

Evaluation of Fracture Parameters using Probabilistic Fracture Mechanics

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Abstract - Probabilistic fracture mechanics (PFM) is becoming increasingly popular for realistic evaluation of fracture response and reliability of cracked structures. Standardized fracture specimens are being used for fracture analysis and the results are correlated to the actual geometry through fracture models. The probabilistic fracture-mechanics analysis of linear elastic cracked specimens using ANSYS Probabilistic Design System is carried out. Finite element method to Linear Elastic Fracture-Mechanics (LEFM) analysis and probabilistic analysis are integrated. The probabilistic analysis is performed with respect to uncertain parameters like stresses, crack size on stress intensity factor for mode I. The variation of the stress intensity factor (K_I), given the uncertainty of the crack dimensions, material properties, and applied stress are determined. The probability that the maximum stress concentration factor remains below a certain value is determined. The limiting values of the output for a given target probability is also determined to ensure the reliability of the design system. Cumulative distribution curves for the parameters are produced. Sensitiveness of results with respect to the variability of the input parameters is quantified. Theoretical results are also obtained and compared with ANSYS PDS results.

Keywords: Probabilistic Fracture Mechanics (PFM), Stress Intensity Factor (SIF), Centre crack specimen, linear elastic fracture mechanics (LEFM), Finite element analysis (FEA).

I. INTRODUCTION

1.1. probabilistic fracture mechanics (PFM)

Probabilistic fracture mechanics (PFM) is becoming increasingly popular for realistic evaluation of fracture response and reliability of cracked structures [5]. Using PFM, statistical uncertainties can be incorporated in engineering design. The ability to account for uncertainties in loading, material properties, initial crack size, fatigue crack growth parameters, etc., in a systematic, theoretically sound manner is essential to structural design and analysis. The safety of the structural component in the presence of cracks can be assured, if the intensity of stress at the crack tip is less than the fracture toughness of the material. [1] The theory of fracture mechanics provides a relationship between the maximum permissible load acting on a structural component to the size and location of a crack in the component [4]. Probability theory determines how the uncertainties in crack size, loads, and material properties, when modeled accurately, affect the

safety of cracked structures. PFM blends these two theories, and hence, describe the actual behavior and reliability of structures than traditional deterministic methods. Most research has focused on advanced probabilistic algorithms coupled with finite element method as the structural analysis tool.

1.2 Ansys Probabilistic Design System (PDS)

The ANSYS Probabilistic Design System (PDS) analyzes a component or a system involving uncertain input parameters. These input parameters (geometry, material properties, boundary conditions, etc.) are defined in the ANSYS model. The variations of these input parameters are defined as random input variables and are characterized by their distribution type[6] (Gaussian, lognormal, etc.) and by their distribution parameters (mean values, standard deviation, etc.). Any interdependencies between random input variables are defined as correlation coefficients. The results are defined as random output parameters. During a probabilistic analysis, ANSYS executes multiple analysis loops to compute the random output parameters as a function of the set of random input variables. The values for the input variables are generated either randomly (using Monte Carlo simulation) or as prescribed samples (using Response Surface Methods).

1.3. Deterministic Vs. Probabilistic

In traditional deterministic analyses, uncertainties are either ignored or accounted for by applying conservative assumptions. Uncertainties are typically ignored if the analyst knows for certain that the input parameter has no effect on the behavior of the component under investigation. In deterministic analysis, only the mean values or some nominal values are used in the analysis. However, in some situations the influence of uncertainties exists but is still neglected. Probabilistic analysis allows us to determine the extent to which uncertainties in the model affect the results of a finite element analysis [5]. Here, statistical distribution functions such as the Gaussian or normal distribution, the uniform distribution, etc. describe uncertain parameters. In ANSYS

PDS, [2] several probabilistic analysis methods like Monte Carlo simulation, Response surface methods are available.

2. FINITE ELEMENT METHOD FOR FRACTURE MECHANICS

Fracture analysis in actual engineering components is more involved, might be due to size, complexity of shape and the ease with which it is made available. Hence standardized fracture specimens are being used for fracture analysis and the results are correlated to the actual geometry through fracture models. Finite Element method is a powerful analysis tool for engineering analysis in various fields. Fracture mechanics deals with the study of how a crack or flaw in a structure propagates under applied loads. Solving a fracture mechanics problem using FEM involves performing a linear elastic or elastic-plastic static analysis and then using specialized post processing commands or macros to calculate desired fracture parameters.

