Estimation of the Design Concrete Strength from Core Tests: Modified Tolerance Factor Approach

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***Abstract*— Several statistical methods were developed to estimate the design compressive strength of concrete elements from core tests. The Tolerance Factor Approach adopted by the ACI committee 214.4R assumes concrete compressive strengths are normally distributed. The assumption of a normal distribution has many significant drawbacks, in which it incorporates negative values on the left hand tail of the distribution. In this paper a modified tolerance factor approach for modeling concrete compressive strength is developed. This approach is based on a reliability analysis and the incorporation of a lower bound value for the equivalent in place concrete compressive strength. The incorporation of a lower bound is done through the use of a left truncated normal distribution. An experimental formulation of the proposed approach is carried out and compared to results of the Tolerance Factor Approach. Results indicate that the modified tolerance factor approach gives more reliable estimates of equivalent design compressive strength especially for data with high coefficient of variation. A comparison between tolerance factor, modified tolerance factor and Bartlett and Macgregor approaches is made. Results from the tolerance factor approach and Bartlett and Macgregor tend to diverge especially when the coefficient of variation is high. It is shown that the use of the modified tolerance factor approach bridges the gap between the two approaches.**

Keywords*— Core Compressive Strength; ; Tolerance Factor Approach; Normal Distribution; Lower Bound; Left Truncated Normal Distribution; Equivalent Design Compressive Strength.

I. INTRODUCTION AND BACKGROUND

The compressive strength of concrete is a major required parameter for conducting structural analysis/design of concrete structures. Reliable estimates of the compressive strength are required as input for recommendations for repair or demolition, design of the rehabilitation/strengthening systems, assessment of the structural capacity and other structural design problems. Strength evaluation of existing structures is also important in determining whether a structure meets the minimum building code strength requirements (Hanson, 2007)[1]. In-situ concrete compressive strength is determined using several approaches including destructive and non-destructive testing. However; current testing methods used to estimate concrete compressive strength have failed to advance with concrete industry (Rojas-Henao *et al*, 2012)[2]. ACI committee 228.2R[3] suggests that non-destructive testing are mainly used to determine defects in concrete structures, concrete quality and to evaluate different concrete structural elements. A verification of non-destructive testing by means of coring or probing is often required to make a sound evaluation. The most commonly adopted method to estimate the equivalent design compressive strength is by means of core sampling through a probabilistic approach and using a certain statistical distribution (Chen *et al* 2014)[4].

There are many studies in the literature that correlated concrete core compressive strength with normal or lognormal distributions, starting with the early works of Campbell and Tobin (1967)[5] and Soroka (1968)[6], passing through works of Hindo and Bergstrom (1985)[7], Bartlett and Macgregor (1995)[8] and Chmielewski and Konapka (1999)[9] and reaching recent works of Graybeal and Davis (2008)[10]. Common findings of these researches indicate that for data with coefficient of variation less than 20% a normal distribution is the best fit for modeling core compressive strength data. However; for coefficient of variations higher than 20% a lognormal distribution was found to have a better fit for the modeled core compressive strength data. Soroka (1968)[6] discussed the application of statistical methods to quality control of concrete, he illustrates the use of such methods in the Israeli specifications. Soroka argued that lognormal distribution should be used for the strength distribution of concrete, but states that it is safe to use the normal distribution as given by the Central State Theorem if the coefficient of variation is less than 0.1. To strengthen his argument, he formed relations between the specified nominal strength X_o and the required concrete mean strength X_1, these relations established the criteria of "good", "fair", and "poor" concrete. Hindo and Bergstorm (1985)[7] proposed the tolerance factor approach to estimate the equivalent design compressive strength from core tests. Their approach is based on modeling concrete compressive strength data with a normal distribution. Hindo and Bergstorm used the tolerance factor K, based on a non-central t distribution, to estimate the 10% fractile value which can accommodate uncertainties in sample standard deviation and mean. They also used the normal distribution to obtain the Z factor which represents the uncertainty attributed to the strength correction factors. Bartlett and Macgregor (1995)[8] argued that the tolerance factor approach is too conservative and results in low values of equivalent design concrete compressive strength. They found out that core data can be used to estimate the average in-place strength and a lower bound for this average strength for a particular structure. Similar to the Tolerance Factor Approach, Bartlett and Macgregor used the normal distribution to calculate the mean in-place strength, the standard deviation and to obtain the Z factor; however they proposed the Student's t distribution to obtain the factor T, which represents the effect of the sample size on the uncertainty of the mean in-place strength and standard deviation. Chmielewski and Konapka (1999)[9] conducted a

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 5 Issue 01, January-2016

statistical analysis for concrete core strength data. Results from Chmielewski and Konapka indicate that for a 14% coefficient of variation no discrepancies between actual distribution and normal distribution for concrete are noticed. As for a 23% coefficient of variation, no difference between actual distributions against lognormal distribution is observed.

