

# Empirical Equation Correlate California Bearing Ratio (CBR) with Dry Density for Granular Soil

A. Kareem Khaleel Al Saffar<sup>1</sup>, Shamil Abdul Majeed Behaya<sup>2</sup>  
Hassanean S. H. Jassim<sup>3</sup>, Harith K K Ajam<sup>4</sup>  
<sup>1,2,3,4</sup>Department of Civil Engineering,  
Collage of Engineering, University of Babylon, Iraq.

**Abstract:-** In this study a 12 test results of CBR test granule mixed soil was taken from soil laboratories, and a statistical method was used to find out a relationship between the CBR and dry density of the soil, desiring to obtain a straight line relationship and a formula combining the previous two variables. A reasonable straight-line relationship is obtained, and CBR could be founded directly by using the formula predicted in this study. In other hand, the laboratory time and efforts are minimized.

**Keywords:** Granular soil; California Bearing Ratio (CBR); dry density; least squares methods.

## 1. INTRODUCTION

There were many soil tests, and these engineering tests were taken place for many engineering purposes and there were one or more manners to test one of the engineering features of soil (P.N.Khanna, 1979). These manners have founded because of the improvement of the used equipment and scientific progressing. (Baraja, 2007).

C.B.R. test one of these tests which can find out the pressure act on the soil related to the California soil capacity. C.B.R. test took a large field in civil engineering designs and especially pavement designs.

It was an empirical test found and improved by Porter, then it was used by (U.S Army Corps of engineers) in 1942 and it is considered the first test in world which gives right and considerable results, besides the pavement designs which depends C.B.R. test was practical and suitable, so it was depended by American Association of standards for Test and Materials ASTM (Leelavthamma, 2005). According to ASTM 1883 Results of the soil test completed after at least four days (ASTM, 2004).

In this study, the data collected from the laboratory tests for maximum dry density and California Bearing Ratio (CBR) for a number of samples were taken to obtain a relationship between CBR value and the maximum dry density of granule soil. Statistical method by linear regression was used to correlate these variables in an empirical equation may be used to find out the value of CBR from the value of dry density directly to minimize the time and efforts.

## 2. SAMPLING

Twelve samples of two granular mixed soils (class C and class B) were chosen cooperated with Babylon central Laboratory and Engineering collage laboratory having a laboratory test results for their maximum dry density and corresponding California Bearing Ratio (CBR). Non real samples were excluded aiming to have realistic results. Table (1) shows sieve analysis for the soil samples and Table (2) shows the maximum dry density for the twelve samples and the corresponding CBR testing results.

Table (1): Granular soil (sub base) mixed Class B & Class C.

Sieve No.	%passing	
	class B	class C
2	100	-
1	75-95	100
3/8	40-75	50-85
No. 4	30-60	35-65
No. 8	21-47	26-56
No. 50	14-28	14-28
No. 200	5-15	5-15

Table (2): Maximum dry density Vs. CBR test.

$\gamma_d$ kN/m <sup>3</sup>	20.4	20.6	20.9	21.3	21.5	21.8	22.0	22.2	22.3	22.5	22.6	22.7
CBR %	25.0	28.5	30.2	33.0	35.0	36.2	37.5	39.4	40.5	41.8	43.3	45.0

## 3. LINEAR REGRESSION EQUATION

The two variables here are the maximum dry density (independent) and the CBR test result (dependent) and can be associated by plotting them on (x-y) axis. Plotting line result define the relationship type whether linear (straight line) or nonlinear (curved line) if the points are scattering.

Figure (1) shows the type of the relationship linking the two observed variables is tendency to a straight line with no or negligible scattering, so it is a linear relationship.

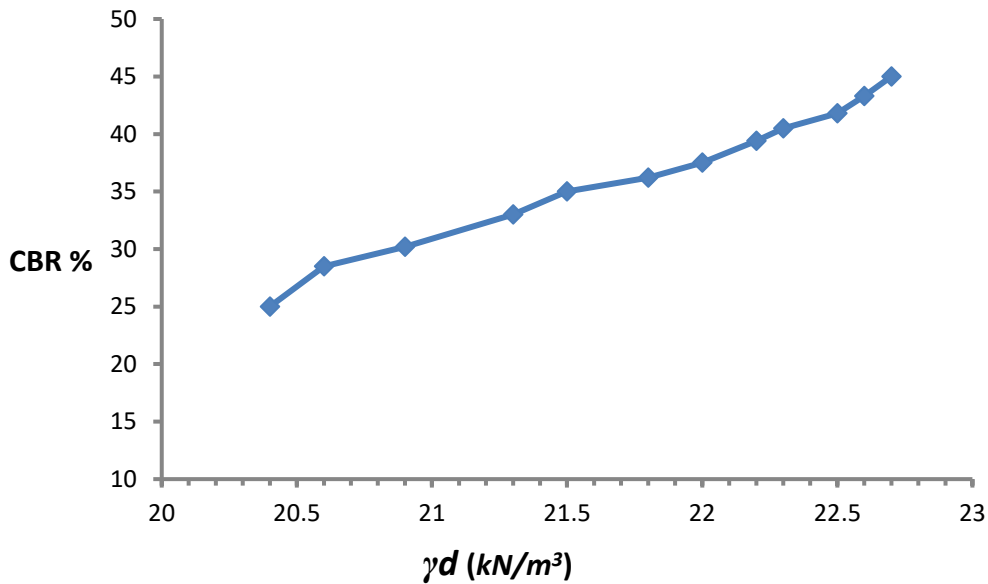


Figure (1): Observed data ( $\gamma_d$  Vs. CBR).

