# Energy Efficient MIMO Wireless Communication Systems Performance Analysis 

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#### Abstract

Multiple Input Multiple Output (MIMO) systems with multiple antenna elements at both Transmitter and Receiver ends are an efficient solution for future wireless communications systems. They provide high data rates by exploiting the spatial domain under the constraints of limited bandwidth and transmit power. Space-Time Block Coding (STBC) is a MIMO transmit strategy which exploits transmit diversity and high reliability. The proposed work presents a comprehensive performance analysis of orthogonal space-time block codes (OSTBCs) with transmit antenna selection under uncorrelated Rayleigh fading channel employing Alamouti's code. The transmitted symbols belong to BPSK, QPSK and Quadrature amplitude modulation (QAM). The objective of this paper is to analyze the performance of MIMO wireless systems in Rayleigh channel using various modulations schemes such as BPSK, QPSK, 16-QAM, and 64-QAM


## 1. Introduction

Multiple Input Multiple Output (MIMO) technology holds the promise of higher data rates with increased spectral efficiency. All commercial wireless systems operate in high multipath environments and it is the benefit of multipath that provides the performance improvement when using multiple antenna configurations [1]. Diversity is often used in wireless channels to combat multipath fading effect. The main idea behind "diversity" is to provide different replicas of the transmitted signal to the receiver [2]. If these different replicas fade independently, it is less probable to have all copies of the transmitted signal in deep fade simultaneously and the receiver can reliably decode the transmitted signal. The replica of the
transmitted signal can be sent through different means, MIMO utilizes Antenna diversity to combat fading by combining signals from two or more independently faded channels. [3].In this paper, we have used Alamouti Space Time Coding (STC) that sends the same user data to both transmit antennas. The remainder of the paper is organized as follows: Section 2 defines the MIMO system model; section 3 provides a theoretical analysis. Section 4 shows the performance analysis of the channel and its impact on the performance of OSTBC .In section 5, we summarize the study.

## 2. MIMO System Model

In a MIMO system where there are M transmit and N receive antennas as shown in fig. 1.0 below, the transmitted data signals pass through multiple paths to get to the receiving antennas. Though not shown on the diagram, but there is also noise that interferes with the data signals along the paths [4].

In order to design efficient communication algorithms for MIMO systems and to understand the performance limits, it is important to understand the nature of the MIMO channel. For a system with $M$ transmit antennas and $N$ receive antennas, assuming frequency-flat fading over the bandwidth of interest, the MIMO channel at a given time instant may be represented as an $\mathrm{M} \times \mathrm{N}$ matrix.


Figure 1.0 Basic M-transmit by N-receive MIMO system

Figure 1.0 above shows a MIMO transmission systems consisting of M transmit antennas and N receive antennas. The channel ' $\mathbf{H}$ ' is presumed to be a rich scattering environment. MIMO uses the multi antenna spatial diversity at both ends of the link, treating the multiplicity of the different scattering paths as separate parallel sub channels.

The received signal vector $\mathbf{y}$ can be expressed in terms of the channel matrix $\mathbf{H}$ as:

$$
\begin{equation*}
\mathbf{y}=\mathbf{H x}+\mathbf{n} \tag{1.1}
\end{equation*}
$$

Where,
$\mathbf{y}$ is the received signal vector
$\mathbf{x}$ is the transmitted signal vector
$\mathbf{n}$ is the additive white Gaussian noise vector with zero mean and a variance of $\sigma^{2}$

Equation 1.1 can also be represented as a system of linear equations given by:-

$$
\left[\begin{array}{c}
y_{1}  \tag{1.2}\\
y_{2} \\
\vdots \\
\vdots \\
y_{N}
\end{array}\right]=\left[\begin{array}{c}
x_{1} \\
x_{2} \\
\vdots \\
\vdots \\
x_{M}
\end{array}\right]\left[\begin{array}{ccccc}
h_{11} & h_{12} & \cdots & \cdots & h_{1 M} \\
h_{21} & h_{22} & \cdots & \cdots & h_{2 M} \\
\vdots & \vdots & \vdots & & \vdots \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
h_{N 1} & h_{N 2} & \cdots & \cdots & h_{N M}
\end{array}\right]+\left[\begin{array}{c}
n_{1} \\
n_{2} \\
\vdots \\
\vdots \\
n_{M}
\end{array}\right]
$$

