Empirical Wavelet Transform & its Comparison with Empirical Mode Decomposition: A review

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***Abstract*— In signal processing, adaptive representation of signal is very important. Empirical Wavelet Transform is a new adaptive signal decomposition technique. This technique is very useful for de-noising, decompression etc. This paper presents a review and comparison of Empirical Wavelet Transform with Empirical Mode Decomposition. Illustration demonstrates the comparison of these methods on one dimensional signal.**

Keywords*— Empirical Wavelet Transform, Empirical Mode Decomposition, Adaptive data analysis

I. INTRODUCTION

In recent years, a growing field of research has been in "adaptive systems" which has resulted in variety of signal processing techniques. These developments have made it clear that significant performance gains can be achieved beyond those achievable using standard adaptive filtering approaches. Signal analysis in adaptive manner finds a variety of applications in the field of signal processing. In general, a signal can be represented as a linear combination of basis functions. In Fourier and wavelet transform these basis functions are predefined, but in adaptive data analysis techniques these functions are derived from the information enclosed in the signal. The adaptive data analysis method that has gained popularity in signal processing since last decade is the algorithm called "Empirical Mode Decomposition" (EMD) proposed by Huang et al. [1]. The purpose of this method is to decompose the signal into its principal "modes" (a mode corresponds to a signal which has a compactly supported Fourier spectrum). It has proven to be quite versatile in a broad range of applications for extracting signals from data generated in noisy nonlinear and non-stationary processes. These include study of wide variety of data including rainfall [2], earthquakes detection [3], Sunspot number variation, heart-rate variability, financial time series, and ocean waves [4], fault diagnosis [5], signal denoising, image processing [6], biomedical signal processing ,speech signal analysis [7], pattern recognition [8]. Experiments by Flandrin [9] show that EMD behaves like an adaptive filter bank. However, the main issue of the EMD approach is its lack of mathematical theory.

A new adaptive data analysis method having a similar goal like EMD is the Empirical Wavelet Transform (EWT), proposed by Gilles [10] which explicitly builds an adaptive wavelet filter bank to decompose a given signal into different modes. It is a new approach to build adaptive wavelets capable of extracting Amplitude modulated-Frequency modulated components of a signal which have a compact support Fourier spectrum. Separating various modes corresponds to segmentation of the Fourier spectrum and to apply some filtering corresponding to each detected support. The EWT performs local maxima detection of the Fourier spectra of the signal, then performs spectrum segmentation based on detected maxima and, finally, constructs a corresponding wavelet filter bank. The segmentation mechanism of the Fourier spectrum is important as this step provides the adaptability with respect to the signal under analysis. In this paper a review of EWT and EMD is presented and their comparison using 1D signals.

II. EMPIRICAL MODE DECOMPOSITION

In 1998, Huang et al. [1] proposed an adaptive data analysis method called Empirical Mode Decomposition (EMD) which decomposes a signal into specific modes. The EMD works in temporal space directly rather than in the corresponding frequency space; it is perceptive, direct, and adaptive, with an a posteriori defined basis derived from the data. The decomposition is based on a simple assumption that, at any given time, the data may have many coexisting simple oscillatory modes of significantly different frequencies, one superimposed on the other.EMD has proven to be the most promising method and widely applied since the last decade [4-6, 11-14]. EMD aims to decompose a signal f(t) as a (finite) sum of N + 1 Intrinsic Mode Functions (IMF) fk (t) such that

$$f(t) = \sum_{k=0}^{N} f_k(t) \quad (1)$$

An IMF is an amplitude modulated-frequency modulated function which can be written in the form

$$f_k(t) = F_k(t)cos(\varphi_k(t)) \quad (2)$$

where $F_k(t), \varphi'_k(t) > 0 \quad \forall\ t$

The main assumption is that amplitude and frequency variations are much slower than phase variations. This definition of IMF was proposed by [11]

The original definition of IMF proposed by [1] that a function is considered to be an IMF if it satisfies two conditions:

i. The number of zero-crossings of local extreme point must be equal or a difference of one at most in the whole data set
ii. The mean value of the envelope of the local maxima and minimum must be zero at any point.

