Estimation of Hydrodynamic Forces and Motion Characteristics of a Trimaran Light Aboard Ship using Finite Element Method

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***Abstract*— The Trimaran ship design has become attractive to naval architects due to its superior sea going performance. Trimaran vessels are developed for different applications and hydrodynamic behavior of such vessels is different than usual mono hulls. Consequently, the trimaran has become a high performance vehicle type, especially in the design of high-speed commercial and naval ships. In the present work, linear diffraction- radiation theory has been used to formulate the problem and the differential equations have been solved using three-dimensional finite element method to obtain the wave loads and moments on the given vessel at zero forward speed in different sea states. Subsequently the motion characteristics have been obtained by solving the equation of motion. Solution is done by a FORTRAN program based on the theoretical and finite element formulation of the wave structure interaction problem. Computations have been performed for different wave parameters.**

***Keywords*— Multi hull ships; Trimaran; Fluid structure interaction problem; Finite Element Method.**

I.INTRODUCTION

The study of ocean waves and the effects of wave environment and the estimation of hydrodynamic forces that ocean waves may exert on marine structures and vehicles have received considerable mathematical and engineering interests among scientists and engineers. Of the environmental loadings acting on them waves account for the most of the structural loading and since they are time dependent, produce dynamic effects tending to increase the stresses and damage the long-term behavior of the system. Therefore accurate prediction of these wave forces on these structures and their responses to waves are extremely important for their design, installation and operation purposes.

Marine vehicles are exposed to various dynamic forces generated by environmental phenomenon like waves and winds. The hydrodynamic forces on the body and the motions exhibited by them characterise the fluid-structure interaction. By imposing suitable restrictions on the motion amplitude and wave steepness, the fluid-body interaction problem can be linearised, which facilitates analytical/numerical solutions in a wide variety of practical cases. The solution of the linear body-fluid interaction problem is greatly enhanced if one appeals to the principle of linear super position and identifies a diffraction problem, wherein the body is assumed constrained and exposed to the incident wave field and a serious of radiation problems in each of which the body is allowed to undergo motion in one of the degrees of freedom. Thus the linear hydrodynamic problem is henceforth referred to as the diffraction-radiation problem [1, 9].

The urge for safety and operational efficiency has been leading mankind to venture into innovative ideas and its implementation on marine vehicle design and development. One of the arenas in this regard has been the evolution of mutli-hull vessels, from catamarans to trimaran and then to pentamaran. Trimarans share most of the characteristics of catamarans, but in few aspects, trimarans are more efficient than catamarans. Lyakhovitsky compared a trimaran with a mono-hull and catamaran of same characteristics and showed that the trimaran is better in hydrodynamic performance compared to other alternatives. In addition trimarans have some other privileges such as: extended deck, lower draft and better transverse stability compared with single body vessels [4].

In the present study, trimaran concept is used for the transport of ship bare hulls (with no machinery and outfits) built in one part of the world to another part through ocean, with three individual ships connected rigidly to form the base (carrier) trimaran and other hulls loaded on it. This complex structure is towed using tugs at a speed of 3 to 4 knots. The objective of present work is to study the hydrodynamic performance (estimation of wave loads and motion characteristics) of the trimaran light aboard ship numerically, using three dimensional finite element method.

II. PROBLEM FORMULATION

A. Theoretical Formulation

The fluid (water) is assumed to be incompressible and inviscid and the flow is considered as irrotational, so that the fluid velocity may be represented as the gradient of a scalar potential ϕ. A right – handed Cartesian coordinate system (x,y,z) is defined with origin at the still water level and *z*- axis positive upwards (Figure 1).The fluid in the domain(Ω) is bound by the free surface (S_F),bottom surface (S_{Bot}),body surface(S_B) and far boundary surface(S_R) .

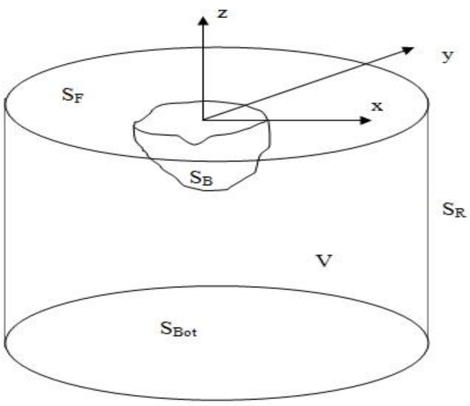


Figure 1: Definition of the geometry

Both incident and scattered waves are assumed as harmonic and small amplitude. Hence, following Haskind's decomposition, the complex velocity potential may be written as [2]:

$$\Phi(x,y,z,t)=\phi(x,y,z)e^{i\omega t}=\left\{\phi_I+\phi_S+\sum_{j=1}^{6} i\omega\xi_{aj}\phi_j\right\}e^{i\omega t} \qquad (1)$$

Where, Φ - total wave velocity potential, ϕ_I - incident wave velocity potential, ϕ_s - scattered wave velocity potential, ξ_{aj} - response amplitude of the body in j-th degree of freedom or mode, ϕ_j - velocity potential of the radiated wave due to body motion with unit velocity amplitude in the j-th mode.

ω - circular frequency of wave, t - time.

The governing equations of the three-dimensional hydrodynamic problem referred above are as follows.

Laplace Equation

$\nabla^2\phi_m = 0$ within the fluid domain Ω
(2)

Where the subscript m =I, s, j; I denotes the incident wave, s represents the scattered wave and j (j =1,…, 6), radiation wave potentials.

Linearised Free Surface Condition:

By combining the kinematic and dynamic conditions, we get the linearised free surface condition for a harmonic wave as [3]:

$$\frac{\partial\phi_m}{\partial z}-\frac{\omega^2}{g}\phi_m = 0 : m=I,s,j(j=1,...,6) \text{ on the free surface } S_F \qquad (3)$$

Where g is the acceleration due to gravity.