Different statistical approaches to strength evaluation of concrete structures were also studied by many researchers. Examples include works of Kilinc et al (2012)[11] and Chen et al (2014)[4]. Chen et al (2014) analyzed core compressive strength data from eight concrete mixes ranging between 10 and 50 MPa for different concrete beams. Data were fitted to different statistical models and CS values, KS distances and LK values were calculated. Results of Chen et al. indicates that the three parameter Weibull distribution and the normal distribution are the most appropriate in some cases. Whereas the two-parameter Weibull distribution is the most appropriate in some other cases. They concluded that the appropriate statistical distribution is based on the strength property of concrete.

In this paper a modification of the tolerance factor approach is described. This modification is developed by introducing a lower bound concrete compressive strength into the normal distribution to model concrete compressive strength data. The paper is organized as follows: first it summarizes previous works of several researchers on statistical approaches in estimating equivalent design concrete compressive strength from core tests. Second, a summary of the modified tolerance factor approach is presented. In the third part, an experimental comparison of the proposed approach versus the tolerance factor approach is conducted. Finally the paper concludes with major results and findings.

II. MODIFIED TOLERANCE FACTOR APPROACH

In many civil engineering applications the capacity is modeled using a normal distribution. This is mainly due to its simplicity and physical worth of its parameters. The Tolerance Factor Approach adopted by ACI committee 214.4R[12] assumes concrete compressive strengths are normally distributed. However, the assumption of a normal distribution accepts concrete compressive strength values below and near zero on the left hand tail of the distribution to be incorporated in the calculations. This is not possible from a physical and engineering point of view when discussing the compressive strength of concrete. This drawback is significant especially if the uncertainty in the capacity is relatively large, which is the case in the core compressive strength of concrete. Another drawback of the tolerance factor approach is that it overestimates the true variation of the core compressive strength (Bartlett and Macgregor, 1995)[8].

A. Outlying Observations

Outliers in a given data set are described as values that diverges significantly from other observed values in this data set. These outlying values can divert and flaw the findings and results of the test. To be able to obtain accurate results, all outliers should be detected and ignored from the data set. ASTM E178-02 (2002) “Practice for Dealing with Outlying Observations”[13] is used in this study to detect and remove the flawed values.

B. Strength Correction Factors

Conversion of core strength to and equivalent in-place strength is through the incorporation of strength correction factors. Strength correction factors account for length to diameter ratio, diameter, moisture conditions and effect of core drilling for cores. Strength correction factors are based on the adopted approach of ACI committee 214.4R (2010)-Table 9.1.

C. Lower Bound Concrete Compressive Strength

Any structural concrete element must exhibit a minimum compressive strength to be able to safely carry its own weight, to ensure bonding between concrete and reinforcing steel bars, to ensure no shear failure and in general to resist any failure mechanism of concrete. Statistics indicate that reinforced concrete buildings can have a wide range of low-strength concrete varying between 4 and 20 MPa (Ahmad et al, 2014)[14]. Thus, the value of 4 MPa is taken thereafter as a lower bound confined compressive strength for the equivalent in place concrete strength. A simple approach in incorporating a lower bound on the measured concrete compressive strength is the use of a left truncated normal distribution.

D. Characteristics of a Left Truncated Normal Distribution

Lower bound capacity can be incorporated in estimating the equivalent design compressive strength through the use of a left truncated normal distribution (Figure 1). The point of truncation (lower bound capacity) $X_L = 4\ \text{MPa}$, as applied to concrete core compressive strength data, is denoted by K_L as given by:

$$K_L = \frac{X_L - \bar{f}_c}{s_c} = \frac{4 - \bar{f}_c}{s_c} \tag{1}$$

Where, $\bar{f}_c$ and s_c are the mean and the standard deviation of the equivalent in place strength, after applying the strength correction factors, and are based on a non-truncated normal distribution. Mathematical expressions for the mean and variance of a left truncated normal distribution applied to concrete compressive strength data are given by equations 2 and 3 below:

$$\bar{f}_{c,LT} = \bar{f}_c + \left[\frac{s_c}{\sqrt{2\pi}} e^{-\frac{1}{2}K_L^2}\right] \frac{1}{1-\emptyset(K_L)} \tag{2}$$

$$s_{c,LT}^{\;2} = \bar{f}_c^{\;2} + \left[\frac{2\bar{f}_c s_c}{\sqrt{2\pi}} e^{-\frac{1}{2}K_L^2}\right] \frac{1}{1-\emptyset(K_L)} + \left[\frac{s_c^{\;2} K_L}{\sqrt{2\pi}} e^{-\frac{1}{2}K_L^2}\right] \frac{1}{1-\emptyset(K_L)} + s_c^{\;2} - \bar{f}_{c,LT}^{\;2} \tag{3}$$

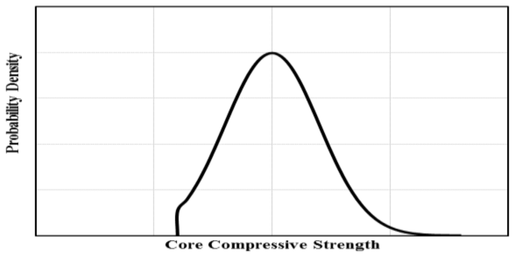

Figure 1. Left Truncated Normal Distribution

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 5 Issue 01, January-2016

E. Tolerance Factor "K"

The correction factor for number of cores K is calculated using Natrella (1963)[15] equations and is based on the 10% fractile of the in place strength data.

$$a = 1 - \frac{z_\gamma^{\,2}}{2(n-1)} \tag{4}$$

$$b = z_p^{\,2} - \frac{z_\gamma^{\,2}}{n} \tag{5}$$

$$K = \frac{z_p + \sqrt{z_p^{\,2} - ab}}{a} \tag{6}$$

Where, γ and P are the confidence level and percentage of population that will lie respectively and n is the number of cores extracted. Z values for a left truncated normal distribution, depending on the truncation point, were developed by Johnson (2001)[16] through a reformulation of the standard "t" variate of a left truncated normal distribution in terms of standard "z" variate of a standard normal distribution. Applying equation 6 above, tolerance factors for some values of n, γ, P and K_L are covered in Table I below. Values of K_L ranges between -3 and -0.2. It's good to note that variation of K_L below -3 is not significant. The confidence level γ is taken as 75% based on the recommendation of ACI committee 228.1R[17]. The percentage of population that will lie is taken as 90% which corresponds to the 10% fractile of the in place strength data.

F. Equivalent Design Strength

The equivalent design strength, which is derived from a standard left truncated normal distribution and following the modified tolerance factor method is obtained from the equation below

$$f'_{c,eq} = \bar{f}_{c,LT} - \sqrt{\left(Ks_{c,LT}\right)^2 + (Zs_a)^2} \tag{7}$$

The second factor under the square root is to account for uncertainties in the strength correction factors. Z values are based on a normal distribution and may be taken from Table 9.3 of the ACI 214.4R (2010) Guide depending on a specified confidence level, s_a is the standard deviation of the strength correction factors and may be calculated using equation 9-4 of ACI 214.4R (2010) guide.

III. Experimental Formulation

Core samples were extracted from slab elements of two, two-story old buildings. 23 cores were extracted from each building. Cores measured 10cm in diameter and 20cm in length and were soaked for 48 hours prior to crushing. Results of core compressive strength are shown in Table II below.