In this paper, a center cracked tensile specimen is modeled in Finite element Analysis package (ANSYS 10) and analyzed for stress intensity factor using singular element method[2]. Stress Intensity factor (SIF) is a parameter to measure severity of stress at the crack tip. Critical stress intensity factor is the limit on SIF such that if SIF exceeds beyond the critical stress intensity factor, the crack may grow. Thus to predict the growth of a crack in a component, designer should know two values (i) the SIF determined through analysis for the geometry of the component, crack configuration and applied loads and (ii) the critical SIF determined through experiments for the material of the component.

Stress intensity factor (SIF) is a parameter, to characterize a crack. For a given geometry (Fig. 1) of finite plate of width W with a centre crack of length $(2a)$ with far field stress (σ) , and thickness of the specimen (t) , there are three variables, the far field stress (σ) , the crack length a and a function $f(a/W)$. The stress concentration factor is proportional to all these variables. The expression for stress intensity factor [1] is presented herewith – for a centre crack tension specimen shown in figure 1.

$$K_I = \sigma \sqrt{\pi a} f(\lambda) \quad (1)$$

$$f(\lambda) = 1 + 0.256 \lambda - 1.152 \lambda^2 + 12.200 \lambda^3 \quad (2)$$

$$\lambda = a/w \quad (3)$$

Where

K_I	=	Stress intensity factor, $\text{MPa}\sqrt{\text{mm}}$
σ	=	Far field stress, MPa
a	=	Semi crack length, mm
W	=	width of the plate, mm

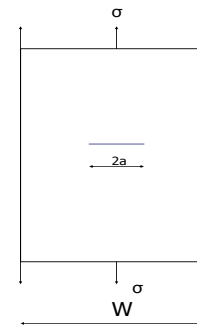


Fig. 1 Centre Crack specimen

In this paper ANSYS 10 finite element software, which has the capability to determine the stress intensity factor is used for the deterministic analysis. The specimen geometries are directly modeled in the ANSYS software. While modeling for finite element analysis, symmetric boundary condition is taken into consideration to minimize the memory and processing time. Element type Plane82 2D with eight nodes can be used. The element at the crack tip is specially defined due to the singularity problem. The mid node of the element in line with the crack moves to the quarter position and thereby it alleviates the singularity problem associated with the element. The analysis was carried out by specifying the stress of the corresponding specimen and linear elastic - static analysis option was chosen since determination of stress intensity factor fell within the validity limits of the LEFM. Once the static analysis is completed, using the general postprocessor (POST1), SIF is calculated. The POST1 **KCALC** command calculates the stress intensity factors KI, KII, and KIII. Thus FEM is used for determining SIF in center crack specimen.

3. INTEGRATING ANSYS WITH PROBABILITY ANALYSIS (ANSYS PDS)

The coupling between ANSYS and ANSYS PDS is done through simple interface routines. The PDS is a general-purpose probabilistic tool in the sense that it can utilize all deterministic analysis capabilities of ANSYS itself. An ANSYS input file containing a complete analysis sequence (preprocessing, solution, and post processing) is prepared. This is known as the Analysis file. The file must contain a parametrically defined model using parameters to represent all inputs and outputs to be used as Random Input variables RVs and Random Output parameters RPs. Here the log file created to determine SIF is the analysis file. This file is saved as a macro file so that it gets looped during the probabilistic analysis. ANSYS PDS automatically creates the loop file via analysis file to perform analysis loops. The integrated data flow [2] is as shown in fig. 2

T6Aluminum [8] ($K_f = 758.94 \sqrt{\text{mm}}$ 24 MPa $\sqrt{\text{m}}$) a center crack tensile specimen is done.

