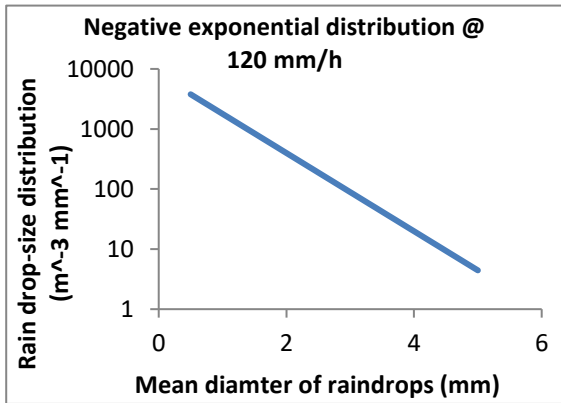


Fig. 1d. Drop size distribution @ 120 mm/h

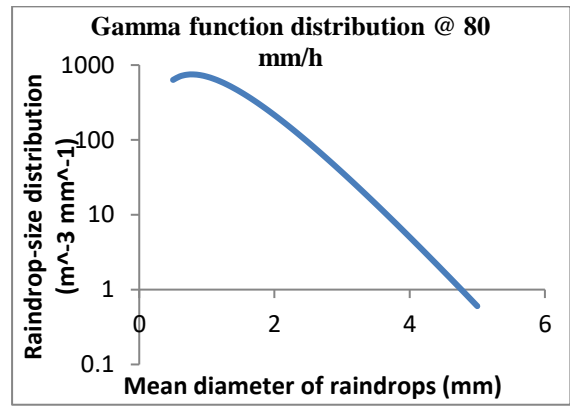


Fig. 2c. Gamma drop size distribution @ 80 mm/h

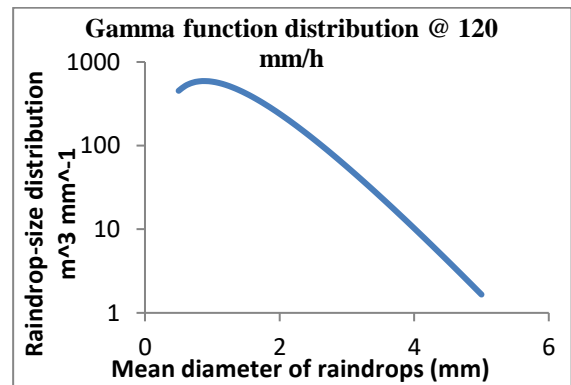


Fig. 2d. Gamma drop size distribution @ 120 mm/h

From the radar data obtained from Heipang, Nigeria, rainfall lasted for the period ranging from 1:08 to 2:23AM (18th August 2020) with the highest recorded rain rate and liquid water content (LWC) of 120mm/h and 28.55 respectively which occurred around 1:22 AM. The contribution formed by raindrop diameter intervals to the overall raindrop size distribution for different rain rates is illustrated in Figures (1a), (1b), (1c), (1d), (2a), (2b), (2c), (2d). The contribution of larger drop diameters as observed in the results show smaller raindrop size distributions and hence lesser attenuation values. For instance, in Figure (2d), the highest contribution in the diameter range $3.5 \text{ mm} \leq D \leq 5.0 \text{ mm}$ is 24.44 for a rainfall rate of 120mm/h considering the Gamma function distribution.