An additional analysis of this definition was done by Yang and Yang [15] and the conclusion was that condition i can be deduced from condition ii and an improved definition was given as an Intrinsic Mode Function (IMF) is a function that satisfies the condition that at any time instant, the mean value of the upper envelope as defined by the local maxima and the lower envelope as defined by the local minima is zero.

The concrete steps of EMD decomposition known as the sifting process are as follows:

i. Locate the local maxima F_{max} and minimum F_{min} of signal f*(t)*.

ii. The upper and lower envelopes are found using F_{max} and F_{min} via cubic spline or some other interpolation method respectively.

iii. The local mean m_1 of original data *f(t)* is obtained by averaging upper and lower envelope at individual points, the difference between original data and local mean is defined as:

$$h_1 = f(t) - m_1 \qquad (3)$$

iv. if h_1 is an IMF, then f1=h1, else Replace h1 with f(t), and repeat step (i-iii) until standard deviation (SD) of two consecutive screening

$$SD = \sum_{t=0}^{T} \frac{|h_{k-1}(t) - h_k(t)|^2}{h_{k-1}^2(t)} \qquad (4)$$

is smaller than standard setting (generally between 0.2 to 0.3), which is considered that h_k is an IMF component.

v. Repeat step i-iv to obtain all modes or residue becomes a monotonic function, the EMD decomposition end.

The stoppage criteria mentioned in the sifting process, Standard deviation, is difficult to implement this criterion for the following reasons: First, how small is SD needs an answer. Second, this criterion is not based on the definition of the IMFs, the squared difference might be small, but there is no guarantee that the function will have the same numbers of zero crossings and extrema. To remedy the shortcomings of SD criteria, Huang et al. [16] proposed the second type of criterion, termed the S stoppage. With this type of stoppage criterion, the sifting process stops only after the numbers of zero crossings and extrema are either equal or at most differ by one and stay the same for S consecutive times. Extensive tests by Huang and Wu [13] suggest that the optimal range for S should be between 4 and 8, but the lower number is favored. Every choice is ad hoc, and a rigorous justification is needed.

The interesting fact about this algorithm is that it is highly adaptable and is able to extract the non-stationary part of the original function. The main drawback of the EMD approach is that it is a pure nonlinear algorithmic procedure and the obtained representation is implementation dependent (e.g it depends on how the envelopes are detected, which interpolation process is used and the chosen stopping criteria). Moreover due to the nonlinear aspects, no theoretical background supports this method. Consequently it is difficult to really understand what the EMD provides. For example, some problems appear when some noise is present in the signal. To deal with this problem, an Ensemble EMD (EEMD) was proposed in [17]. The authors propose to compute several EMD decompositions of the original signal corrupted by different artificial noises. Then the final EEMD is the average of each EMD. This approach seems to stabilize the obtained decomposition but it increases the computational cost.

III. EMPIRICAL WAVELET TRANSFORM

In 2013, Jerome Gilles [10] introduced a new adaptive data analysis method called Empirical Wavelet Transform which explicitly builds an adaptive wavelet filter bank to decompose a given signal into different modes. EWT also aim's like the EMD, to extract AM-FM components from a signal. The EWT works in frequency space unlike EMD which works in temporal space; it is intuitive, direct, and adaptive algorithm supported by a strong mathematical background. EWT proposes a method to build a family of wavelets adapted to the processed signal. If Fourier point of view is considered, this construction is equivalent to building a set of band pass filters. The idea consists of defining a bank of N wavelet filters (one low pass and $N-1$ band- pass filters corresponding to the approximation and details components, respectively) based on well chosen" Fourier supports (meaning by selecting relevant modes in the signal spectrum). One way to reach the adaptability is to consider that the filters' supports depend on where the information in the spectrum of the analyzed signal is located. Indeed, the IMF properties are equivalent to say that the spectrum of an IMF is of compact support and centered around a specific frequency (signal dependent).