From which the received signal vectors are;

$$
\begin{aligned}
& y_{1}=x_{1} h_{11}+x_{2} h_{12}+\ldots \ldots \ldots \ldots \ldots+x_{M} h_{1 M}+n_{1} \\
& \begin{array}{ccc}
y_{2}=x_{1} h_{21}+x_{2} h_{22}+\ldots \ldots \ldots \ldots .+x_{M} h_{2 M}+n_{2} \\
\vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots
\end{array} \\
& y_{N}=x_{1} h_{N 1}+x_{2} h_{N 2}+\ldots \ldots \ldots \ldots \ldots+x_{M} h_{2 M}+n_{N}
\end{aligned}
$$

For a $2 \times 2$ MIMO system, the expression for equation (1.1) reduces to;

$$
\left[\begin{array}{l}
y_{1}  \tag{1.3}\\
y_{2}
\end{array}\right]=\left[\begin{array}{ll}
h_{11} & h_{12} \\
h_{21} & h_{22}
\end{array}\right]\left[\begin{array}{l}
x_{1} \\
x_{2}
\end{array}\right]+\left[\begin{array}{l}
n_{1} \\
n_{2}
\end{array}\right]
$$

## 3. Space-Time Block Coding (STBC)

One of the methodologies for exploiting the capacity in MIMO system consists of using the additional diversity of MIMO systems, namely spatial diversity, to combat channel fading. This can be achieved by transmitting several replicas of the same information through each antenna. By doing this, the probability of losing the information decreases exponentially [5]. The antennas in a MIMO system are used for supporting a transmission of a SISO system since the targeted rate of is that of a SISO system. The diversity order or diversity gain of a MIMO system is defined as the number of independent receptions of the same signal. A MIMO system with $N_{t}$ transmit antennas and $N_{r}$ receive antennas has potentially full diversity (i.e. maximum diversity) gain equal to $N_{t} x N_{r}$. The different replicas sent for exploiting diversity are generated by a space-time encoder which encodes a single stream through space using all the transmit antennas and through time by sending each symbol at different times. This form of coding is called SpaceTime Coding (STC). Due to their decoding simplicity,
the most dominant form of STCs is space-time block codes (STBC).

### 3.1 The Alamouti $2 \times 2$ MIMO Scheme

The encoding and transmission sequence for the two transmit antennas in the Alamouti scheme is shown in Table 1 [6]

Table 1: Encoding and transmission sequence for the Alamouti two transmit antennas.


Figure 2.0 Space Time Coding with Two transmit and two receive antenna

The data input is first modulated using any of the Mary modulation schemes. The encoded symbols are then sent to Alamouti STBC encoder. In this system, two different symbols are simultaneously transmitted from the two antennas during any symbol period. During the first time period t , the first symbol in the sequence, $s_{1}$ is transmitted from antenna 1 while the second symbol, $s_{2}$ is simultaneously transmitted from antenna
2. During the next symbol time $\mathrm{t}+\mathrm{T}$, the signal $-s_{2}^{*}$ is transmitted from antenna 1 and the signal $s_{1}^{*}$ is transmitted from antenna 2 [7].

Where * denotes complex conjugation. A generalized code matrix for Alamouti $2 \times 2$ is shown below.

$$
G_{2}=\left[\begin{array}{cc}
s_{1} & s_{2}  \tag{1.4}\\
-s_{2}^{*} & s_{1}^{*}
\end{array}\right]
$$