TABLE 1- Material Properties

properties	Values
Modulus of Elasticity (E) in Mpa	71000
Poisson's Ratio (v)	0.3400

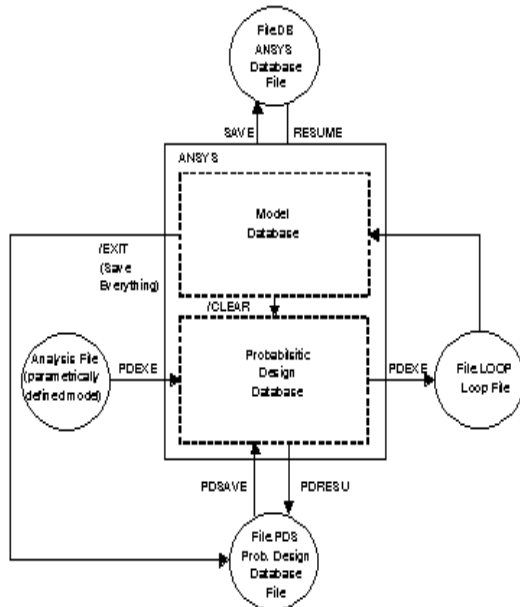


Fig. 2 Integrated data flow

4. STEPS IN PROBABILISTIC ANALYSIS

The usual process for probabilistic analysis consists of the following general steps.

1. Create an analysis file for use during looping. The file should represent a complete analysis sequence and must do the following:
 - a. Build the model parametrically (PREP7).
 - b. Obtain the solution(s) (SOLUTION).
 - c. Retrieve and assign to parameters, the quantities that will be used as random input variables and random output parameters (POST1/POST26).
2. Establish parameters in the ANSYS database which correspond to those used in the analysis file. If this step is skipped, then the parameter names won't be available for selection in interactive mode.
3. Enter PDS and specify the analysis file.
4. Declare random input variables.
5. Visualize random input variables.
6. Specify any correlations between the RVs.
7. Specify random output parameters.
8. Choose the probabilistic design tool or method.
9. Execute the loops required for the probabilistic design analysis.
10. Review the results of the probabilistic analysis

5. PROBABILISTIC ANALYSIS OF CENTRE CRACK SPECIMEN

Fracture analysis in actual engineering components is more involved, might be due to size, complexity of shape and the ease with which it is made available. Hence standardized fracture specimens are being used for fracture analysis and the results are correlated to the actual geometry through fracture models. Here the probabilistic analysis of AA6061-

5.1 Deterministic Model Information

The analysis file created using ANSYS containing the deterministic model is stored. ANSYS PDS executes or "loops through" multiple times during the probabilistic analysis. The

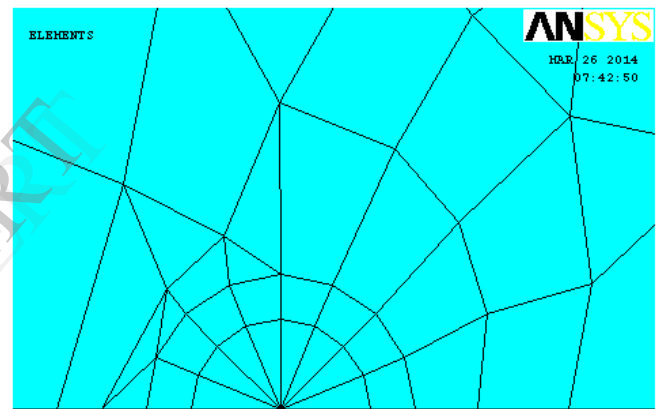


Fig. 3 Finite Element model of the specimen

Model geometry and the finite element mesh are as shown in fig.3. The element type is chosen as PLANE 82, plane strain with thickness option. The K_I value is obtained using singular element method which is the output parameter here.

5.2 Probabilistic Model Information

5.2.1. Random Input Variables:

The deterministic model has 3 parameters namely semi crack length (a), applied stress (σ), thickness of the specimen (t) and half width of the specimen (W) that are regarded as random input variables. A list of these random input variables, the distribution they are subjected to, and their distribution parameters are provided in Table 2. The probability density functions and cumulative distribution functions are shown in Figures 8 to 11.