In classical wavelet transforms, a constant prescribed ratio is used in the subdivision scheme which limits its adaptability ,while in EWT these are determined empirically Figure 1 highlights the difference in both techniques, in EWT the choice of the supports in the Fourier domain is not prescribed to a dyadic tiling but chosen accordingly to the analyzed signal.

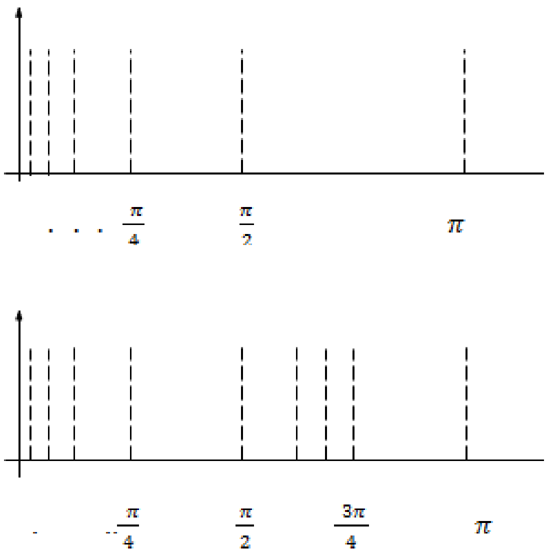

Fig.1: On top: dyadic wavelet tiling of the frequency line. On bottom: a wavelet packet like tiling[10]

Suppose the Fourier support $[0, \pi]$ is divided into N contiguous segments, then N-1 boundaries need to be extracted excluding 0 and π. To find the boundaries, the local maxima in the spectrum are detected and are sorted in decreasing order and boundaries are defined as average between the consecutive maxima's. If we denote ω_n to be the limits between each segments (where ω_0=0 and ω_n=π) and if each segment is denoted as $\wedge_n = [\omega_{n-1}, \omega_n]$, then $\bigcup_{n=0}^{N} \wedge_n = [0, \pi]$.

A transition phase T_n of width $2\tau_n$ (such that $\tau_n = \gamma\omega_n$ where $0 < \gamma < 1$) is defined around the centre of each $\wedge_n$ as shown in figure 2. The empirical wavelets are defined as band pass filters on each $\wedge_n$. For this, the author, Gilles [10] has utilized the idea used in the construction of both Littlewood-Paley and Meyer's wavelets.

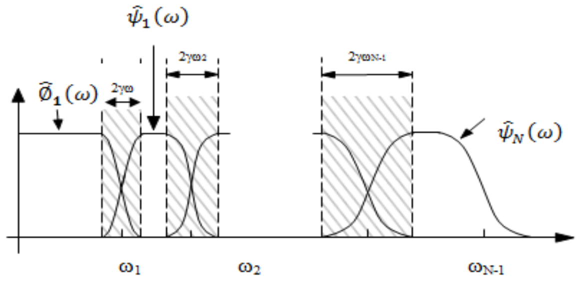

Fig.2: Fourier line decomposition principle and EWT basis construction [18]

Thus, $\forall\, n > 0$ the Fourier transform of empirical scaling function and the empirical wavelets are defined by Equations (5) and (6), respectively.

$$\hat{\emptyset}_n(\omega) = \begin{cases} 1 & \text{if } |\omega| \le (1-\gamma)\omega_n \\ \cos\left[\frac{\pi}{2}\beta\left(\frac{1}{2\gamma\omega_n}(|\omega|-(1-\gamma)\omega_n)\right)\right] & \text{if } (1-\gamma)\omega_n \le |\omega| \le (1+\gamma)\omega_n \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