### 3.2 Orthogonal Space Time Block Coding (OSTBC)

In this section, we consider another type of STBC called orthogonal space time block codes (OSTBC) for the case of 3 and 4 transmit antennas. The transmit diversity scheme designed by Alamouti can be used only in a system with two transmit antennas. It turns out that this technique belongs to a general class of codes named Space-Time Block Codes or, more precisely, Orthogonal STBCs, since they are based on the theory of orthogonal designs. The authors of [8] introduced the theory of generalized orthogonal designs in order to create codes for an arbitrary number of transmit antennas. The general idea behind STBCs construction is based on finding coding matrices $X$ that can satisfy the following condition:

$$
\begin{equation*}
X X^{H}=p\left(\sum_{i=1}^{n}\left|x_{i}\right|^{2}\right) \cdot I_{M_{T}} \tag{1.5}
\end{equation*}
$$

In this equation, $X^{H}$ is the Hermitian of $X, p$ is a constant, $I_{M_{T}}$ is the identity matrix of size $M_{T} x M_{T}$, $M_{T}$ represents the number of transmit antennas, and $n$ is the number of symbols $x_{i}$ transmitted per transmission block in $X$. The generalized theory of orthogonal design is exploited to provide codes that satisfy Equation 1.5 . The orthogonality property of STBCs is reflected in the fact that all rows of $X$ are orthogonal to each other. In other words, the sequences transmitted from two different antenna elements are orthogonal to each other for each transmission block. [8]

The Alamouti scheme discussed in Section 3.1 of this paper is part of a general class of STBCs known as Orthogonal Space-Time Block Codes (OSTBCs) [8]. The authors of [9] apply the mathematical framework of orthogonal designs to construct both real and
complex orthogonal codes that achieve full diversity. For the case of real orthogonal codes, it has been shown that a full rate code can be constructed [9]. However, for the case of complex orthogonal codes, it is unknown if full rate and full diversity codes exist for $N_{t}>2$ [8]. Complex modulation techniques are of interest in this paper and therefore real orthogonal codes are not discussed.

### 3.3 Orthogonal Space-Time Block Codes for $\mathbf{N t}=3$ and $\mathbf{N t}=4$

For any arbitrary complex signal constellation, there are OSTBCs that can achieve a rate of $1 / 2$ for any given number of Nt transmit antennas. For example, the code matrices $G_{3}$ and $G_{4}$ are OSTBCs for three and four transmit antennas, respectively and they have the rate 1/2 [9].

$$
G_{3}=\left[\begin{array}{ccc}
s_{1} & s_{2} & s_{3}  \tag{1.6}\\
-s_{2} & s_{1} & -s_{4} \\
-s_{3} & s_{4} & s_{1} \\
-s_{4} & -s_{3} & s_{2} \\
s_{1}^{*} & s_{2}^{*} & s_{3}^{*} \\
-s_{2}^{*} & s_{1}^{*} & -s_{4}^{*} \\
-s_{3}^{*} & s_{4}^{*} & s_{1}^{*} \\
-s_{4}^{*} & -s_{3}^{*} & s_{2}^{*}
\end{array}\right]
$$

With the code matrix $G_{3}$, four complex symbols are taken at a time and transmitted via three transmit antennas in eight time slots. Thus, the symbol rate is $1 / 2$

$$
G_{4}=\left[\begin{array}{cccc}
s_{1} & s_{2} & s_{3} & s_{4}  \tag{1.7}\\
-s_{2} & s_{1} & -s_{4} & s_{3} \\
-s_{3} & s_{4} & s_{1} & -s_{2} \\
-s_{4} & -s_{3} & s_{2} & s_{1} \\
s_{1}^{*} & s_{2}^{*} & s_{3}^{*} & s_{4}^{*} \\
-s_{2}^{*} & s_{1}^{*} & -s_{4}^{*} & s_{3}^{*} \\
-s_{3}^{*} & s_{4}^{*} & s_{1}^{*} & -s_{2}^{*} \\
-s_{4}^{*} & -s_{3}^{*} & s_{2}^{*} & s_{1}^{*}
\end{array}\right]
$$

With the code matrix $G_{4}$, four symbols are taken at a time and transmitted via four transmit antennas in eight time slots, resulting in a transmission rate of $1 / 2$ as well.