The statistical method can be used to fitting straight line equation objectively by least squares methods. Since that we can obtained number of lines from the data values, least squares method principle states that the series of observations can be fitting with a best line that the sum of the squares of the departures (the difference between the line and the observed data values) are at minimum. The departures of the dependent variable are usually used and the relationship has the equation:

$$Y = A + b X$$

This equation may be obtained by solving for  $A$  and  $b$ , the two normal equations:

$$\sum Y = n A + b \sum X$$

$$\sum XY = A \sum X + b \sum X^2$$

where  $n$  is number of observed values of variables  $X$  and  $Y$ .

Table (3) showing the calculations for least square method to find the best equation linking the two variable observation data, where max dry density as independent variable ( $X$ ) and CBR test result as dependent variable ( $Y$ ).

Table (3): Calculations for least square equation.

$\gamma_d$	CBR	$X^2$	$XY$
20.4	25.0	416.16	510.00
20.6	28.5	424.36	587.10
20.9	30.2	436.81	631.18
21.3	33.0	453.69	702.90
21.5	35.0	462.25	752.50
21.8	36.2	475.24	789.16
22.0	37.5	484.00	825.00
22.2	39.4	492.84	874.68
22.3	40.5	497.29	903.15
22.5	41.8	506.25	940.50
22.6	43.3	510.76	978.58
22.7	45.0	515.29	1021.50
<b><math>\sum X = 260.8</math></b>	<b><math>\sum Y = 435.4</math></b>	<b><math>\sum X^2 = 5674.94</math></b>	<b><math>\sum XY = 9516.25</math></b>

$$Y = A + b X$$

$$A = \frac{\sum Y \sum X^2 - \sum X \sum XY}{n \sum X^2 - (\sum X)^2}$$

$$= \frac{435.4 \times 5674.94 - 260.8 \times 9516.25}{12 \times 5674.94 - (260.8)^2} = -132.734$$

$$b = \frac{n \sum XY - \sum X \sum Y}{n \sum X^2 - (\sum X)^2}$$

$$b = \frac{12 \times 9516.25 - 260.8 \times 435.4}{12 \times 5674.94 - (260.8)^2} = 7.777$$

$$Y = -132.734 + 7.777X$$

$$CBR = 7.777 \gamma_d - 132.734 \dots\dots\dots (1)$$

Straight line equation (1) represent the relation between the maximum dry density and the California Bearing Ratio (CBR) of granular soil and to measuring the degree of association of two linearly dependent variables of it we can used the most commonly used statistical parameter which is the correlation coefficient (*r*).

$$r = \frac{\sum(\Delta x \cdot \Delta y)}{\sqrt{\sum(\Delta x)^2 \cdot \sum(\Delta y)^2}} = \frac{\sum xy - n \bar{x} \bar{y}}{(n - 1)\sigma_x \sigma_y}$$

Where  $\Delta x = x - \bar{x}$ ,  $\Delta y = y - \bar{y}$ .

$\sigma_x$ ,  $\sigma_y$  = standard deviations of *x* and *y*, respectively.

$$\sigma_y = \sqrt{\frac{\sum(y - \bar{Y})^2}{n - 1}} = \sqrt{\frac{\sum(\Delta y)^2}{n - 1}}$$

Also the standard error of estimate of *y* with respect to *x* which represents the scatter about the regression line can be obtained from the equation:

$$S_{y.x} = \sigma_y \sqrt{1 - r^2}$$

Calculations to estimates the correlation coefficient and standard error of estimate are listed at Table (4) below with the error percentage for the predicted (BCR) values.

Table (4): Calculations for correlation coefficient.

$\gamma_d (x)$	Predicted BCR ( $y'$ )	$(y'-y)^2/y$	$\Delta y$	$\Delta x$	$(\Delta y)^2$	$(\Delta x)^2$	$x y$
20.4	25.917	0.042	-10.3693	-1.45086	107.523	2.104983	525.6475
20.6	27.472	0.030	-8.81393	-1.00081	77.68536	1.001623	569.558
20.9	29.805	0.003	-6.48083	-0.78222	42.00116	0.611865	624.4435
21.3	32.916	0.000	-3.37003	-0.42218	11.3571	0.178237	701.468
21.5	34.472	0.005	-1.81463	-0.16501	3.292882	0.027229	743.4798
21.8	36.805	0.014	0.51847	-0.01071	0.268811	0.000115	799.479
22.0	38.360	0.025	2.07387	0.156448	4.300937	0.024476	839.6781
22.2	39.915	0.010	3.62927	0.400758	13.1716	0.160607	883.4766
22.3	40.693	0.002	4.40697	0.5422	19.42138	0.293981	906.4457
22.5	42.249	0.007	5.96237	0.70936	35.54986	0.503192	948.1548
22.6	43.026	0.001	6.74007	0.902236	45.42854	0.81403	973.9069
22.7	43.804	0.027	7.51777	1.12083	56.51687	1.256259	1001.086
$\Sigma =$ 260.8	$\Sigma =$ 435.434	$\Sigma$ Error 0.165			$\Sigma =$ 416.5175	$\Sigma =$ 6.976597	$\Sigma =$ 9516.823