and

$$\hat{\psi}_n(\omega) = \begin{cases} 1 & if\ (1+\gamma)\omega_n \le |\omega| \le (1-\gamma)\omega_{n+1} \\ \cos\left[\frac{\pi}{2}\beta\left(\frac{1}{2\gamma\omega_{n+1}}(|\omega|-(1-\gamma)\omega_{n+1})\right)\right] & if\ (1-\gamma)\omega_{n+1} \le |\omega| \le (1+\gamma)\omega_{n+1} \\ \sin\left[\frac{\pi}{2}\beta\left(\frac{1}{2\gamma\omega_n}(|\omega|-(1-\gamma)\omega_n)\right)\right] & if\ (1-\gamma)\omega_n \le |\omega| \le (1+\gamma)\omega_n \\ 0 & otherwise \end{cases} \quad (6)$$

where β(x) is an arbitrary C^k ([0,1]) function such that

$$\beta(x) = \begin{cases} 0 & x \le 0 \\ 1 & x \ge 0 \\ x^4(35 - 84x + 70x^2 - 20x^3) & x \in [0,1] \end{cases} \quad (7)$$

and

$$\beta(x) + \beta(1-x) = 1 \quad \forall\, x \in [0,1] \quad (8)$$

Having defined the the empirical wavelet and scaling function, the empirical wavelet transform, $W_f^{\epsilon}(n,t)$ of a signal f(t) is defined in a way similar to the classic wavelet transform. The detail coefficients are given by the inner products with the empirical wavelets.

$$W_f^{\epsilon}(s,t) = \langle f, \psi_n \rangle = \int f(\tau)\overline{\psi_n(\tau - t)}d\tau \quad (9)$$

And the approximation coefficients by the inner product with the scaling function

$$W_f^{\epsilon}(0,t) = \langle f, \emptyset_1 \rangle = \int f(\tau)\overline{\emptyset_{1n}(\tau - t)}d\tau \quad (10)$$

A. Segmentation of Fourier Transform

The segmentation of the Fourier spectrum is important as it is the step which provides the adaptability with respect to the analyzed signal in EWT. The aim is to separate different portions of the spectrum which correspond to modes e.g. centered around a specific frequency and of compact support. In Gilles [10],the author assumed that the number of modes in a signal, N, is given. This implies that a total of N +1 boundary are needed, but 0 and π are always included and consequently N − 1 extra boundaries are to be found out. To find the boundaries the author utilized local maxima method n which all the local maxima in the spectrum are detected and sorted in decreasing order (0 and π are excluded). Let us assume that the algorithm found M maxima. If the number of detected maxima is M, then two cases can arise

i. M > N, then only the first N-1 maxima are kept;
ii. M < N, then all the detected maxima are kept and N is set to appropriate value.

Now, equipped with this set of maxima plus 0 and π, the boundaries ω_n of each segment is defined as the center between two consecutive maxima.

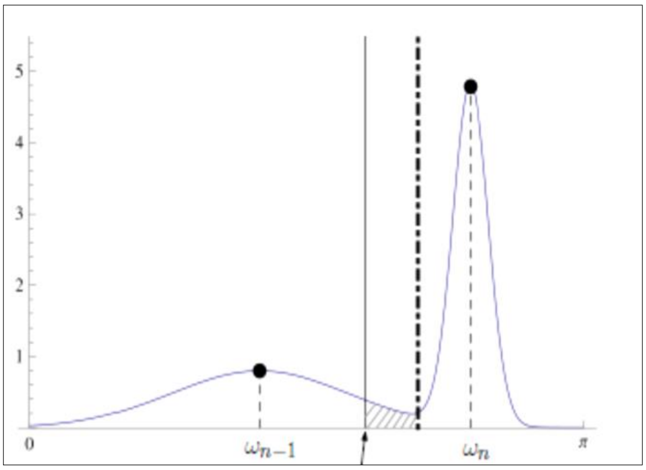

Fig.3: Flat mode issue [18]