TABLE 2 - Random Input Variable Specifications

Variable	Mean	STD	Distribution
Stress (σ) in MPa	100	10	Lognormal
Semi Crack length (a) in mm	5	0.5	Gaussian
Half width (w) in mm	25	2.5	Gaussian
Thickness(t) in mm	1.6	0.6	Gaussian

Distributions generated using the tabulated values are shown below:

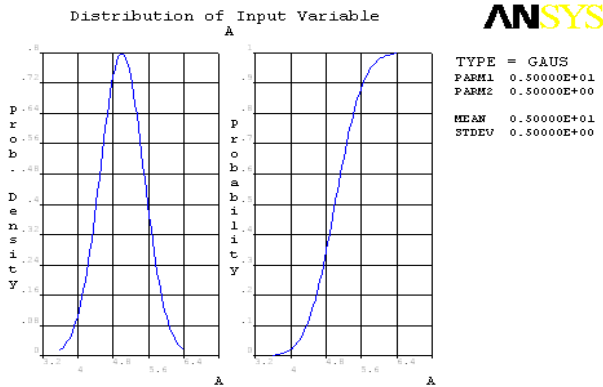


Fig.4 Distribution of input variable 'a' (Semi crack Length)

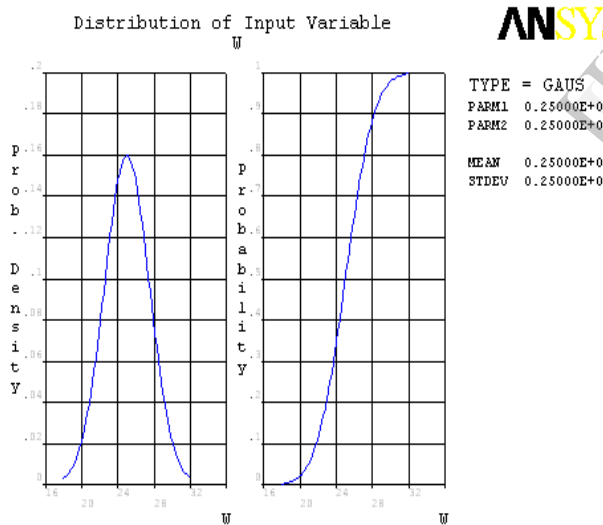


Fig.5 Distribution of input variable 'W' (Half width)

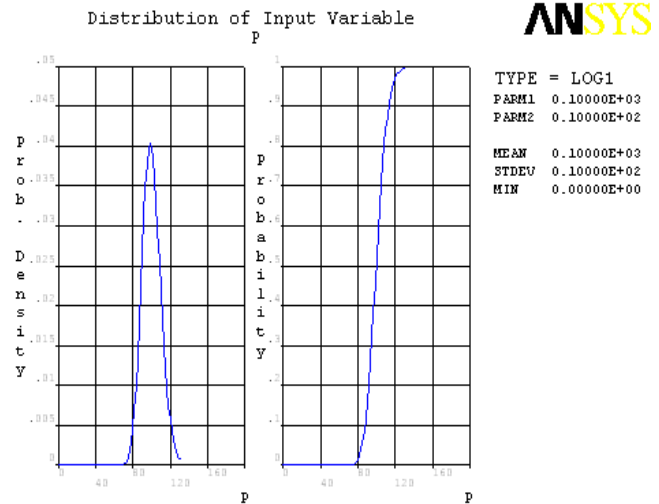


Fig.6 Distribution of input variable ' σ '(Stress)

5.3 Probabilistic Analysis

The probabilistic analysis is performed using Monte Carlo simulation technique. ANSYS PDS also offers a probabilistic method selection wizard which helps in determining the fastest method for probabilistic analysis, number of samples, looping time based on the information given. In Monte Carlo method, either direct sampling or Latin hyper cube sampling can be employed. The Latin Hypercube Sampling (LHS) technique is a more advanced and efficient form for Monte Carlo Simulation methods [5]. The only difference between LHS and the Direct Monte Carlo Sampling technique is that LHS has a sample "memory," meaning it avoids repeating samples that have been evaluated before (it avoids clustering samples). Here LHS method is used for sampling. The results of each probabilistic analysis can also be stored in a separate solution set indicated with a solution label. The number of samples taken for analysis is 5000.

6. POST PROCESSING PROBABILISTIC ANALYSIS RESULTS

There are two groups of post processing functions in the probabilistic design system: statistical and trend. A statistical analysis is an evaluation function performed on a single probabilistic design variable; for example, a histogram plots of a random output parameter. The ANSYS PDS allows statistical evaluation of either random output parameters or random input variables. A trend analysis typically involves two or more probabilistic design variables; for example, a scatter plot of one probabilistic design variable versus another.

