Empirical Determination and Analysis of Hourly Solar Heat Gain Factors in Wa, Ghana

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Abstract

The main purpose of the air conditioning system is to provide suitable internal thermal conditions of comfort to occupants or manufacturing applications. It is therefore necessary to perform cooling load calculations. By proper design and orientation of the building, the overall energy cost (initial and operational) can be reduced. Solar heat gain, through fenestration, particularly on vertical surfaces, is therefore a significant factor in determining the cooling load of commercial buildings; thus enabling proper sizing of air-conditioning equipment. SHGFs are site-dependent. In this work, SHGFs have been obtained from empirical correlations on the horizontal surface and for surfaces facing the four main cardinal points: North, South, East and West (N, S, E and W) for Wa (latitude 10.01°N, longitude 2.5°W) using design days of each month. Results show that SHGFs are dependent on the sun's position in the sky and the direction of scattering of its radiation. It was also found out that horizontal and north-facing surfaces receive maximum annual mean solar heat gains; values of about 418 W/m^2. We also found out that substantial amount of solar heat gains could be avoided if more windows are placed in the southern and western directions. The SHGFs obtained will help architects and building engineers in Wa to take into account solar geometry for calculating air-conditioning cooling loads.

Keywords: Solar Heat Gain Factors, horizontal surface, vertical surface, sun's radiation.

1. Introduction

Ghana has been experiencing a steady growth in electric power demand as a result of increasing economic activity and improvement in the standard of living such as the use of air conditioners (A/C) in both commercial and residential buildings. As a result, demand for electricity today far outstrips supply resulting in unstable and poor quality supply and load shedding. To meet this increasing demand

for electricity and improve supply quality requires two approaches, namely, generate more power and/or economize the use of available power by avoiding wastage [1]. From surveys carried out by the Energy Foundation in Ghana, among others, air conditioners represent over 60% of the power usage in air-conditioned buildings [2]. Thus, energy-efficient air-conditioner usage in buildings will contribute immensely to reduce energy misuse in buildings and thereby reduce the strain on the national electricity grid. Solar heat gains come about as a result of direct solar radiation being transmitted through glazing to the building interior and indirect admittance of solar radiation to the building interior due to absorptance of solar radiation by the glazing. In the design of space to be air-conditioned, most architects and building engineers in Wa do not take into account solar geometry for the location as an energy consideration for the building's orientation. This usually accounts for high energy requirements of the air conditioning equipment.It is necessary to have SHGF data that represent the prevailing weather conditions [3]. This is because SHGFs can be used to estimate solar heat gains for calculating air-conditioning cooling loads. The American Society of Heating, Refrigeration and Air-Conditioning Engineers Handbook(ASHRAE) [4] provides solar heat gain factors on cloudless days for daylight hours of 21st day of each month for a given window orientation at a particular latitude and time of year [3]. However, the effects of the sun's position and the prevailing climatic conditions are the two essential variables for SHGF analysis. Hence, SHGFs are site-dependent.

In this paper, we determined monthly hourly SHGFs empirically using the solar radiation data measured at Wa Polytechnic weather station on horizontal surface based on the formulae by [3], using monthly design days of Wa given by [5]. The monthly hourly SHGFs were determined for the horizontal surface and the four principal vertical orientations (N,S,E and W), based on available weather data from 7 am to 4 pm. We then explained the trend in the hourly values based on the sun's position, the hour of the day and the atmospheric conditions.

2. Empirical Relations

2.1 Horizontal Surface

Hourly SHGF (W/m^2) for a horizontal surface, $SHGF_h$, is given by [3]:

$$SHGF_h = (I - I_d)(\tau_b + N_i\alpha_b) + I_d(0.799 + 0.0544N_i) \quad (1)$$

where

I is hourly horizontal radiation (W/m^2), I_d is hourly diffuse radiation (W/m^2), τ_b is transmittance of the reference glazing for direct radiation, α_b is absorptance of the reference glazing for direct radiation and N_i is the inward flowing fraction of the absorbed radiation. The weather station at Wa Polytechnic measures the hourly horizontal radiation. The diffuse component of I, I_d is determined empirically from a number of relations, outlined below. [6] determined

$$\frac{I_d}{I} = \begin{cases} 1 - 0.09 \text{ for } k_T \le 0.22 \\ 0.9511 - 0.1604k_T + 4.388k_T^2 - 16.638k_T^3 \\ \quad + 12.336k_T^4 \text{ for } 0.22 \prec k_T \prec 0.80 \\ 0.165 \text{ for } k_T \succ 0.80 \end{cases} \quad (2)$$