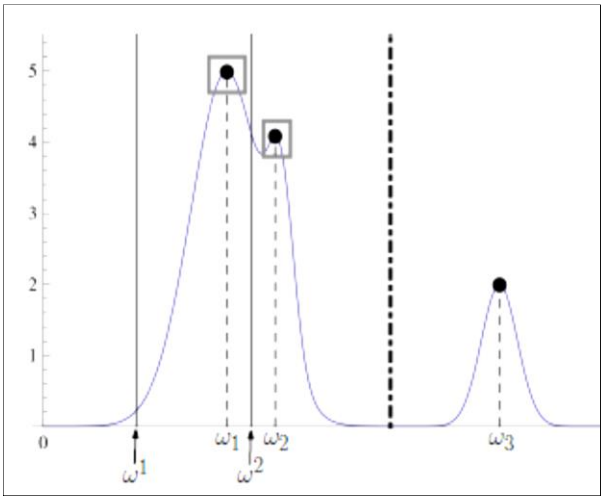

Fig.4: Global Vs local modes [18]

Gilles et al. [18] reviewed the concept of segmentation Fourier spectrum to detect useful supports in EWT. With local maxima method, the problems of flat picked modes and global Vs local modes occur. Flat picked modes occur where two close consecutive modes where one has a wide support while the other has a narrow support. Thus the corresponding boundary obtained from the local maxima detection will fall in the largest support of the first mode. The problem of global Vs local modes occurs when several local maxima belong to the same mode and are larger than other modes. The fact that the segmentation method considers only local information but it should be better to also take into account the spectrum global trend to avoid such issues. To eliminate the problem of flat picked modes (as shown in figure 3) , lowest minima method has been proposed and for global Vs local modes(shown in figure 4), global trend removing approach is followed where in place of spectrum its logarithmic is considered to detect the supports. The methods of segmentation of Fourier spectrum are discussed in [18, 19] in detail.

B. Frame

Concerning the choice of τ_n, several options are possible. The simplest is to choose τ_n proportional to ω_n , i.e. $\tau_n = \gamma\omega_n$ where $0 < \gamma < 1$. The parameter γ must be chosen properly in order to obtain a tight frame. The parameter allows us to ensure that two consecutive transitions areas (dashed regions in Figure 2) do not overlap. A necessary condition on γ is proved by Gilles in order to have the tight frame property is as follows.

$$\gamma = \min_n \left(\frac{\omega_{n+1} - \omega_n}{\omega_{n+1} + \omega_n}\right) \qquad (11)$$

C. Automatic Detection of the number of modes

To estimate the appropriate number of modes, where there is no prior information of the signal, a more robust method is required. EWT can decompose a signal into its components, but a user input of number of modes present in a signal is required. The Gilles et al. [18] in paper has proposed a technique for automatic detection of number of modes, Fine to Coarse histogram segmentation but it is computationally expensive. In cases where no information is there about the signal, it should be interesting to estimate the appropriate number of modes. A parameter less scale space approach is presented in [19] by Jerome Gilles in which a parameter less algorithm to automatically find meaningful modes in an histogram or spectrum. The approach is based on a scale-space representation of the considered histogram which permits to define the notion of "meaningful modes" in a simpler way by defining a threshold on the length of scale space curves. The methods to define these thresholds discussed by author are probabilistic approach (half normal distribution), Otsu's method and K-means clustering (binary). It is shown that finding N (where N itself is unknown) modes is equivalent to perform a binary clustering. This method is simple and runs very fast.

IV. COMPARISON EMD VS EWT

While the EMD automatically estimate the number of modes, we fix a priori the number of modes, N, for the EWT. Also it is observed that, the EMD always overestimates the number of modes and then separates some information which is originally part of the same component. Except for the high frequencies, it is difficult to interpret the EMD outputs compared with the known "true" components constituting the test signals. Although EWT, can detect the presence of modes in the spectrum and provides different components which are close to the original ones.