Radiation Condition:

T The scattered wave is assumed to propagate radially outward at infinity, which is represented by the Sommerfeld radiation condition [11]:

$$\sqrt{kr}\left(\frac{\partial\phi_m}{\partial r}-ik\phi_m\right) = 0; m=s, j(j=1,..,6) \text{ as } r\to\infty \qquad (4)$$

Where k denotes the wave number.

Bottom Boundary Condition:

An impervious bottom is assumed and hence the normal velocity of the particle at the bottom boundary is zero.

Kinematic Body Boundary Condition:

i) Diffraction Problem

$$\frac{\partial\phi_m}{\partial n} = 0; \ m=I,s,j(j=1,..,6) \text{ on the sea bed}, S_{Bot} \qquad (5)$$

ii) Radiation problem

$$\frac{\partial\phi}{\partial n}=\bar{n}_j \quad \text{on } S_B \qquad (7)$$

Where $\{\bar{n}\}$ is the generalized normal vector associated

$$\frac{\partial\phi_S}{\partial n} = -\frac{\partial\phi_I}{\partial n} \quad on\ S_B \qquad (6)$$

with a point (*x*,*y*,*z*) on the hull.

Hydrodynamic forces and coefficients

From the linear unsteady form of the Bernoulli's equation, the hydrodynamic pressure can be obtained as:

$$P=-\rho\frac{\partial\phi}{\partial t} \qquad (8)$$

From Eqn. (1), it can be simplified as

$$p=-i\omega\rho\phi \qquad (9)$$

The generalized complex (harmonic) hydrodynamic forces on the wetted body surface may be obtained as:

$$f_k=\int_{S_B} p\bar{n}_k dS \qquad (10)$$

For the diffraction problem, the hydrodynamic exciting force in the j-th mode can be written as,

$$f_{dj}=\int_{S_B} p\bar{n}_j dS=-i\rho\omega\int_{S_B}(\phi_I+\phi_S)\bar{n}_j dS \qquad (11)$$

For the radiation problem, the complex hydrodynamic reaction force amplitude may be written as:

$$f_{jk}=\rho\omega^2\int_{S_B}\phi_j\bar{n}_k dS;\ \text{j,k=1,2\&6} \qquad (12)$$

I. FINITE ELEMENT FORMULATION

The governing differential equation and the boundary conditions may be cast in the variational or weighted residual form to obtain the finite element equations. The assembled finite element formulation of a diffraction-radiation problem is [7-8, 10, 12]:

$$[K]\{\phi\}=\{f\} \qquad (13)$$

The solution is done by reduction and back substitution.

B. Motion Analysis

Motion analysis is done by solving the equation of motion. The equation for the structure describing the dynamic equilibrium between the inertia, damping, restoring and exciting forces in matrix form can be written as [5-6]:

$$[M^1]\{\ddot{x}\}+[B]\{\dot{x}\}+[K]\{x\}=\{F\} \qquad (14)$$

Where $[M^1]=[m]+[A]$; [m] – Mass matrix, [A]- Added mass matrix, [B]- Damping matrix, [C]- Restoring coefficient matrix, {F} - Excitation force.

For a harmonic motion, the displacement can be represented as:

$$\{x\}=\{x_0\}e^{-i\omega t} \qquad (15)$$

Therefore Eq. (14) becomes

$$\{-\omega^2([M]+[m])-i\omega[B]+[C]\}\{x_0\}=\{F_0\} \qquad (16)$$

We get the response vector $\{x_0\}$ from the above equation by knowing wave excitation force, added mass and damping coefficients for a given wave frequency.

III. GEOMETRIC MODELLING OF THE VESSEL

The carrier vessel geometry is modeled using the known offset table, which gives the y & z values (coordinates) of different stations or frames, using the 3D graphic package ANSYS ICEM. The three hulls of the catamaran carrier vessel, whose particulars given in Table 1, are rigidly connected by placing and fixing rectangular box, with cross-section dimensions of depth by width = 4.0m x 1.0m, running throughout the parallel body region. The bottom plan and the isometric views of which are shown in Figure 2.

Table 1: Carrier Vessel Particulars

Length Overall = 135.0m	Moulded Breadth of combination = 47.0m
Moulded breadth of single hull = 15.0m	Depth = 5.39m
Loaded draft: 4.0 m	Inter-hull box size= 77m length, 1.0m width, and 4.0 m depth from deck downward

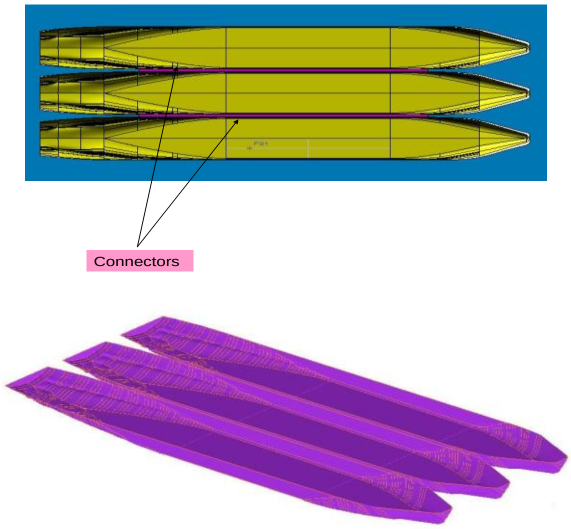
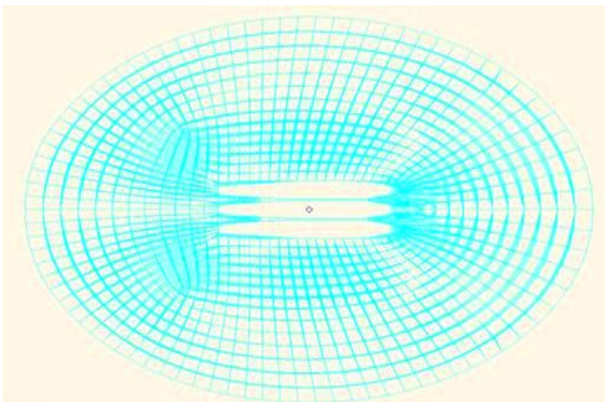