### 3.4 OSTBC with a rate of $\mathbf{3 / 4}$

The following code matrices $H_{3}$ and $H_{4}$ are complex generalized designs for OSTBC with rate $3 / 4$ for three and four transmit antennas, respectively [9]

$$
H_{3}=\left[\begin{array}{ccc}
s_{1} & s_{2} & \frac{s_{3}}{\sqrt{2}}  \tag{1.8}\\
-s_{2}^{*} & s_{1}^{*} & \frac{s_{3}}{\sqrt{2}} \\
\frac{s_{3}^{*}}{\sqrt{2}} & \frac{s_{3}^{*}}{\sqrt{2}} & \frac{\left(-s_{1}-s_{1}^{*}+s_{2}^{*}\right)}{2} \\
\frac{s_{3}^{*}}{\sqrt{2}} & \frac{-s_{3}^{*}}{\sqrt{2}} & \frac{\left(s_{2}+s_{2}^{*}+s_{1}-s_{1}^{*}\right)}{2}
\end{array}\right]
$$

As can be observed, (1.8) transmits 3 symbols every 4 time intervals, and therefore has rate $3 / 4$.
$H_{4}=\left[\begin{array}{cccc}s_{1} & s_{2} & \frac{s_{3}}{\sqrt{2}} & \frac{s_{3}}{\sqrt{2}} \\ -s_{2}^{*} & s_{1}^{*} & \frac{s_{3}}{\sqrt{2}} & \frac{-s_{3}}{\sqrt{2}} \\ \frac{s_{3}^{*}}{\sqrt{2}} & \frac{s_{3}^{*}}{\sqrt{2}} & \frac{\left(-s_{1}-s_{1}^{*}+s_{2}-s_{2}^{*}\right)}{2} & \frac{\left(-s_{2}-s_{2}^{*}+s_{1}-s_{1}^{*}\right)}{2} \\ \frac{s_{3}}{\sqrt{2}} & \frac{-s_{3}}{\sqrt{2}} & \frac{\left(s_{2}+s_{2}^{*}+s_{1}-s_{1}^{*}\right)}{2} & -\frac{\left(s_{1}+s_{1}^{*}+s_{2}-s_{2}^{*}\right)}{2}\end{array}\right]$

Obviously, some transmitted signal samples are scaled linear combinations of the original symbols.

### 4.0 Simulations

Simulations were done in MATLAB using the Rayleigh channel model. The Rayleigh model assumes NLOS, and is used for environments with a large number of scatterers. The Rayleigh model has independent identically distributed (i.i.d.) complex, zero mean, unit variance channel elements and is given by [10]. We simulate $G_{2}, G_{3}, G_{4}, H_{3}$, and $H_{4}$, for the case of $N_{t}=1$ up to $N_{r}=4$. We modulate using BPSK, QPSK, 4-QAM, 16-QAM, and 64-QAM gray mapping constellations. For each sample, blocks of 104 symbols are simulated until at least 100 bit errors are obtained, or until 104 blocks are simulated. The simulation is stopped when the SNR reaches 40 dB or after simulating 104 blocks without errors. Consequently, each value obtained for the bit-error rate (BER) of $10^{-7}$ with 100 errors has a $99.9 \%$ confidence level.

### 4.1Simulation Results and Analysis

Keeping all other variables the same, the results obtained for BPSK and QPSK are nearly identical, and we therefore present data for QPSK and omit that of BPSK. Since the data is nearly identical, the reader can safely assume that the performance of BPSK is that of QPSK. We study the performance of each block code discussed earlier for the different cases of constant $N_{t} N_{r}$, rate, and diversity order


Figure 3: Bit error rate versus SNR of OSTBC for $\mathbf{N r}=1$.

For the case $N_{r}=1$. The result is shown in Figure 3 above. As expected, for each different code blocks, the performance degrades as more bits per symbol are transmitted. It can be observed that for a particular modulation and high SNR, the best performance is obtained by $G_{4}$ followed by $H_{4}, G_{3}, H_{3}$, and $G_{2}$. However, for any modulation and low SNR, $G_{3}$ out performs $H_{4}$ even when $H_{4}$ greater gain has. The result is that the best performance at low SNR is obtained by $G_{4}$ followed by $G_{3}, H_{4}, H_{3}$, and $G_{2}$.