For a chirp signal while EWT can detect the modes, EMD can't. EMD fails in case of chirp signals as there is no variation in amplitude so the output is chirp signal only. The results for EWT are as shown in figure:

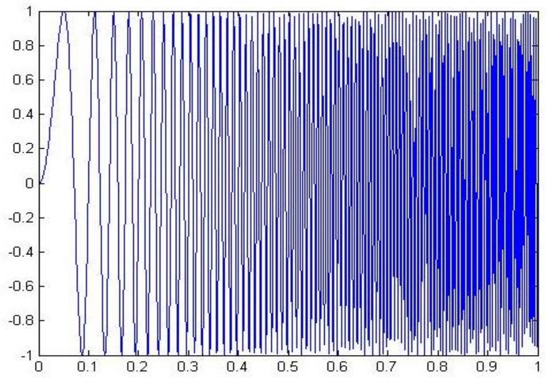

Fig.5: Chirp input

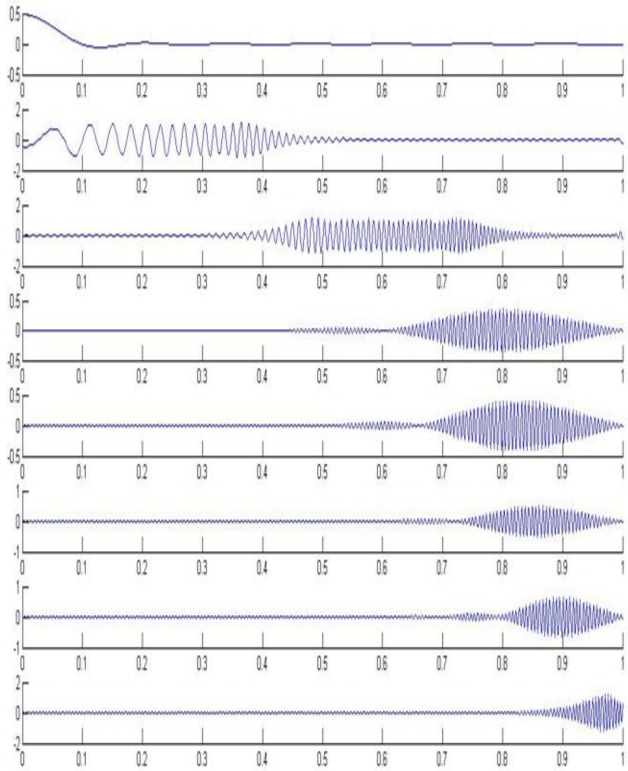

Fig.6: EWT Decomposition of chirp input

As discussed in [20] ,experiments on the real ECG signal seem to give the advantage to the EWT because the EMD provides too many modes. Typically, the EMD modes six to nine are really difficult to interpret as such behavior is clearly not visible in the signal itself. A contrary, the EWT focuses on the oscillating patterns. Also EWT is computationally faster than EMD

However, in some cases, the EWT can fail (like any other wavelet approaches) compared to the EMD. For instance, if the input signal is composed of two chirps which overlap in both the time and frequency domains then the EWT will not be able to separate them while the EMD is supposed to be able to extract the most oscillating part first and the lowest one next. Such cases should probably be addressed by building adaptive frames with enough redundancy.

V. CONCLUSION

In this paper a review of EWT & EMD is presented. The key idea is to build a wavelet filter bank based on Fourier supports detected from the information contained in the processed signal spectrum. A comparison with the Empirical Mode Decomposition (EMD) showed that the EWT gives a more consistent decomposition while, generally, the EMD exhibits too much modes, which are sometime really difficult to interpret. Another advantage of the EWT compared to the EMD is that we can adapt the classic wavelet formalism to understand it. The EWT has one obvious advantage over EMD, that it is a computationally fast algorithm as compared to EMD. EMD & EWT both are promising adaptive time frequency representation techniques, where, EMD has already been explored in vast engineering and related applications and proves to be potential, it still lacks mathematical background. EWT is an emerging technique with immense potential to be explored, it still needs to be fully adaptive. The automatic detection of number of modes is an area where it still holdup

REFERENCES

[1] Huang, N.E., et al. ‘The empirical mode decomposition and the Hilbert spectrum for nonlinear and non-stationary time series analysis’, in Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences. 1998.