Figure 2: Geometry of the carrier vessel - Bottom view & isometric view

IV. FINITE ELEMENT MODELLING

The fluid domain for the finite element analysis has been identified based on past experience and logical observations. Then, the finite element model has been created in ANSYS ICEM and the discretisation of the fluid domain has been done by the software ALTAIR HYPERMESH. Twenty noded isoparametric hexahedral elements were used to model the fluid domain. The meshing scheme employed is a uniformly graded mesh with finer mesh in regions closed to the body. Surface code marking, which is an elaborate process involving the identification of thousands of 20-noded hexahedral elements, has been done. The finite element mesh consists of 23932 elements and 102973 nodes. There are 1126 elements on the free surface, 1548 elements on the radiation surface and 648 elements on the body boundary. The top view (free surface) and isometric view of the mesh are shown in Figure 3. A FORTRAN computer code, based on the theoretical and finite element formulation of the wave–structure interaction problem for the estimation of wave forces and moments, was modified to include the calculation of a floating body motion characteristics.



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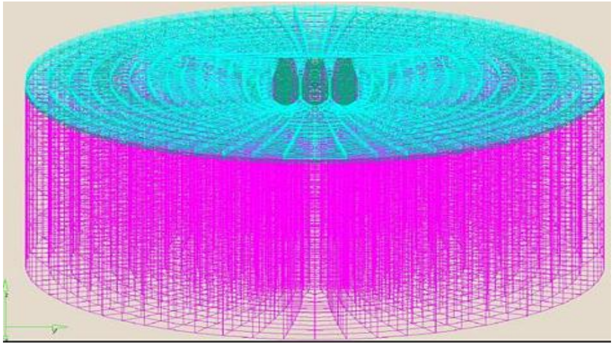

Figure 3: Finite element mesh- free surface & isometric views

V. RESULTS AND DISCUSSION

The numerical estimation of wave forces and motion responses in different sea states have been carried out using finite element method. The hydrodynamic force distribution over the wetted region of the hull and global forces and moments acting on the bottom ships has been obtained from the above estimation.

A. *Wave Force and Moments*

Beam sea (μ = 90 deg)

The wave exciting forces per unit wave amplitude (transfer function) on the vessel in sway and heave modes are shown in Figure 4 & 5 and the wave exciting moments per unit wave amplitude (transfer function) on the vessel in roll, pitch and yaw are shown in Figure 6, 7& 8. In beam seas, the surge exciting force is insignificant and hence not presented here. Unlike the mono-hull vessels, it may be noted that the roll exciting moment exceeds the pitch moment. This must be due to the lower length/breadth ratio of the trimaran vessel. Pitch moment is significant here even though it is a beam sea condition. Here heave force and pitch moment are not exhibiting a frequency turning effect.