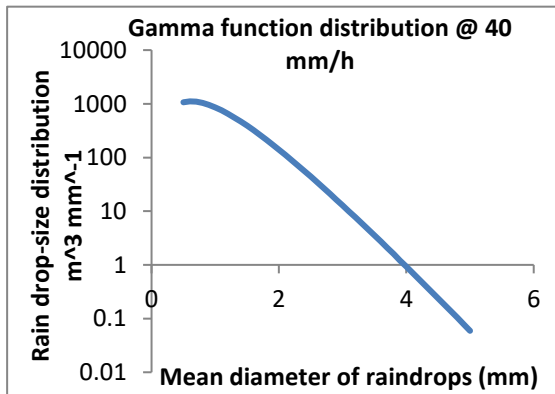


Fig. 2a Gamma drop size distribution @ 40 mm/h

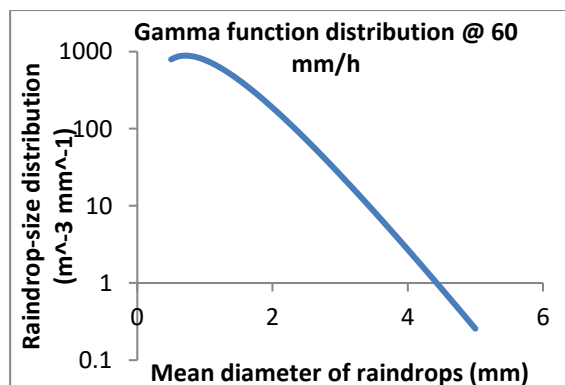


Fig. 2b Gamma drop size distribution @ 60 mm/h

Considering the negative exponential distribution proposed by [10] for the range $3.0 \text{ mm} \leq D \leq 5.0 \text{ mm}$ we obtained maximum raindrop size distribution of 88.87 for a rainfall rate of 120mm/h as shown in Figure (1d). Figures (1a), (1b), (1c), (1d), (2a), (2b), (2c) and (2d) show that for drop diameters ranging $0.5 \text{ mm} \leq D \leq 3.0 \text{ mm}$ we realize high drop size distributions of 452.38 and 55.87 for drop diameters 0.5mm and 3mm respectively. This implies that the largest contributions to attenuation are due to drop diameters not exceeding 3.5 mm for all rainfall regimes at all frequencies and this confirms the results obtained in other tropical locations such as Malaysia and Singapore. Hence, the diameter ranges $0.5 \text{ mm} \leq D \leq 3 \text{ mm}$ are critical to attenuation in Nigeria being a tropical region characterized by convective rainfall.