Table I. K values for a 75% confidence level

K_L / n	-3	-2.8	-2.6	-2.4	-2.2	-2	-1.8	-1.6	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2
3	2.1	2.1	2.1	2.1	2.12	2.13	2.16	2.21	2.27	2.36	2.48	2.64	2.86	3.15	3.55
4	1.91	1.91	1.91	1.92	1.93	1.94	1.97	2	2.05	2.12	2.21	2.33	2.49	2.69	2.95
5	1.81	1.81	1.81	1.82	1.83	1.84	1.86	1.89	1.93	1.99	2.07	2.17	2.30	2.47	2.68
6	1.75	1.75	1.75	1.76	1.76	1.78	1.79	1.82	1.86	1.91	1.98	2.07	2.19	2.34	2.52
7	1.7	1.7	1.7	1.71	1.72	1.73	1.74	1.77	1.8	1.85	1.92	2.00	2.11	2.24	2.41
8	1.67	1.67	1.67	1.67	1.68	1.69	1.71	1.73	1.76	1.81	1.88	1.95	2.05	2.17	2.33
10	1.62	1.62	1.62	1.62	1.63	1.64	1.66	1.68	1.71	1.75	1.80	1.88	1.97	2.08	2.22
12	1.58	1.58	1.59	1.59	1.6	1.6	1.62	1.64	1.67	1.71	1.76	1.83	1.91	2.02	2.14
15	1.55	1.55	1.55	1.55	1.56	1.57	1.58	1.6	1.63	1.66	1.71	1.77	1.85	1.95	2.07
18	1.52	1.52	1.52	1.53	1.53	1.54	1.55	1.57	1.60	1.63	1.68	1.74	1.81	1.90	2.01
21	1.5	1.5	1.5	1.51	1.51	1.52	1.53	1.55	1.57	1.61	1.65	1.71	1.78	1.87	1.98
24	1.48	1.49	1.49	1.49	1.5	1.5	1.51	1.53	1.56	1.59	1.63	1.69	1.76	1.84	1.94
30	1.46	1.46	1.46	1.47	1.47	1.48	1.49	1.51	1.53	1.56	1.60	1.65	1.72	1.80	1.90

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 5 Issue 01, January-2016

Table II. Core compressive strength results

Building	Core Compressive Strength (*MPa*)				
First Building	5.3 6.8 11.1 9.3 13.6	19.7 6.9 17.7 12.3 11.1	17.5 14.3 9.1 10.3 15.7	18.2 7.1 10.6 18.6	9.7 14.2 8.6 8.5
Second Building	5.8 20.1 22.2 8.7 12.4	16.3 7.2 7.4 21.5 11.3	12.2 10.3 26 17.3 23	7.2 11.6 24.4 7.8	17.3 14.2 9.2 16.2

Outlying observations were detected in accordance to ASTM E-178-02 (2002). Core compressive strength data were corrected to account for several factors including length to diameter ratio, diameter, moisture content, and damage due to drilling as per recommendations of ACI committee 214.4R. Equivalent specified strength was calculated by multiplying each core compressive strength by a factor of 1.16.

A. *Equivalent Design Compressive Strength*

The equivalent design compressive strength for the two buildings was calculated based on the modified tolerance factor approach and the tolerance factor approach. Results are shown in Table III below. Results shows that the modified tolerance factor approach gives more reliable and higher estimated design strength values of concrete than the tolerance factor approach.

B. *Discussion and Analysis*

Findings indicate that for data with high coefficient of variation, the tolerance factor approach can be too conservative. The effect of subtracting the (Ks_c) factor from the mean compressive strength, can reduce the mean by orders of 50% in some cases. The modified tolerance factor can correct this drawback by increasing mean compressive strength, decreasing the standard deviation and a possible decrease in K values. The combined effect of these factors can increase equivalent design strength results of the tolerance factor approach by orders of 30% in some cases. Table IV below shows percentage increase of equivalent design strength when using the modified tolerance factor approach.

Table IV. Percentage increase in equivalent design strength

Building	Coefficient of Variation	Percentage Increase in Equivalent Design Strength
First	35%	13.5%
Second	42%	28%

Results of the design compressive strength from the two approaches are compared now to Bartlett and Macgregor (1995) approach. It's good to note here that Bartlett and Macgregor approach is also adopted by ACI committee 214.4R as a second approach for estimating equivalent design compressive strength from core tests. Table V below shows estimated equivalent design strength based on Bartlett and Macgregor and percentage difference between tolerance factor approach and Bartlett and Macgregor approach and between modified tolerance factor approach and Bartlett and Macgregor approach respectively.

Table III. Equivalent design strength

Building	$\bar{f}_c$ (*MPa*)	s_c	K_L	$\bar{f}_{c,LT}$	$s_{c,LT}$	Equivalent Design Strength (Modified Tolerance Factor Approach) (*MPa*)	Equivalent Design Strength (Tolerance Factor Approach) (*MPa*)
First Building	13.9	4.86	-2.04	14.2	4.57	7.43	6.58
Second Building	16.62	7.05	-1.8	17.2	6.46	7.63	5.97

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 5 Issue 01, January-2016

Table V. Comparison between different statistical approaches

Building	Equivalent Design Strength (Bartlett and Macgregor) (MPa)	Percentage Difference (Tolerance Factor and Bartlett and Macgregor)	Percentage Difference (Modified Tolerance Factor and Bartlett and Macgregor)
First	11.07	68%	49%
Second	13.1	119%	71%

It can be directly inferred from Table V that the tolerance factor approach and the approach proposed by Bartlett and Macgregor and which are both adopted by the ACI committee 214.4R as methods to estimate the equivalent design strength from core tests, tend to diverge when the coefficient of variation of the core compressive strength data is high. The percentage difference between the two approaches is very high, and in some cases exceeds 100% which is statistically unacceptable. The modified tolerance factor approach can bridge the gap and significantly decrease the percentage difference between the two approaches adopted by the ACI committee 214.4R.