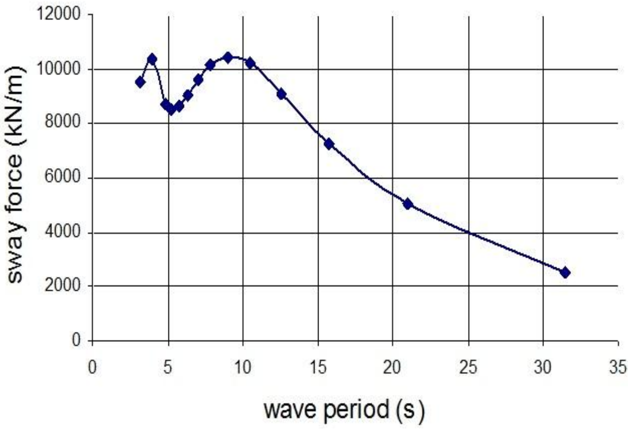

Figure 4: Variation of sway force with wave period

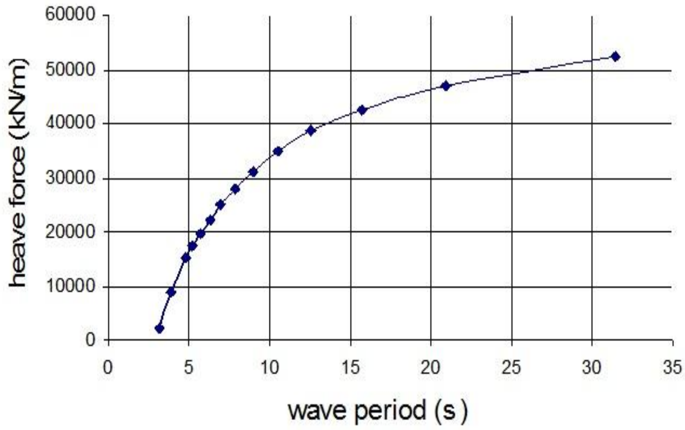

Figure 5: Variation of heave force with wave period

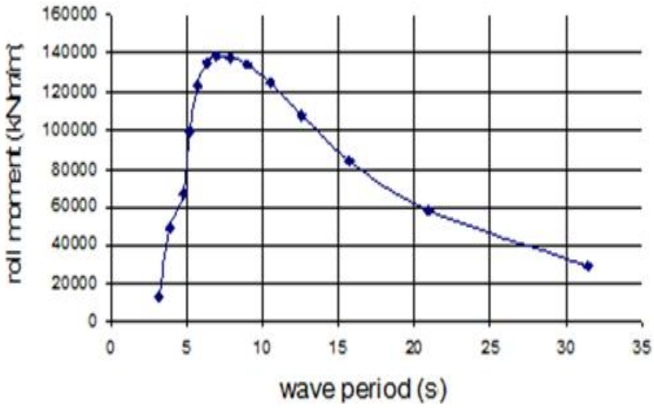

Figure 6: Variation of roll moment with wave period

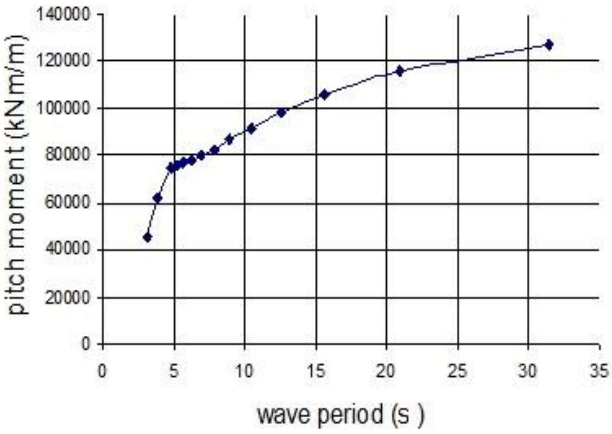

Figure 7: Variation of pitch moment with wave period

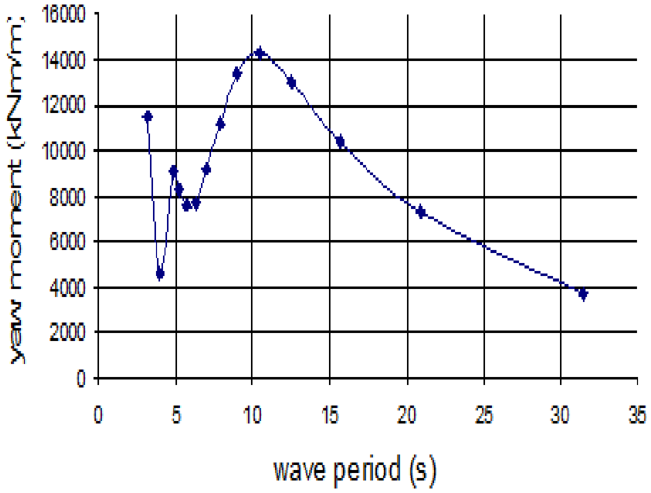

Figure 8: Variation of yaw moment with wave period

Oblique sea (μ = 135 deg)

A quartering sea is a case where all modes of motion become relevant. Surge and sway exciting forces initially increase with wave period, reach a maximum value and then decreases. Figure 9, 10 & 11 shows the wave exciting force per unit wave amplitude for surge, sway and heave modes and Figure 12,13 &14 represents the wave exciting moment per unit wave amplitudes for roll, pitch and yaw motions respectively. Roll & pitch exciting moment amplitudes are found to have a similar trend. Heave force is observed to increase over wave period. The pitch exciting moment is very significant as expected in an oblique sea.