Figure 4: Bit error rate versus SNR of OSTBC for $\mathrm{Nr}=4$.

Figure 4 shows the case where Nr is fixed to 4 . As can be observed, for a particular modulation, the best performance is obtained by $G_{4}$ followed by $G_{3}, H_{4}, H_{3}$, and $G_{2}$. This order is the same as for the case of $\mathrm{Nr}=1$ and low $\operatorname{SNR}$ where $G_{3}$ outperforms $H_{4}$ even with $H_{4}$ having higher gain. One possible reason for this behaviour is that the higher rate of $H_{4}$ causes lower channel gain per symbol and therefore higher BER for a particular SNR.

The BER curve for the case of keeping $\mathrm{Nt}=4$ constant while varying Nr from 1 to 4 for different modulations is depicted in Figure 5 below.


Figure 5: Bit error rate versus SNR of OSTBC for $\mathbf{N t}=4$.

It can be observed that for any modulation and block code, the gain of using 3 more antennas is approximately 14 dB . However, between $\mathrm{Nr}=1$ and Nr $=2$ the gain is approximately 8 dB , between $\mathrm{Nr}=2$ and $\mathrm{Nr}=3$ the gain is approximately 4 dB , and between Nr $=3$ and $\mathrm{Nr}=4$ the gain is approximately 2 dB . This result suggests diminishing returns as Nr increases. Another observation is that for any Nr and modulation scheme, $G_{3}$ and $G_{4}$ has a 3 dB gain over $H_{3}$ and $H_{4}$ respectively. An interesting observation is that the performance of $G_{4}$ with $\mathrm{Nr}=2$ is similar to that of $H_{4}$ with $\mathrm{Nr}=3$, while $G_{4}$ with $\mathrm{Nr}=1$ is outperformed by $H_{4}$ with $\mathrm{Nr}=2$, and $G_{4}$ with $\mathrm{Nr}=3$ outperforms $H_{4}$ with $\mathrm{Nr}=4$.


Figure 6: Bit error rate versus SNR of OSTBC at 3 bits/sec/Hz.

In order to fairly compare all the block code schemes, a comparison with equal data rate and a comparison with equal diversity gain is needed. For the case of equal data rate, we simulate $H_{3}$ and $H_{4}$ with constellation 16-QAM, and $G_{3}$ and $G_{4}$ with constellation 64 QAM. Since $H_{3}$ and $H_{4}$ have code rate 3/4, using 16-QAM (4 bits/symbol) leads to $3 \mathrm{bits} / \mathrm{sec} / \mathrm{Hz}$. Similarly, since $G_{3}$ and $G_{4}$ have code rate $1 / 2$, using 64-QAM (6 bits/symbol) leads to $3 \mathrm{bits} / \mathrm{sec} / \mathrm{Hz}$. The result for $\mathrm{Nr}=$ $1, \mathrm{Nr}=2$, and $\mathrm{Nr}=4$ is presented in Figure 7. As expected, having more receive diversity leads to better performance on all cases. In general, there is a 3 dB gain when using the 16-QAM lower order constellations with higher code rate $3 / 4$ over using the 64-QAM higher order constellation with lower code rate $1 / 2$ for the same number of transmit antennas. It is particularly interesting to notice that at high SNR and $\mathrm{Nr}=1,64-\mathrm{QAM} G_{4}$ outperforms 16-QAM $H_{3}$ while at low SNR and $\mathrm{Nr}=1,16-\mathrm{QAM} H_{3}$ outperforms 64QAM $G_{4}$


Figure 7: Bit error rate versus SNR of OSTBC with spatial diversity 12.