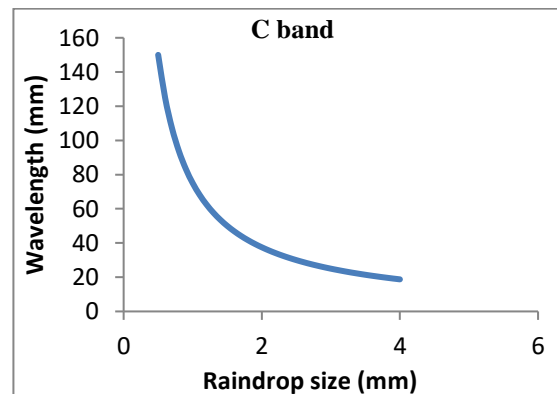


Fig. 3a. Wavelength large factor for C band

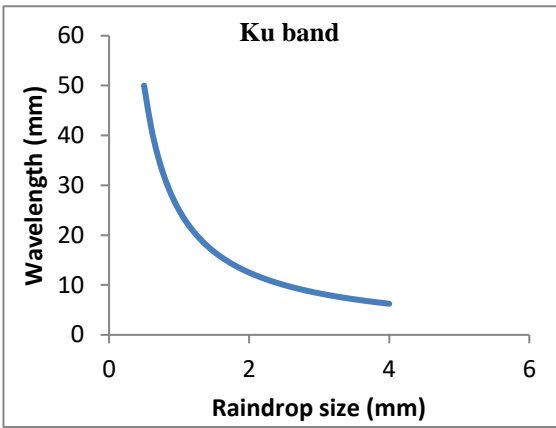


Fig. 3b. Wavelength large factor for Ku band

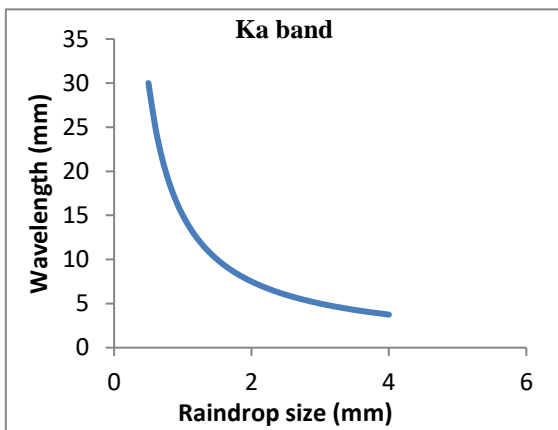


Figure. 3c. Wavelength large factor for Ka band

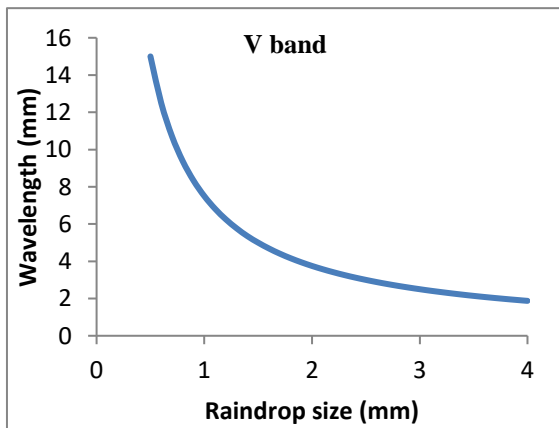


Fig. 3d Wavelength large factor for V band

The wavelength large factors shown in Figures (3a), (3b), (3c), and (3d) are obtained by dividing the wavelength range for the respective bands (C band, Ku band, Ka-band, and V band) and their corresponding raindrop size. The wavelength large factor is denoted by lambda (λ) and is given as follows:

$$\text{wavelength large factor } (\lambda) = \frac{\text{wavelength range (mm)}}{\text{raindrop size (mm)}} \quad (9)$$

The wavelength range for the C band, Ku band, Ka-band, and V bands is 7.5 cm to 3.75 cm, 2.5 cm to 1.67 cm, 11.1 mm to 7.5 mm, 500 nm to 700 nm respectively. Reference raindrop sizes ranged from 0.5 mm to 4.0 mm. From Figure (3a), we observe that the wavelength is large by a factor ranging between 150 to 75 for a raindrop size range of 0.5 mm to 1mm. Hence the effect of the raindrop size on the link can be seen as insignificant. For raindrop size ranging 1.1 mm to 1.6 mm a λ large factor of 68.1818 to 46.8750 is realized. For raindrop sizes ranging from 1.2 mm to 4.0 mm, λ large factor ranged between 62.5 to 18.75. Due to the large λ factors obtained as shown in Figure (3a), it can be said that the raindrop size effect on the C band is insignificant. Considering Figure (3b) λ large factors ranged between 50 to 10.869 for raindrop size of 0.5 mm to 2.3 mm. Figure (3b) further suggests that for a raindrop size of 2.3 mm the effect of the raindrop begins to be significant on the link and could result in to link outage. For the raindrop size range of 2.4 mm to 4.0 mm, a continuous drop in λ large factor is realized which ranged between 10.4166 to 6.25. For the Ka-band link shown in Figure (3c), raindrop size begins to severely affect the link at a size of 1.5 mm. Drop sizes ranging between 1.5 mm to 4.0 mm produced a λ large factor ranging between 10 to 3.75. Figure (3d) suggests that the drop size effect on link outage for V bands starts at approximately 0.7 mm. This is because of a λ large factor that ranged between 10.714 to 1.875.

IV. CONCLUSION

This study compares and discusses the effect of raindrop diameters for raindrop size distributions in Nigeria. The effect of raindrop size on radio links for the C, Ku, Ka, and V bands is investigated. Results suggest that the V band is prone to severe attenuation effect as raindrop size as small as 0.7 mm could result to link outage. Furthermore, results suggest that for the convective rain types, the maximum drop diameters lead to a reduced raindrop size distribution suggesting that increasing rain rates of the convective rains arise from the combined increases of the raindrop diameters across all frequencies. Results from this study provide a good understanding of raindrop diameters as it relates to drop size distribution and rainfall attenuation characteristics which can be helpful to radio engineers for proper network design and planning.

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