IV. CONCLUSIONS

In this study a proposed modified tolerance factor approach was developed to estimate concrete design compressive strength from core tests. The approach is based on the incorporation of a lower bound to the equivalent in place concrete compressive in the analysis and modeling core compressive strength results with a left truncated normal distribution. An experimental formulation of the proposed approach was conducted and based on theoretical/experimental results, the following conclusions could be made:

- Modeling core compressive strength data with a normal distribution has many significant drawbacks such as accepting and incorporating negative concrete strength data in the calculations which is not acceptable from physical and engineering point of view.
- Incorporating a lower bound confined strength of concrete gives more reliability to results especially for data with high coefficient of variation.
- The two approaches adopted by the ACI committee 214.4R to estimate equivalent design compressive strength of concrete from core tests tend to diverge with increasing coefficient of variation. Percentage difference between the two approaches may go beyond 100% which is not acceptable statistically. The modified tolerance factor approach can bridge the gap between the two approaches.

REFERENCES

[1] J. Hanson, B. Noble, "Survey of practice to determine strength of in situ concrete from core tests." J. Perform. Constr. Facil, vol. 21, pp. 22-27, February 2007.

[2] L. Rojas-Henao, J. Fernandez-Gomez, and J.C. Lopez-Agui, "Rebound hammer, pulse velocity, and core tests in slef consolidating concrete." ACI Mater. J. , pp. 235-243, 2012.

[3] ACI Committee 228, "Report on nondestructive test methods for evaluation of concrete in structures (ACI 228.2R-13)", American Concrete Institute, 2013.

[4] X. Chen, S. Wu, and J. Zhou, "Variability of compressive strength of concrete cores", J. Perform. Constr. Facil, Vol. 28, August 2014.

[5] R. H. Campbell, and R. E. Tobin, "Core and cylinder strengths of natural and lightweight concrete", ACI J., Vol. 64, pp. 190-195, 1967.

[6] I. Soroka, "An application of statistical procedures to quality control of concrete", Mater. Struct., Vol. 1, pp. 437-441, 1968.

[7] K. R. Hindo, and W. R. Bergstorm, " Statistical evaluation of the in-place compressive strength of concrete", Concrete International, Vol. 7, pp. 44-48, February 1985.

[8] F. M. Bartlett, and J. G. MacGregor, "Equivalent specified concrete strength from core test data", Concrete International, Vol. 17, pp. 52-58, March 1995.

[9] T. Chmielewski, and E. Konapka, "Statistical evaluations of field concrete strength", Mag. Conr. Res, Vol. 51, pp. 45-52, February 1999.

[10] B. Graybeal, and M. Davis, "Cylinder or cube: strength testing of 80-200 MPa (11.6 to 29 ksi) ultra-high-performance fiber-reinforced concrete", ACI Mater. J., Vol 105, pp. 603-609, November 2008.

[11] K. Kilinc, A.O. Celik, M. Tuncan, A. Tuncan, G. Arslan, and O. Arioz, "Statistical distributions of in-situ microcore concrete strength", Construction and Building Materials, Vol. 26, pp. 393-403, January 2012.

[12] ACI Committee 214, "Guide for obtaining core and interpreting compressive strength results (ACI 214.4R-10)", American Concrete Institute.

[13] ASTM (2002). "Practice for dealing with outlying observations". ASTM E178.

[14] S. Ahmad, K. Pilakoutas, Q. U. Z. Khan, and K. Neocleous, "Stress-strain model for low-strength concrete in uni-axial compression", Arab J. Sci. Eng., Vol. 40, pp. 313-328, 2015.

[15] M. Natrella, "Experimental statistics", Handbook No. 9, National Bureau of Standards, United States Governmnet Printing Office, Washington, 1963.

[16] A. Johnson, "On the truncated normal distribution: characteristics of singly and doubly truncated populations of application in management science", Unpublished PhD thesis, University of Illinois, May 2001.

[17] ACI Committee 228, "In-place methods to estimate concrete strength (ACI 228.1R-03), American Concrete Institute, 2003.