6.1 Statistics of Probabilistic Analysis

TABLE 3 - Statistics of the Random Input Variables

Variable	Mean	Standard Deviation	Min	Max
Stress (σ) (MPa)	100	10.004	68.32	150.13
Semi Crack length (a) (mm)	5	0.5	3.219	6.864
Half Width (w) (mm)	25	2.5	14.236	33.956
Thickness(t) (mm)	1.6	0.599	-0.563	3.850

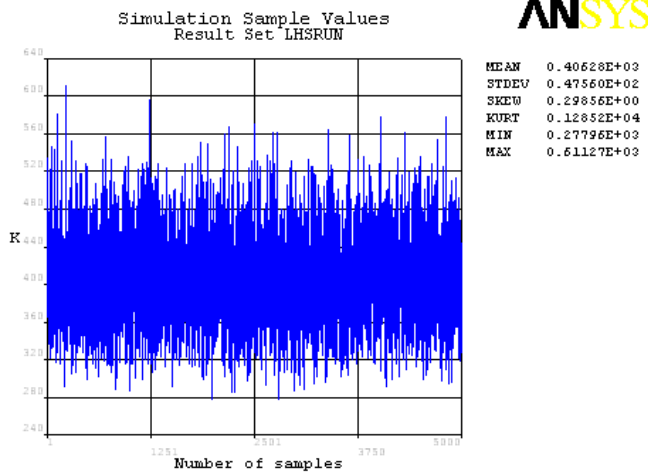


Fig.7. Result Set – Statistics of the random Output parameter

TABLE 4 – Result Set - Statistics of the Random Input Variables

Name	Mean	Standard Deviation	Minimum	Maximum
K_1 MPa(mm) ^{0.5}	406.28	47.56	277.96	611.27

6.2 Sample History

The sample history values are a function of the number of simulation loops. Here we can review the simulation values, the mean, minimum maximum values and the standard deviations. It is most helpful to review the mean values and standard deviation history if we want to decide if the number of simulation loops was sufficient. If the number of simulation loops was sufficient, the mean values and standard deviations for all random output parameters should have converged.

Mean Values of Samples Result Set LHSRUN

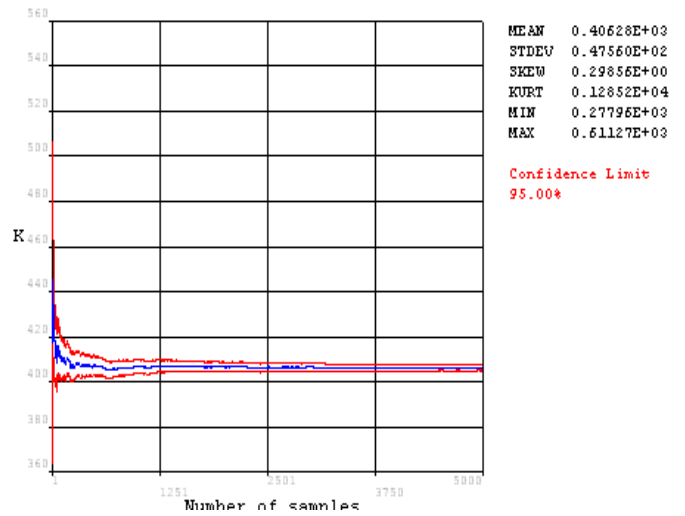


Fig. 8 Mean Value History for Output Parameter K_1

Standard Deviation of Sample Result Set LHSRUN

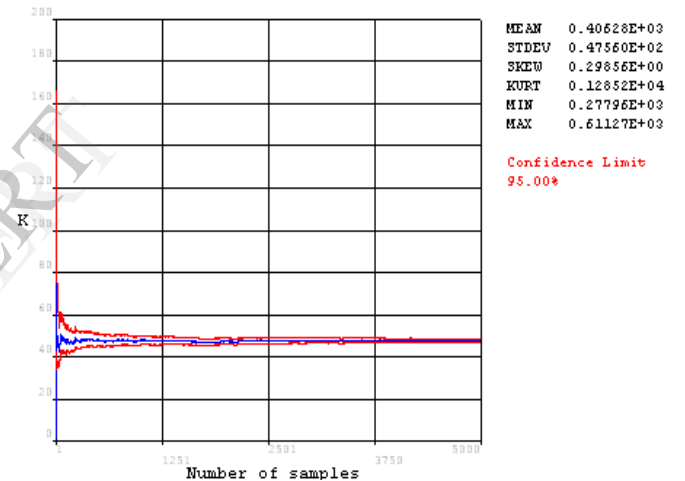


Fig. 9 Standard Deviation History for Output Parameter K_1

6.3 Sensitivity Plots

The evaluation of the probabilistic sensitivities is based on the correlation coefficients between all random input variables and a particular random output parameter. Either Spearman rank order correlation coefficients or Pearson linear correlation coefficients can be used. To plot the sensitivities of a certain random output parameter, the random input variables are separated into two groups: those that are significant (important) and those that are insignificant (not important) for the random output parameter.