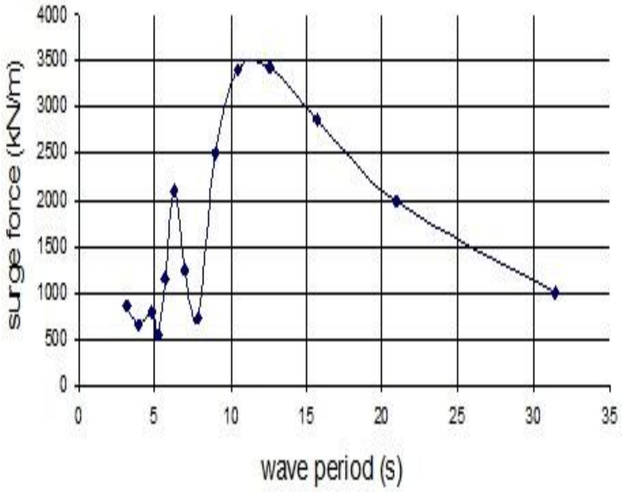

Figure 9: Variation of surge force with wave period

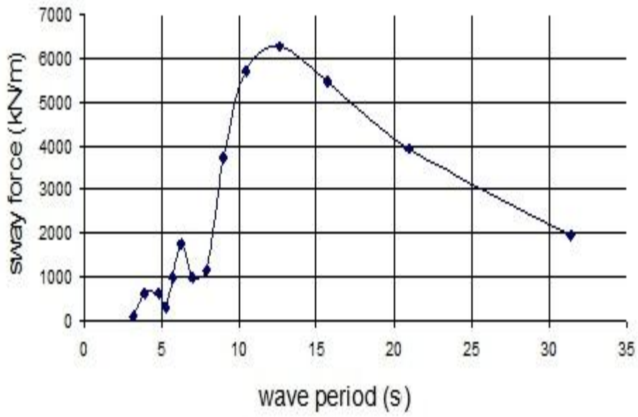

Figure 10: Variation of sway force with wave period

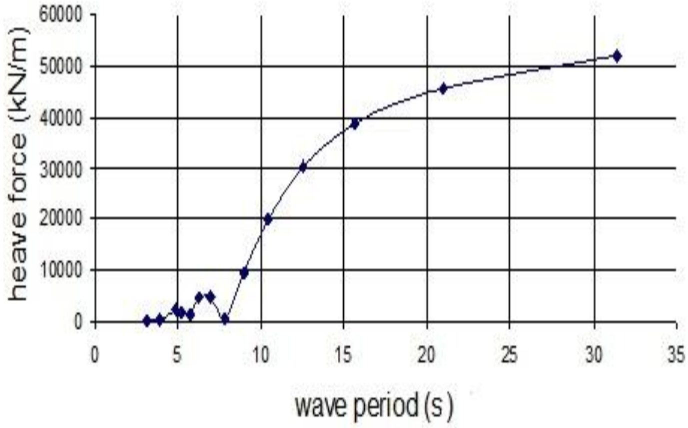

Figure 11: Variation of heave force with wave period

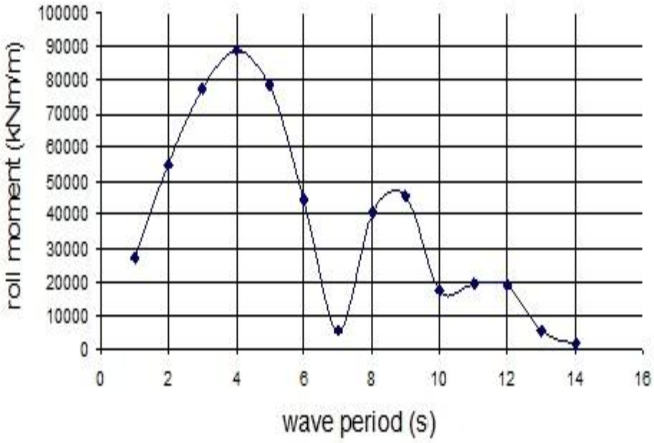

Figure 12: Variation of roll moment with wave period

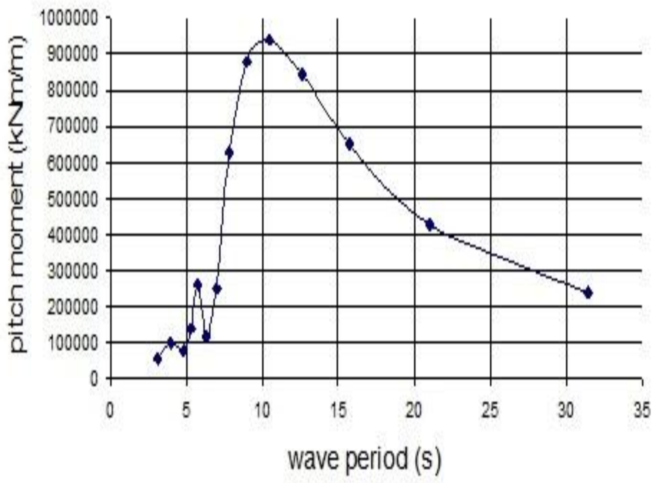

Figure 13: Variation of pitch moment with wave period

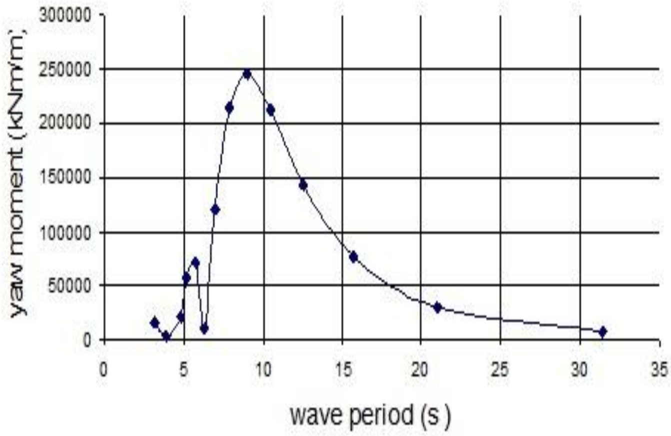

Figure 14: Variation of yaw moment with wave period

Head sea (μ = 180 deg)

Figure 15 &16 shows the wave exciting force per unit wave amplitude for surge and heave modes and Figure 17 represent the wave exciting moment per unit wave amplitude for the pitch motion. Surge force amplitude is less compared to heave exciting force. Heave force is found to increase over wave period. Pitch moment amplitude initially increases with wave period, attain a peak value and then decreases. In Head seas, the sway exciting force, roll and yaw moments are insignificant.

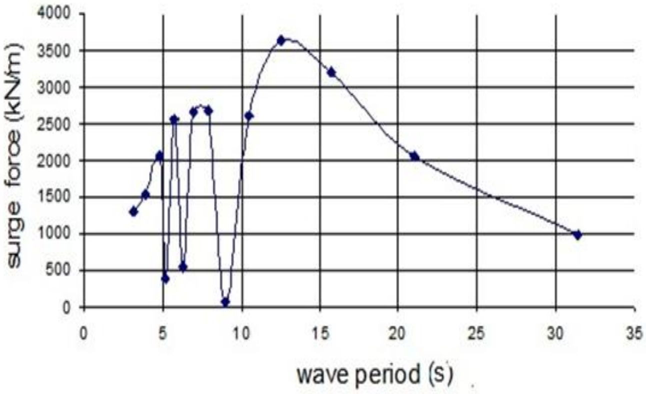

Figure 15: Variation of surge force with wave period

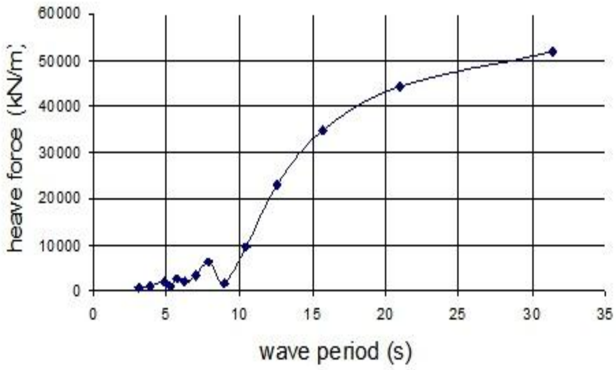

Figure 16: Variation of heave force with wave period

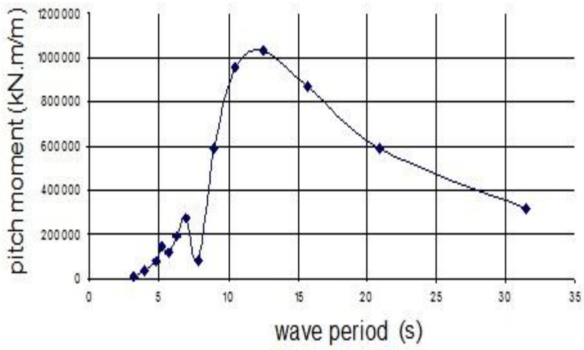

Figure 17: Variation of pitch moment with wave period

The maximum values of wave exciting forces and moments per unit wave amplitude for different sea conditions are analyzed and results are shown in Table 2.

