

Estimation of Zero Moment Point using Centre of Gravity based Method

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Abstract—Centre of gravity and Zero Moment Point are necessary for the balancing of a biped system. Forward Kinematics has been used for the estimation of co-ordinates of the end points and C.G of links. The location of the centre of gravity of each of the link can be estimated by using geometric method derived in the paper. The reference frames can be translated using matrix transformations to find out all co-ordinates from one base reference frame. The co-ordinates of the CG can be further used to find out the location of the ZMP by simple mathematical calculation. Creating a program, ZMP point location at every position of the leg in the specified walking gait pattern can be easily obtained.

Keywords—Biped; robot; Zero Moment Point; ZMP; forward kinematic; gravity; robotics; gait pattern

I. INTRODUCTION

A Humanoid Bipedal Robotic Legged system is a Robot walking on two leg mechanisms similar to a Human Being. The Legged System is a 6 Degree of Freedom Mechanism consisting of 3 Degree of Freedom Robotic manipulators as each of its legs. There is a Base plate attached above the two legs on which the controller drives, a balancing mechanism and a camera unit is mounted. Each of the joints used are Rotary or Revolute joints and a DC Servo motor is used to actuate these joints using an appropriate speed reduction drive to obtain the Suitable joint torques and Speed. The Legged Structure of the Bipedal System is shown in figure 1.



Figure 1: Legged structure

The Gait defines the manner of walking of a human being. This gait pattern determines the position of the end effector which is the lowest point of the foot of the Bipedal unit. A cycloidal gait pattern has been selected for simplicity and productive walking motion. From the position of gait pattern which is known to us from the gait pattern, we can calculate the angles of each of the joints using the Inverse Kinematics. Figure 2 shows the gait trajectory of the end point of the foot.

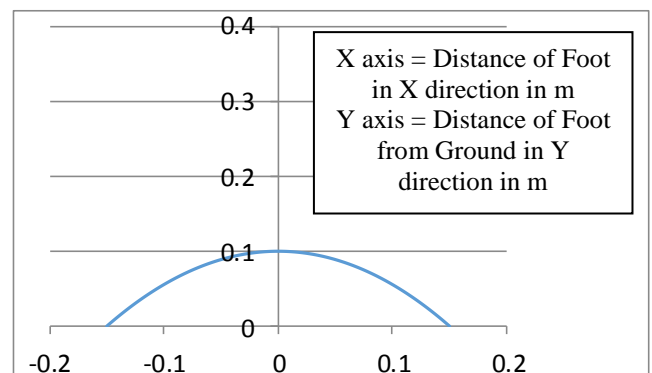


Figure 2: Gait Pattern of the foot

The Forward Kinematics is used to find out the position of the ends of the links from the given joint angles. The centre of Gravity is the point at which the whole mass of the given body is assumed to be concentrated. Stability is maintained by controlling a non-physical degree of freedom, which is called “Zero Moment Point” (ZMP). This criteria was proposed first by Professor Miomir Vukobratovic. The dynamics of the whole body is taken into account. The criteria affirms that the biped does not fall down if the ZMP is maintained inside of the convex hull during the walking motion. Zero moment point and Centre of gravity are essential in maintaining the static and dynamic stability of the system respectively [1]. The method for calculation of the Zero Moment point for the Biped System using the geometric method of forward Kinematics is as given below in the succeeding sections.

II. FORWARD KINEMATICS USING GEOMETRIC METHOD

The forward kinematics problem for a serial-chain manipulator is to find the position and orientation of the end-effector relative to the base given the positions of all of the joints and the values of all of the geometric link parameters. Forward Kinematics deals with obtaining the position of the end point of foot of a given Bipedal system using the joint angles of rotation [2]. The Base plate is considered as fixed and the location or position of the end point is calculated. For Obtaining the Centre of gravity of first link, only the portion up to that particular link is considered assuming the C.G as the end effector in each case.