The sensitivity plots will only include the significant random input variables.

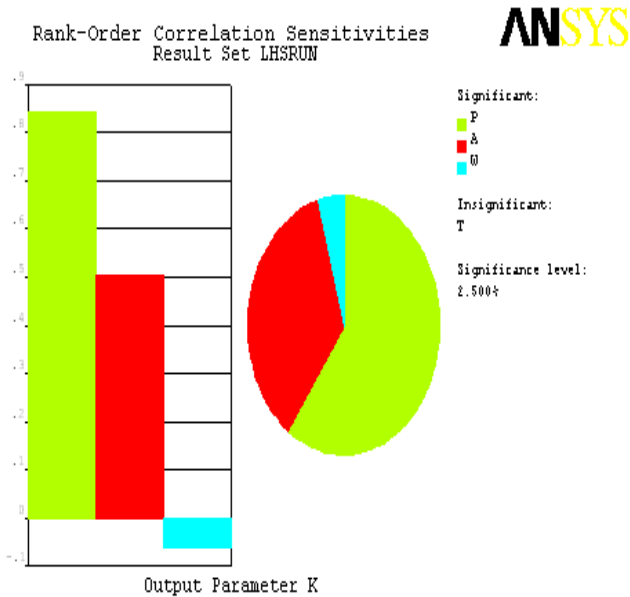


fig. 10 sensitivity plot for output parameter (k_i)

6.4 Histogram Plots

ANSYS PDS calculates an appropriate number of classes based on the number of samples. The number of classes is equal to the number of bars shown in the histogram. The range between the smallest and largest sample value has been divided into classes of equal width. A histogram is derived by counting the number of hits in the individual classes and dividing this number by the total number of samples. Hence, a histogram represents the relative frequencies of the random quantity it is plotted for.

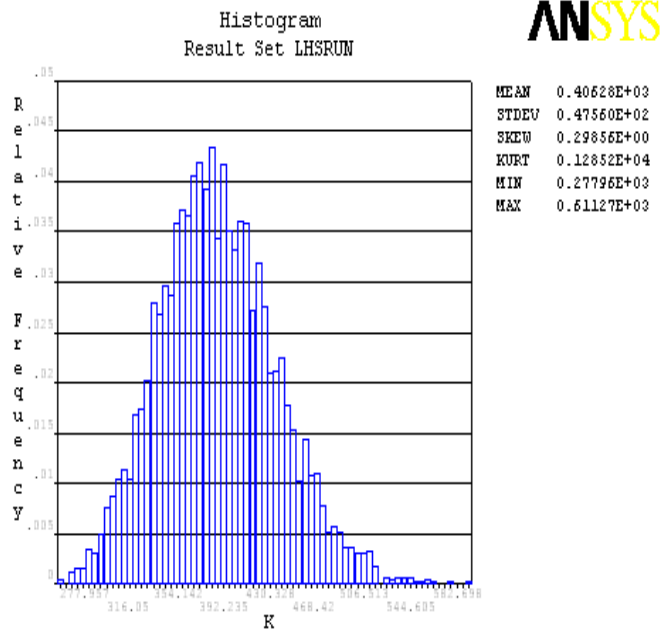


Fig. 11 Histogram of Output Parameter (K_i)

7. THEORETICAL EVALUATION OF ANSYS PDS RESULTS

The geometry, external loads are treated as random variables. In most cases, the random variables are assumed to follow normal distribution for computational simplicity. The reliability index Z_R [7] can be given as,

$$z_R = \left(\frac{\bar{x}_s - \bar{x}_\sigma}{\sqrt{(S_s^2 - S_\sigma^2)}} \right) \tag{4}$$

Where, x_s and S_s (x_σ and S_σ) denoting respectively, the mean and standard deviation of the strength (load). By applying partial derivative rule [6], the standard deviation of stress factor is found.