[2] Molla, M.K.I., et al., “Empirical mode decomposition analysis of climate changes with special reference to rainfall data’, Discrete Dynamics in Nature and Society, 2006.

[3] Loh, C.-H., T.-C. Wu, and N.E. Huang, “Application of the empirical mode decomposition-Hilbert spectrum method to identify near-fault ground-motion characteristics and structural responses”, Bulletin of the seismological Society of America, vol. 91, no. 5, pp: 1339-1357, 2001.

[4] Huang, N.E., Z. Shen, and S.R. Long, “A new view of nonlinear water waves: The Hilbert Spectrum 1”, Annual review of fluid mechanics, vol. 31, no. 1, pp: 417-457, 1999.

[5] Liu, B., S. Riemenschneider, and Y. Xu, “Gearbox fault diagnosis using empirical mode decomposition and Hilbert spectrum”, Mechanical Systems and Signal Processing, vol. 20, no. 3, pp: 718-734, 2006.

[6] Bi, N., et al., “Robust image watermarking based on multiband wavelets and empirical mode decomposition”, IEEE Transactions on Image Processing, vol. 16, no. 8, pp: 1956-1966, 2007.

[7] Yang, Z., D. Qi, and L. Yang. “Signal period analysis based on Hilbert-Huang transform and its application to texture analysis”, in IEEE First Symposium on Multi-Agent Security and Survivability, 2004.

[8] Yang, Z., D. Qi, and L. Yang, “Chinese font recognition based on EMD”, Pattern Recognition Letters, vol. 27, no. 14, pp: 1692-1701, 2006.

[9] Flandrin, P., Rilling, G., & Goncalves, P., “Empirical mode decomposition as a filter bank”, IEEE Signal Processing Letters, vol. 11, no. 2, pp: 112-114, 2004.

[10] Gilles, J., “Empirical wavelet transform”, IEEE Transactions on Signal Processing, vol. 61, no. 16, pp: 3999-4010, 2013.

[11] Daubechies, I., Lu, J., & Wu, H. T., “Synchrosqueezed wavelet transforms: An empirical mode decomposition-like tool”, Journal of Applied and Computation harmonic analysis, vol. 30, No. 2, pp: 243-261, 2011.

[12] Feldman, M., “Analytical basics of the EMD: Two harmonics decomposition”, Mechanical Systems and Signal Processing, vol. 23, no. 7, pp: 2059-2071, 2009.

[13] Huang, N.E. and Z. Wu, “A review on Hilbert-Huang transform: Method and its applications to geophysical studies”, Reviews of Geophysics, vol. 46, no. 2, 2008.

[14] Meignen, S. and V. Perrier, “A new formulation for empirical mode decomposition based on constrained optimization”, Signal Processing Letters, IEEE, vol. 14, no. 12, pp: 932-935, 2007.

[15] Yang, Z. and L. Yang, “A new definition of the intrinsic mode function. in Proceedings of World Academy of Science, Engineering and Technology. 2009.

[16] Huang, N.E., et al. “A confidence limit for the empirical mode decomposition and Hilbert spectral analysis”, in Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences. 2003.

[17] Wu, Z. and N.E. Huang, “Ensemble empirical mode decomposition: a noise-assisted data analysis method”, Advances in adaptive data analysis, vol. 1, no. 1, pp: 1-41, 2009.

[18] Gilles, J., G. Tran, and S. Osher, “2D empirical transforms. wavelets, ridgelets, and curvelets revisited”, SIAM Journal on Imaging Sciences, vol. 7, no. 1, pp: 157-186, 2014.

[19] Gilles, J. and K. Heal, “A parameterless scale-space approach to find meaningful modes in histograms—Application to image and spectrum segmentation”, International Journal of Wavelets, Multiresolution and Information Processing, vol. 12, no. 6, 2014.

[20] Singh, O. and R.K. Sunkaria, “Powerline interference reduction in ECG signals using empirical wavelet transform and adaptive filtering”, Journal of medical engineering & technology, vol. 39, no. 1, pp: 60-68, 2015.