The clearness index, k_T is given by

$$k_T = I / I_0 \quad (3)$$

I_0 is the hourly extraterrestrial radiation on a horizontal surface given by [7]

$$I_0 = \frac{12 \times 3600}{\pi} \times G_{sc}\left(1 + 0.033\cos\frac{360n}{365}\right) \times (\cos\phi\cos\delta(\sin\omega_2 - \sin\omega_1) + \pi/180(\omega_2 - \omega_1)\sin\phi\sin\delta) \quad (4)$$

where $\omega_2 > \omega_1$. G_{sc} is the solar constant (1367 W/m^2), ϕ is the latitude of the location n is the day of the year, $1 \le n \le 365$, δ is the declination angle given by

$$\delta = 23.45\sin\left(360\frac{284 + n}{365}\right) \quad (5)$$

The hour angle, ω is given by

$$\omega = (15^0 h^{-1})(t_{solar} - 12h) \quad (6)$$

The solar time, t_{solar}, is also given by

$$t_{solar} = t_{std} + 4(L_{st} - L_{loc}) + E \quad (7)$$

where t_{std} is standard time, L_{st} is the standard meridian (longitude) for the local time zone; $L_{st} = 0$ for GMT zones like Ghana. L_{loc} is the longitude of the location (in degree west); $L_{loc} = +2.5°W$ and E is the equation of time, in minutes.

$$E = 9.87\sin 2B - 7.53\cos B - 1.5\sin B \quad (8)$$

and

$$B = (n-1)\frac{360}{365} \quad (9)$$

For an hourly period (e.g. 7 am - 8 am), the hour angle is found at 7 am and at 8 am. ω_2 is the larger of the two values determined. Now [3] determined the following relations;

$$\tau_b = -0.00885 + 2.71235\cos\theta - 0.62062\cos^2\theta - 7.07329\cos^3\theta + 9.75995\cos^4\theta - 3.89922\cos^5\theta \quad (10)$$

$$\alpha_b = 0.001154 + 0.77674\cos\theta - 3.94657\cos^2\theta + 8.57881\cos^3\theta - 8.38135\cos^4\theta + 3.01188\cos^5\theta \quad (11)$$

The angle of incidence, θ is determined from [7]

$$\cos\theta = \sin\delta\sin\phi\cos\beta - \sin\delta\cos\phi\sin\beta\cos\gamma + \cos\delta\cos\phi\cos\beta\cos\omega + \cos\delta\sin\phi\sin\beta\cos\gamma\cos\omega + \cos\delta\sin\beta\sin\gamma\sin\omega \quad (12)$$

where

β=slope; $\beta = 0$ for horizontal surface and $\beta = 90$ for vertical surface. γ=surface azimuth angle; zero due south, east negative and west positive. For a horizontal surface, $\gamma = 0$.

N_i is given by [3]

$$N_i = \frac{8.29}{(8.29 + (16.21V_s^{0.452}))} \quad (13)$$

V_s is the near surface wind speed obtained from the following relations;

$$V_s = 0.68V_w - 0.5 \quad (14)$$

for wind incidence angles of 20-160°, and

$$V_s = 0.157V_w - 0.027 \quad (15)$$

for other wind incidence angles. V_w is the measured wind speed in m/s. Wind data were obtained from the Wa Meteorological station from 2000-2012 since wind data for the Wa Polytechnic weather station could not be traced.

2.2 Vertical Surface

Hourly SHGF (W/m^2) for a vertical surface, $SHGF_v$, is given by [3]:

$$SHGF_v = H_v(\tau_b + N_i\alpha_b) + I_g(0.799 + 0.0544N_i) \quad (16)$$

H_v is hourly direct radiation on the plane of the vertical glazing, W/m^2.

$$H_v = \left(\frac{I - I_d}{\sin\alpha}\right)\cos\theta \quad (17)$$

α is the solar altitude angle given by

$$\alpha = 90 - \theta_z \quad (18)$$

The zenith angle, θ_z is determined from

$$\cos\theta_z = \cos\delta\cos\phi\cos\omega + \sin\delta\sin\phi \quad (19)$$

$$I_g = I_v - H_v \quad (20)$$

I_v is the measured hourly global radiation on the plane of the vertical glazing in W/m^2. Here again, I_v is determined from empirical relations since the weather station at Wa Polytechnic only measures I.

$$I_v = I_{vb} + I_{vd} \quad (21)$$

The diffuse component of I on the vertical surface, $I_{vd}$is given by [7]

$$I_{vd} = I_d\left(\frac{1+\cos\beta}{2}\right) + (I_b + I_d)\times \rho\left(\frac{1-\cos\beta}{2}\right) \quad (22)$$

$I_b$is the hourly horizontal beam radiation (W/m^2) and is found from

$$I_b = I - I_d \quad (23)$$

The beam component of I on the vertical surface, $I_{vb}$is also given by

$$I_{vb} = R_b \times I_b \quad (24)$$

where

$$R_b = \frac{\cos(\phi-\beta)\cos\delta\cos\omega + \sin(\phi-\beta)\sin\delta}{\cos\theta_z} \quad (25)$$

ρ, the ground albedo is taken as 0.22 [8].