Table 2.Maximum values of wave exciting forces & moments for 1.0m wave amplitude

Sea Direction	Surge Force [kN/m]	Sway Force [kN/m]	Heave Force [kN/m]	Roll Moment [kNm/m]	Pitch Moment [kNm/m]	Yaw Moment [kNm/m]
Beam	---	10400	52300	138000	127000	14500
Bow-Quartering	3520	6270	52000	88400	941000	245000
Head	3640	---	51700	---	1030000	---

B. Motion Responses

Motion responses of the trimaran at zero forward speed in different sea states are obtained by solving the equation of motion and obtained results are plotted against wave period.

Beam sea (μ = 90 deg)

The response transfer operators (RAO) for sway, heave, roll and yaw motions are shown in the Figures 18- 21. In Beam seas, roll motion amplitude is high compared to other responses. Heave motion amplitude initially increases, attain a peak value and remains constant with wave period. Roll response initially increases with wave period, attain a maximum value and then decreases. Yaw motion response is found to be negligible.

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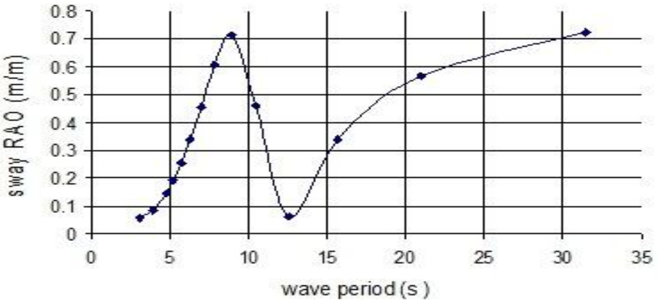

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Figure 18: variation of sway response with wave period

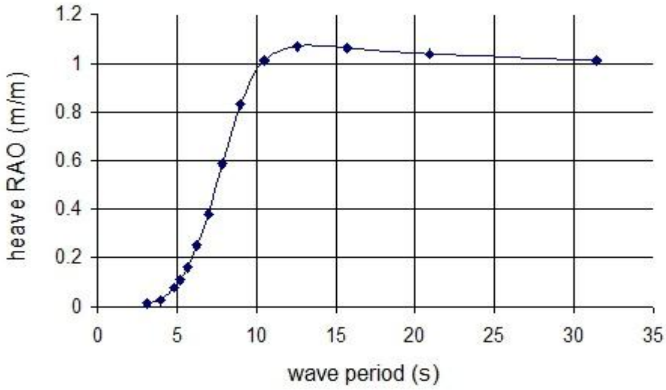

Figure 19: variation of heave response with wave period

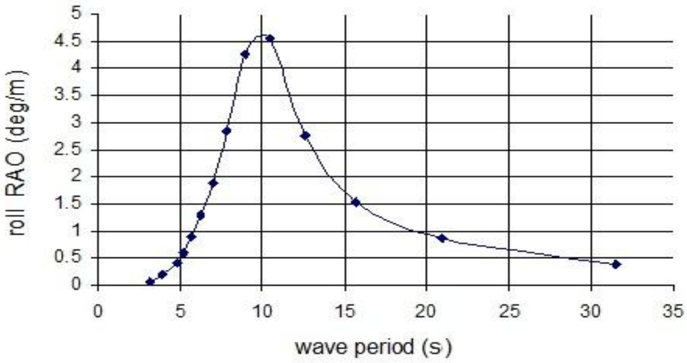

Figure 20: variation of roll response with wave period

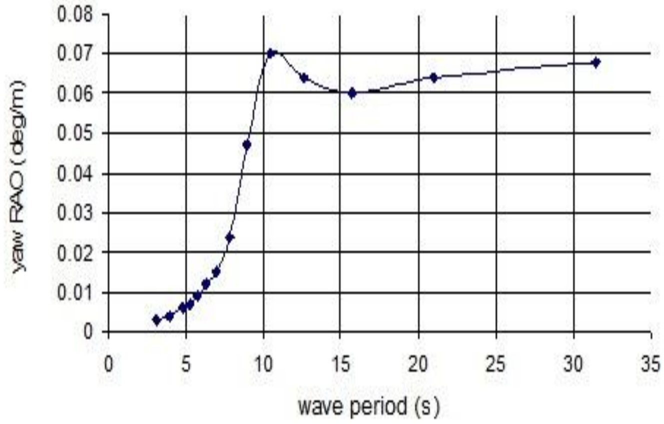

Figure 21: variation of yaw response with wave period

Oblique sea (μ = 135 deg)

The response transfer functions for all six degrees of freedom are shown in Figure 22- 27. In Oblique seas, all motions are significant. Surge motion amplitude is observed to increases over the wave period. Heave motion amplitude is found to increase over wave period and reaches a saturation value. Roll, pitch & yaw motion amplitudes initially increase, attain a peak value and then decreases with wave period.