The joint angles are obtained from the Inverse Kinematics. Using these angles in Trigonometric equations, the x and y co-ordinates of the point can be obtained as given below. The obtained x and y co-ordinates of these points are w.r.t to the Fixed base plate. But, for the calculation of ZMP, we need the co-ordinates of these points w.r.t the starting point of the gait pattern i.e. the initial position of the leg in the gait pattern. For this, the Matrix transformations can be used. The reference frame is translated from the fixed base plate to the initial position of the Leg.

Figure 3 shows the geometric representation of the thigh part of the leg manipulator. L3 denotes the length of link 3 while t3 denotes the angle made by the link w.r.t the vertical axis. xh and yh are the vertical and horizontal components of L3 which also denote the co-ordinates of the point A.

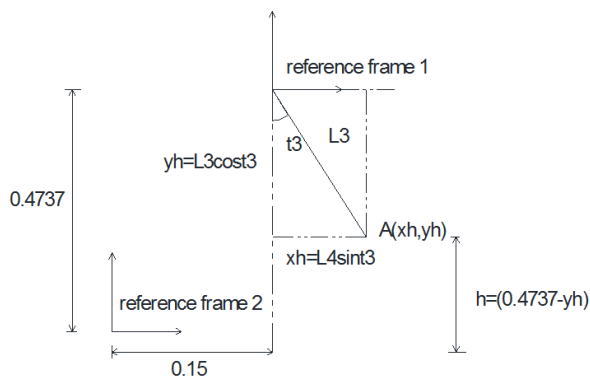


Figure 3: Calculation of C.G using Forward Kinematics

Thus, using these transformations, we get the co-ordinates of C.G of each of the links at all positions of the gait pattern.

$$xh = L3 * \sin t3$$

$$yh = L3 * \cos t3$$

Using Matrices,

Co-ordinates of point A with respect to reference frame 2,

$$P = \begin{bmatrix} 1 & 0 & 0.15 \\ 0 & 1 & 0.4737 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} L3 * \sin t3 \\ -L3 * \cos t3 \\ 1 \end{bmatrix}$$

$$\therefore xhf = L4 * \sin t3 + 0.15$$

$$\therefore xhf = 0.4737 - L3 * \cos t3$$

Xhf and yhf denote the C.G of link 3 w.r.t initial foot position. Similarly, any point coordinates with respect to reference plane 2 on line L3 can be calculated.

Estimation of the co-ordinates of C.G of link 4 can be done in a similar manner. For the Foot, let t5 be the angle the link L4 makes with the vertical. Let point B denote the C.G of the link 4 and xf and yf be the co-ordinates of point B.

$$xf = L4 * \cos t5$$

$$yf = L4 * \sin t5$$

Shifting the axis,

$$xff = L4 * \cos t5 + xg$$

$$yff = L4 * \cos t5 + yg$$

Where, cg and yg are coordinates of gait. xff and yff denote the C.G of link 3 w.r.t initial foot position.

III. CALCULATION OF ZMP

Zero Moment Point is a point on the walking surface about which the Horizontal components of the resultant moment acting on the links are zero. The ZMP determines the dynamic Stability of the System. This ZMP must lie within the surface polygon area of the foot for the system to walk without toppling. There are various methods of calculation of ZMP like Cart Table Method but it leads to tedious calculations for complex systems [3]. The definition of ZMP is used in the geometric method which states that the sum of all moments about the ZMP must be zero. Thus, it is assumed that ZMP lies at a distance of x from the initial position of the leg. The moments are taken and the value of ZMP is taken as Zero. The positions of the C.G of each of the link obtained from the Forward Kinematics are used in this method. The weight is assumed acting downwards at these points and moments are taken about the ZMP. Figure 4 describes the Geometric method of calculation of ZMP.