In determining the $SHGF_v$ for the four main cardinal points, the following parameters were redefined for the particular orientation;γ, β, $\cos\theta$, τ_b, α_b, I_{vd}, I_{vb}, R_b and I_v.

In order to relate the SHGFs obtained with the sun's apparent movement in the sky, the following angles were computed; the solar altitude angle, α_s and the solar azimuth angle, γ_s (as in Fig. 1).

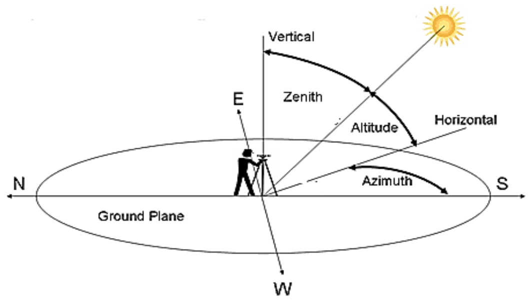

Figure 1: Sun's position in the sky [9]

$$\alpha_s = 90 - \theta_z \quad (26)$$

$$\cos\gamma_s = \frac{\sin\delta\cos\phi - \cos\omega\cos\delta\sin\phi}{\cos\alpha_s} \quad (27)$$

Also, the sun does not rise exactly due east and set exactly due south. Instead, the sun may rise further north of east or further south of east, depending on the location's latitude and longitude. The sun-path across the sky may be seen from Fig. 2.

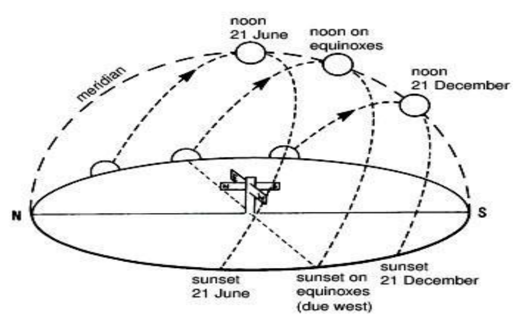

Figure 2. Sun's Rise [10]

A script file in MATLAB was written to aid in the calculations.

3. RESULTS AND DISCUSSION

SHGFs obtained from the empirical procedure outlined from section 2 are in Tables 1-4 for design days given by [5]

Table 1.SHGF (W/m^2): January-March

Month	Solar Time	SHGFs, W/m^2				
		H	N	S	E	W
Jan-15	7	105	252	109	124	0
	8	253	441	203	219	103
	9	399	612	284	295	593
	10	506	705	337	330	657
	11	556	713	357	332	462
	12	540	639	342	356	332
	13	463	497	294	397	283
	14	338	312	215	314	216
	15	192	141	120	133	121
	16	65	45	41	35	41
Feb-12	7	41	29	26	27	10
	8	172	130	107	110	82
	9	339	316	204	214	339
	10	516	565	294	300	622
	11	673	820	361	340	570
	12	676	794	362	386	366
	13	642	763	348	595	333
	14	516	592	294	687	305
	15	291	248	177	246	183
	16	160	125	100	55	104
Mar-03	7	50	126	36	71	0
	8	192	265	89	136	0
	9	401	458	166	216	630
	10	602	634	239	268	927
	11	716	729	279	268	578
	12	772	780	300	360	320
	13	766	784	299	741	296
	14	680	727	271	1143	317
	15	530	624	223	648	300
	16	338	507	166	0	267

Table 2.SHGF (W/m^2): April-June

Month	Solar Time	SHGFs, W/m^2				
		H	N	S	E	W
Apr-29	7	9	5	6	5	1
	8	87	51	52	51	50
	9	191	113	114	114	119
	10	471	214	234	217	328
	11	295	174	176	174	177
	12	371	209	213	214	209
	13	605	193	239	418	195
	14	610	107	201	667	142
	15	448	76	178	170	121
	16	267	39	140	0	81
May-25	7	147	75	95	79	6
	8	330	103	194	122	120
	9	514	99	249	127	478
	10	685	70	271	89	510
	11	804	38	274	40	217
	12	835	21	273	150	31
	13	782	10	271	503	26
	14	669	0	268	734	37
	15	540	0	273	240	0
	16	356	0	294	0	0
June-24	7	9	5	6	5	3
	8	87	50	53	51	52
	9	191	113	116	113	119
	10	472	184	257	189	317
	11	296	172	177	172	176
	12	371	203	218	207	203
	13	607	136	293	350	141
	14	610	35	298	617	67
	15	448	35	268	350	71
	16	270	31	206	0	58