$$\sigma_x = \sqrt{\frac{6.976597}{12 - 1}} = 0.796389$$

$$\sigma_y = \sqrt{\frac{416.5175}{12 - 1}} = 6.153473$$

$$r = \frac{9516.823 - 12 \times 21.73297 \times 36.28613}{(12 - 1) \times 0.796389 \times 6.153473}$$

$r = 0.9935$  Correlation coefficient.

$$S_{y.x} = 6.153473 \sqrt{1 - 0.993516^2} = 0.6\% \text{ Standard error of estimate.}$$

Figure (2) shows the plot for the maximum dry density against the calculated California Bearing Ratio (CBR) from the produced straight line equation.

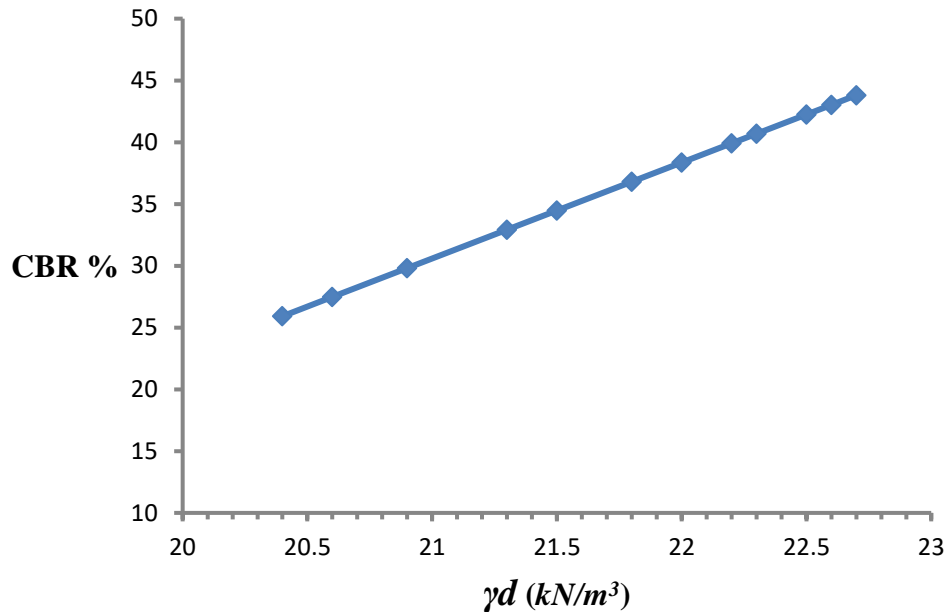


Figure (2): Calculated California Bearing Ratio (CBR) data, ( $\gamma_d$  Vs. CBR).

#### 4. DISCUSSION

1. Referring to the Figure (1) the relationship between CBR and maximum dry density is trends to a straight line that means linear relationship.
2. From Figure (2) CBR Can be found directly from the graph or from the line equation obtained.
3. It is possible to use the equation obtained to determine the amount of California Bearing Ratio (CBR) of the soil by knowing its maximum dry density ( $\gamma_d$ ) without delay when performing a laboratory test.
4. It can also be used in the other direction to find out the appropriate maximum dry density ( $\gamma_d$ ) when determining the required California Bearing Ratio (CBR) by delivering the soil to the appropriate dry density for it.
5. The time (4days) and the effort of the CBR test according to ASTM 1883 are minimized in this study, and CBR value can be obtained directly.
6. This study was taken place and it is might to be reasonable depending upon what the CBR test according to ASTM1883 is an empirical method to obtain CBR.

#### 5. CONCLUSIONS

This study obtained a linear regression equation between California Bearing Ratio (CBR) and maximum dry density ( $\gamma_d$ ) for the granular soil having acceptable summation error percentage of (0.165) with correlation coefficient of (0.9935) and standard error of estimate of (0.9935%) as a following formula:

$$CBR = 7.777 \gamma_d - 132.734$$

Where (CBR) is California Bearing Ratio and ( $\gamma_d$ ) is maximum dry density.

The time and the efforts of finding out California Bearing Ratio (CBR) for the granular soil are minimized.

#### 6. RECOMMENDATIONS

In order to obtain an equation that can be used more widely so that it can be applied to other types of soil (i.e. non granular) cover a wider range, it is recommended to increase the number of soil samples and there types.

It is recommended to include soil particles size as another variable in more spacious concluded equation.

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