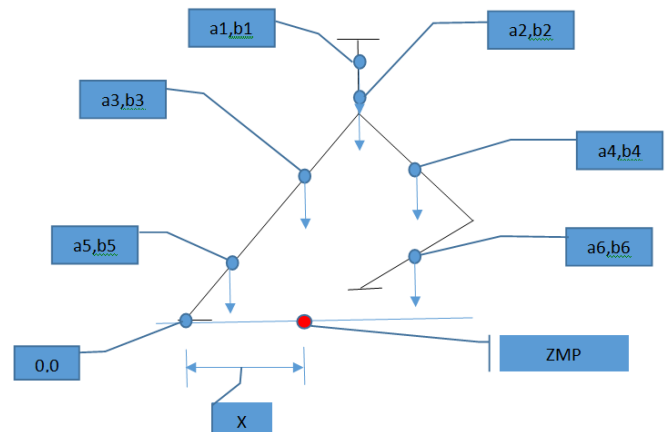


Figure 4: Calculation of ZMP

Calculation of the ZMP is given below: Suppose ZMP lies at a distance x in X-axis.

For ZMP to be at x distance, sum of all moments at that point must be 0. Hence,

$$0 = mg*(a6-x) + mg*(a4-x) + 2*mg*(a1-x) + 2*mg*(a2-x) + mg*(a3-x) + mg*(a5-x)$$

$$0 = a6 - x + a4 - x + 2*a1 - 2*x + 2*a2 - 2*x + a3 - x + a5 - x$$

$$8x = 2*a1 + 2*a2 + a3 + a4 + a5 + a6$$

$$x = \frac{2 * a1 + 2 * a2 + a3 + a4 + a5 + a6}{8}$$

i.e.

$$x = \frac{\text{(Sum of all the distances in Z direction between the C.G.of all links and the origin)}}{\text{Number of links}}$$

Thus we can find the distance x of ZMP in X direction from origin.

Calculations for a certain position are as given below.

$$a1=0.15m, a2=0.15m, a3=0.1127m, a4=0.1719m, a5=0.1257m, a6=0.0507m$$

$$x = \frac{2*0.15+2*0.15+0.1127+0.1719+0.1257+0.0507}{8}$$

$$\therefore x = 0.1326$$

Thus, the geometric method is a simpler and efficient method of calculation of the ZMP. The ZMP location for each of the positions of the leg of the system can be done with the help of Microsoft excel.

IV. CALCULATION OF ZMP

Four Values in a step of 1 second:

At 0.01 seconds - 0.119122 m in X direction.

At 0.25 seconds – 0.13519 m in X direction.

At 0.5 seconds – 0.154712 m in X direction.

At 0.75 seconds – 0.162575 m in X direction.

At 1 second – 0.1566 m in X direction

The variation of ZMP, as the biped completes its gait pattern is represented in figure 5.

Variation of ZMP in X Direction vs Time

x - Time step in centisecond
y - distance in meter

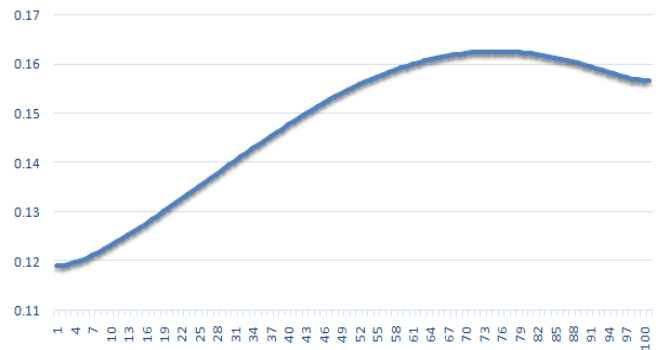


Figure 5: Graph depicting distances of ZMP in X direction vs Time

Where,

Y axis represents distance in meters

X axis represents time.

Figure 5 depicts the variation of ZMP in X direction. As the ZMP of the Bipedal structure ranges from 0.1191 m to 0.1625 m, the foot length of the Bipedal structure must be greater than 0.1625 m such that the ZMP always lies within the foot surface polygon. The designing of the foot must be done accordingly.

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