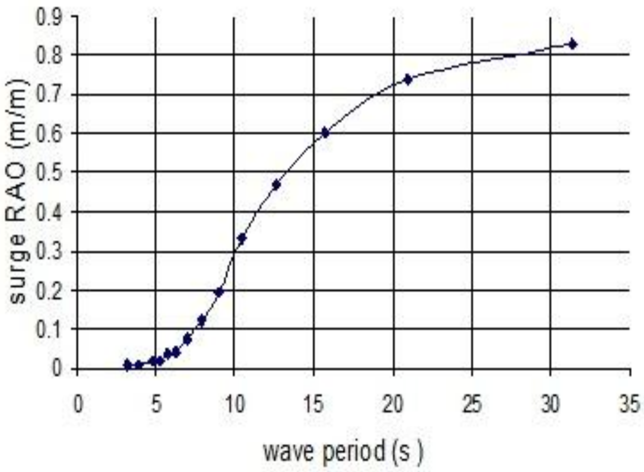

Figure 22: variation of surge response with wave period

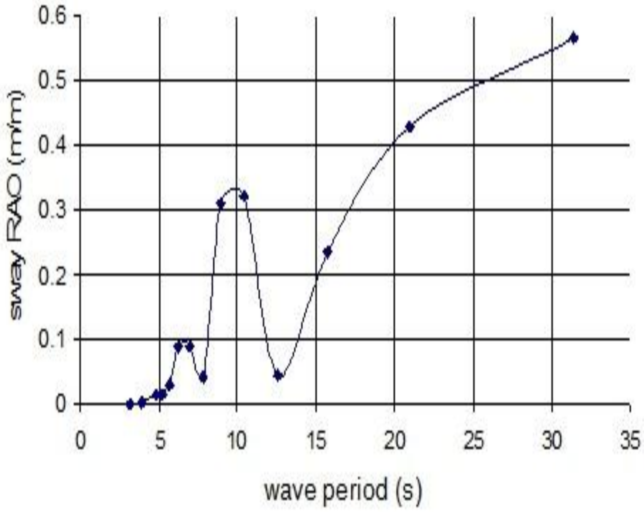

Figure 23: variation of sway response with wave period

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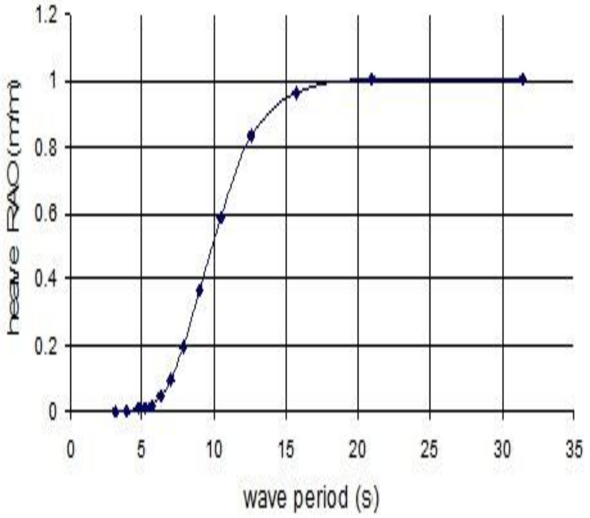

Figure 24: variation of heave response with wave period

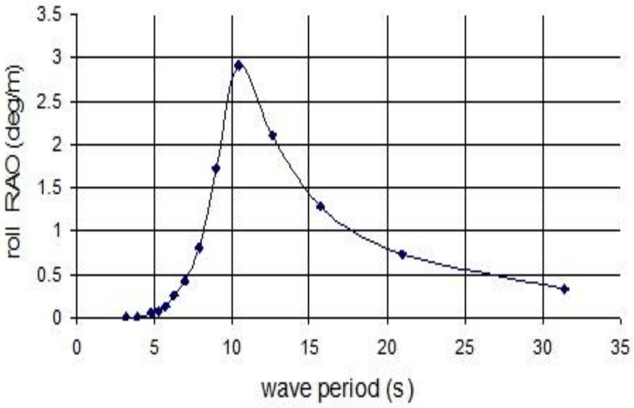

Figure 25: variation of roll response with wave period

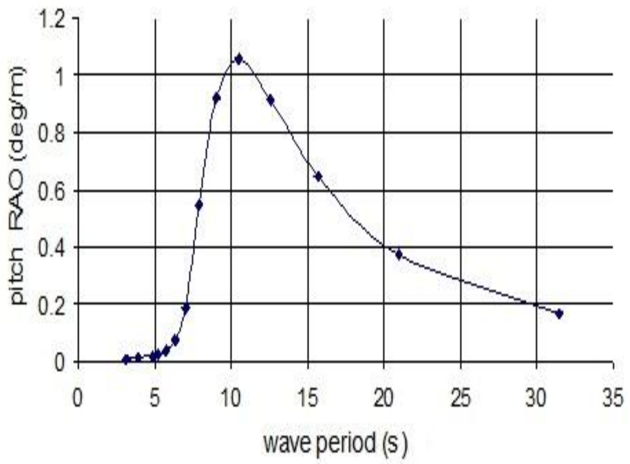

Figure 26: variation of pitch response with wave period

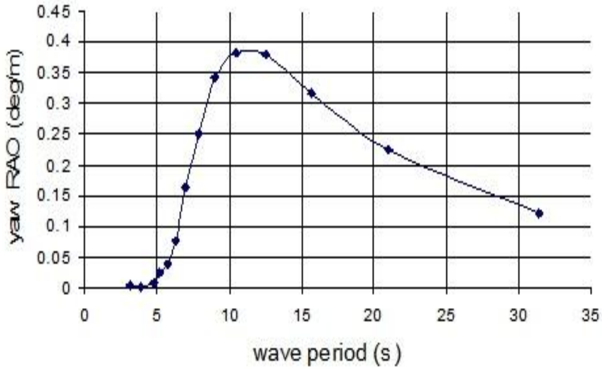

Figure 27: variation of yaw response with wave period

Head sea (μ = 180 deg)

The response transfer functions for surge, heave and pitch motions are shown in Figure 28- 30. Heave motion amplitude initially increases, reach a maximum value, and then remains constant with wave period. Pitch motion amplitude initially increases, attain a peak value and then decreases with wave period. In Head seas, other motions are insignificant.