We have

$$S_\sigma^2 = \left(\frac{\partial s}{\partial a} \right)^2 S_a^2 + \left(\frac{\partial s}{\partial K_I} \right) S_{K_I}^2 \tag{5}$$

Substituting Stress s = K_I / √(πa) and

$$\left(\frac{\partial s}{\partial a} \right) = \left(\frac{K_I}{0.1048} \right) \text{ in (Eq. 5) and assuming to get a reliability}$$

of 99%, we get,

$$K_I = 505.91 \text{ MPa}\sqrt{\text{mm}} \text{ (15.99 MPa}\sqrt{\text{m}})$$

The mean and standard deviations of the output parameter K_I obtained with theoretical values are compared with ANSYS PDS results as shown below.

TABLE 5 – Result Comparison

Method	ANSYS PDS MPa(mm) ^{0.5}	Theoretical MPa(mm) ^{0.5}	% of Error
Mean	526.593	505.91	4.08

8. PROBABILITIES AND INVERSE PROBABILITIES OF THE SYSTEM

The cumulative distribution function of the output parameter is examined by specifying a certain probability level. At which value of the probabilistic design variable, this required probability will occur is determined. This is helpful in evaluating the limits specified so that the probability is not to exceed a certain failure probability or to specially achieve certain reliability for the component. This is known as the Inverse probability. The probability that the response parameter K_I is less than a certain value to achieve target reliability can also be determined from the probabilistic analysis results. The reliability of the design system is thus determined. The inverse probability and probability results for the combination input parameters of this system are determined here and given below.

8.1 Probability Result of Response Parameter ' K_I ':

Given a Stress Intensity Factor of 500 MPa $\sqrt{\text{mm}}$, the reliability of the system is found out using Probability evaluation.

Simulation Method = Monte Carlo with LH Sampling
Number of Samples = 5000

The probability that K is smaller than 500 is 96.78%

8.2 Inverse Probability Result of Response Parameter Stress Intensity Factor K_I :

Simulation Method = Monte Carlo with LH Sampling
Number of Samples = 5000

A 99.00% probability that K_I is smaller than a certain limit value occurs at:

$$K_I = 526.593 \text{ MPa}\sqrt{\text{mm}} \text{ (16.65MPa}\sqrt{\text{m}})$$

9. CONCLUSION

An integral approach to fracture analysis using ANSYS and probabilistic analysis using ANSYS PDS is exercised. The variations in the input random variables are included and corresponding variations in the output parameter are noted. Sensitivities of the output parameter with respect to the variability of the input parameters are quantified. The probability that the stress intensity factor for the given system to achieve target reliability of 99% is determined. The limiting value of the output parameter K_I for a given target probability of 96.78% is also determined to ensure the reliability of the design system. The probability that the response parameter K_I is less than a certain value to achieve target reliability can also be determined. The histogram produced by ANSYS PDS is compared with that of theoretical and considerable similarity is noted. A similar probabilistic approach can be used for determining fracture parameters by including variations in other parameters like orientation, fatigue loads etc. The probabilistic analysis can be carried out for various other specimens so that it can be interpolated in the safe design of structures.

REFERENCES

- [1] Anderson T.L. (1994), "Fracture Mechanics – Fundamentals and Applications" CRC press, New York.
- [2] ANSYS user's manual, version 10.0, Ansys Multiphysics, Ansys Inc.
- [3] Prashant Kumar., (1999), "Elements of Fracture Mechanics", Wheeler Publishing.
- [4] Kare Hellan, (1985) "Introduction to Fracture Mechanics", McGraw Hill Book Company.
- [5] Stefan Reh et al, (2006), Probabilistic finite element analysis using ANSYS, Structural Safety 28 (2006) 17–43
- [6] Singiresu S .Rao, (1991) Reliability based Design, McGraw – hill edition
- [7]. L.S.Srinath, (1991) Reliability Engineering, Affiliated East-West Press Private Limited.
- [8]. P. S. Shinde1, K. K. Singh, V. K. Tripathi, P. K. Sarkar, P. Kumar "Critical J-Integral of Thin Aluminium Sheets Employing a Modified Single Edge Plate Specimen" International Journal of Modern Engineering Research, Vol.2, Issue.3, pp-1360-1365, May-June 2012.