Figure 7 displays a BER curve for the case of equal diversity gain. To accomplish this, we simulate $G_{2}$ with $\mathrm{Nr}=6, G_{3}$ and $H_{3}$ with $\mathrm{Nr}=4$, and $G_{4}$ and $H_{4}$ with $\mathrm{Nr}=3$. The diversity gain in each case is therefore 12. From Figure 7 we see that having fewer number of transmit antennas and more number of receive antennas results in better performance. $G_{3}$ and $H_{3}$ with $\mathrm{Nr}=4$ have a 2 dB performance gain over $G_{4}$ and $G_{4}$ with $\mathrm{Nr}=3$ respectively. It is also interesting to see that $G_{2}$ with $\mathrm{Nr}=6$ has similar performance as to $G_{4}$ with $\mathrm{Nr}=3$. This observation suggests that there is an upper limit at which using few Nt transmit antennas and more Nr receive antennas becomes equivalent to using more Nt transmit antennas and fewer Nr receive antennas. This is conceived to be more economical and practical as, for example, it would only require multiple antennas at the base station in comparison to multiple antennas for every mobile in a cellular communications system

### 5.0 Conclusion

This paper has presented an investigation into the performance of MIMO wireless systems in Rayleigh channel using various modulations schemes such BPSK, QPSK, 16-QAM, and 64-QAM, and has focused on techniques for improving the system performance and spectral efficiency. It has investigated techniques that exploit Space time block coding as a
way of increasing capacity. Employing multiple antennas at transmitter as well as both transmitters and receiver sides enables considerable performance enhancement in wireless communications systems.

We have provided a basic overview of MIMO systems and briefly discussed a basic introduction to SpaceTime Coding provided by presenting Alamouti's scheme. We then discussed block codes schemes with different code rates for the cases of 3 and 4 transmit antennas. Simulation results were then presented. It was observed that higher diversity gain does not always imply better performance. This was observed when $G_{3}$ outperformed $H_{4}$ at low SNR for $\mathrm{Nr}=1$ and at any SNR for $\mathrm{Nr}=2$ up to $\mathrm{Nr}=4$. Similarly, it was observed that equal diversity gain does not imply equal performance. This was particularly demonstrated when $G_{3}$ outperformed all others for equal diversity gain. The penalty of having more transmit antennas, which consequently reduces the energy per transmit antenna was observed. Also, we observed diminishing returns for every scheme as the number of received antennas increased. It was particularly interesting to find that although $H_{3}$ and $H_{4}$ have higher rate than $G_{3}$ and $G_{4}$, the performance of $G_{3}$ and $G_{4}$ is greater and could therefore be preferred in some scenarios.

### 6.0 References

[1]. Carmela Cozzo, and Brian L. Hughes, "Space Diversity in Presence of Discrete Multipath Fading Channel" IEEE Transactions On Communications, Vol. 51, No. 10, October 2003
[2]. Hamid Jafarkhani " Space-Time Coding Theory And Practice ", university Of California, Irvine
[3]. G. J. Foschini and M. J. Gans, "On limits of wireless communication in a fading environment when using multiple antennas," Wireless Pers. Communication., vol. 6, no. 3, pp. 311-355, Mar. 1998.
[4] A. S. Mindaudu and A. M. Miyim "BER Performance of MPSK and MQAM in $2 \times 2$ Alamouti MIMO Systems", International Journal of Information Sciences and Techniques (IJIST) Vol.2, No.5, September 2012
[5] J. Winters, "On the capacity of radio communication systems with diversity in a Rayleigh fading environment," IEEE Journal on Selected Areas in Communications, vol. 5, no. 5 , pp. 871-878, 1987.

[^0][7]. A. J. Paulraj and B. C. Ng, "Space-time modems for wireless personal communications," IEEE Pers. Commun. Mag., pp. 36-48, Feb. 1998
[8] G. Tsoulos, MIMO system technology for wirelesscommunications. CRC Press, 2006.
[9] V. Tarokh, H. Jafarkhani, and A. Calderbank, "Space-time block codes from orthogonal designs," IEEE Transactions on Information Theory, vol. 45, no. 5, pp. 1456-1467, 1999.
[10] G. J. Foschini and M. J. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," Wireless Personal Communications, vol. 6, pp. 311-335, 1998


[^0]:    [6]. S. M. Alamouti, "A simple transmitter diversity scheme for wireless communications," IEEE J. Select. Areas Commun., vol. 16, pp. 1451-1458, Oct. 1998.