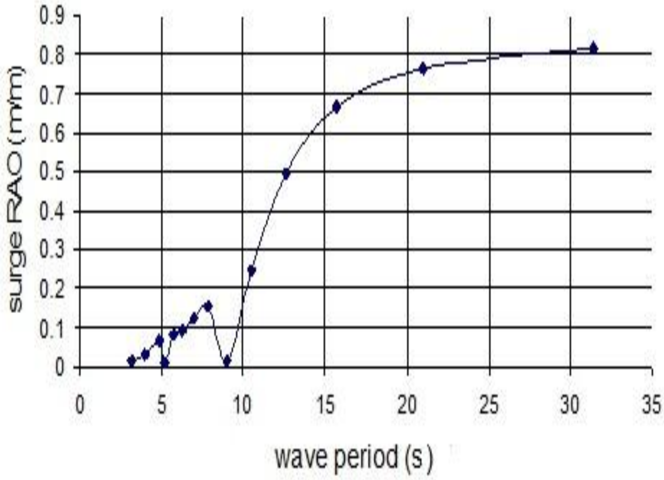

Figure 28: variation of surge response with wave period

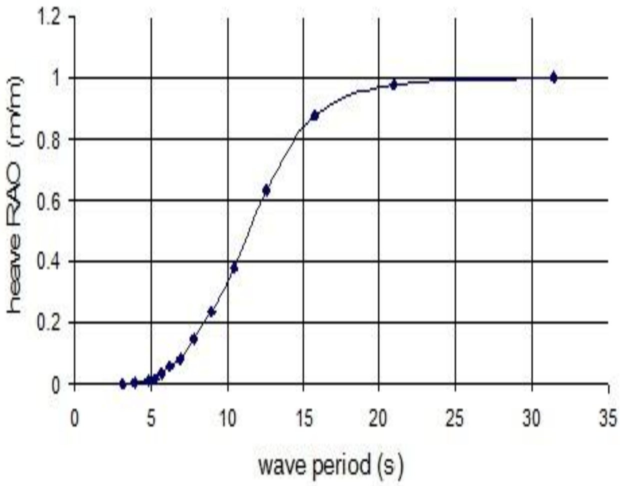

Figure 29: variation of heave response with wave period

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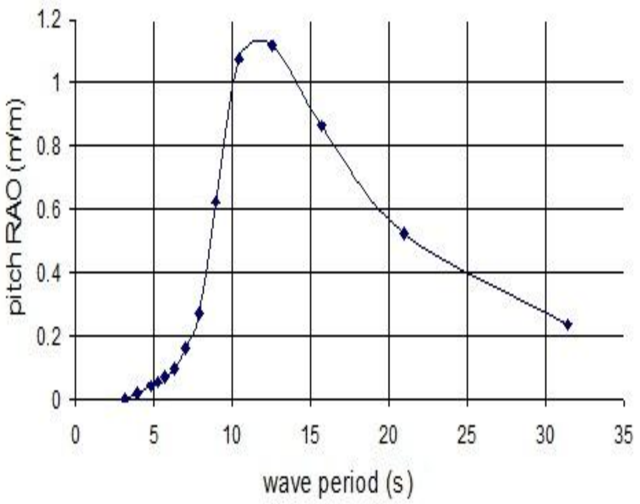

Figure 30: variation of pitch response with wave period

The maximum values of motion responses per unit wave amplitude for different sea conditions are analyzed and results are shown in Table 3.

Table 3. Maximum values of motion response for 1.0m wave amplitude.

Sea Direction	Surge RAO [m/m]	Sway RAO [m/m]	Heave RAO [m/m]	Roll RAO [deg/m]	Pitch RAO [deg/m]	Yaw RAO [deg/m]
Beam	---	0.71	1.06	4.52	---	0.07
Bow-Quartering	0.81	0.55	1.00	2.90	1.07	0.38
Head	0.80	---	1.00	---	1.10	---

VI. CONCLUSIONS

Software packages based on finite element method to estimate wave forces and moments on floating bodies are not commercially available. It is evident from the study that the FORTRAN program based on the theoretical and finite element formulation of the wave – structure interaction problem can be effectively utilized to estimate the wave forces, moments and motion responses of the complicated geometry hull forms like trimaran.

The computationally obtained wave exciting forces and moments on the carrier vessel for different sea directions, such as beam, bow-quartering and head sea are presented in Section 5 A. Table 2 gives the maximum values of wave exciting forces and moments per unit wave amplitude, which when multiplied by the known wave amplitude gives the corresponding wave excitation forces and moments. The heave excitation force is an order above the surge and sway exciting forces, which obviously is due to the broader form of the carrier vessel. Roll moment is insignificant in following and head seas, whereas it is large in beam seas followed by quartering seas. Pitch moment is considerable in head and following seas and also comparable in quartering seas. Due to vessel symmetry about longitudinal vertical central plane the yaw moment is insignificant in head and following seas, and it is maximum in quartering seas.

The motion responses of the trimaran for different sea conditions are showed in Section 5 B. Table 3 shows the peak values of motion response transfer functions, which when multiplied by the known wave amplitude gives the respective motion amplitudes. Roll response is insignificant in head seas where as maximum in beam seas followed by quartering seas. Pitch motion is considerable in head and quartering sea conditions where as insignificant in beam seas. Heave motion of the vessel is considerable in all sea conditions.

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