Table 3.SHGF (W/m^2): July-September

Month	Solar Time	SHGFs, W/m^2				
		H	N	S	E	W
July-01	7	24	7	23	7	3
	8	130	39	82	39	42
	9	254	73	135	74	241
	10	450	111	208	115	347
	11	454	104	199	104	181
	12	537	113	228	143	116
	13	628	91	252	356	93
	14	599	45	249	646	61
	15	478	39	250	418	56
	16	342	27	262	1	40
Aug-13	7	9	5	6	5	3
	8	88	51	53	52	54
	9	192	114	116	114	122
	10	474	198	240	205	350
	11	297	174	177	174	178
	12	373	208	216	212	209
	13	608	177	259	375	181
	14	612	93	237	697	126
	15	451	81	213	498	118
	16	272	61	165	6	89
Sep-30	7	9	9	3	6	9
	8	90	76	28	45	263
	9	203	161	63	84	489
	10	498	387	155	171	644
	11	318	247	99	100	171
	12	398	309	124	215	125
	13	630	490	196	817	217
	14	623	494	193	1503	258
	15	453	383	143	1321	227
	16	248	268	89	258	183

Table 4.SHGF (W/m^2): October-December

Month	Solar Time	SHGFs, W/m^2				
		H	N	S	E	W
Oct-09	7	69	114	35	62	221
	8	217	258	88	127	767
	9	371	397	143	177	949
	10	587	603	221	233	778
	11	758	768	283	278	434
	12	770	781	288	476	279
	13	736	758	277	1054	297
	14	655	707	253	1759	318
	15	486	589	200	1739	293
	16	257	449	136	780	246
Nov-16	7	65	302	87	108	14
	8	211	574	177	200	601
	9	365	794	254	265	910
	10	580	1130	371	348	849
	11	751	1397	464	423	569
	12	763	1425	473	638	424
	13	727	1435	470	1183	449
	14	644	1438	458	1698	485
	15	469	1350	412	1277	472
	16	233	1267	361	0	454
Dec-09	7	62	359	106	126	0
	8	208	641	203	223	493
	9	362	855	285	292	898
	10	576	1189	410	381	902
	11	747	1451	509	455	629
	12	759	1475	517	647	462
	13	723	1495	515	1148	480
	14	641	1520	506	1598	519
	15	467	1454	460	1093	507
	16	233	1388	409	0	487

The solar azimuth angles obtained indicate the sun rises north of east for the months of January, February, March, September, October, November and December. The sun rises south of east for the months of April to August. To avoid repetitive explanations, the trend in SHGF values for the months of January and June representing different rise of the sun have been explained in the next sections.

3.1 January Trend

At 7 am solar time, the solar azimuth and solar altitude angles are 115° and 13° respectively. This means the sun rises 25° north of east and is at a very low elevation. A surface facing North at this hour then experiences a higher solar heat gain than the horizontal surface because of the high diffuse component of the hourly radiation. There is also a high value of the heat gain for the east-facing surface compared to the horizontal surface (Figs. 3 and 4).

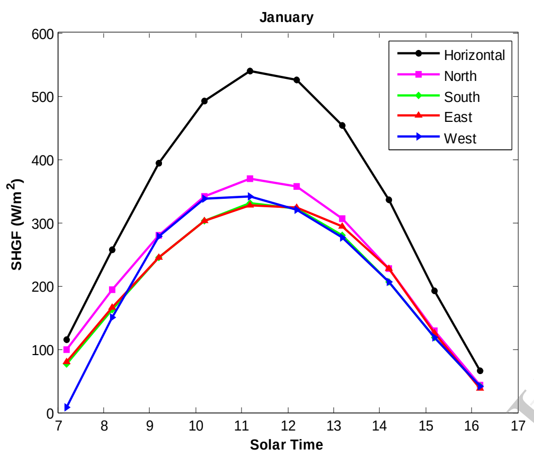

Figure 3. SHGFs for horizontal and four vertical surfaces in January

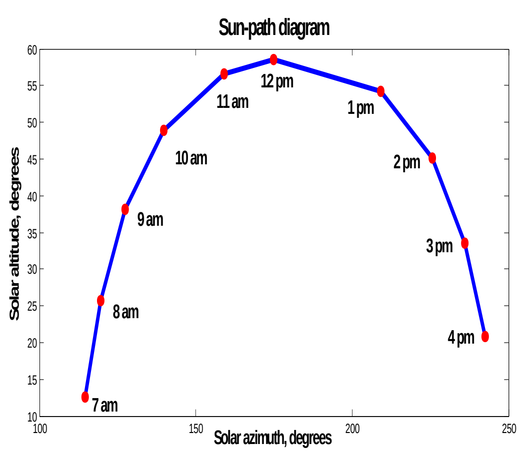

Figure 4. Sun-path diagram: 15th January

A south-facing surface obviously will have a low solar heat gain because of the rise of the sun at this hour. The value of the west-facing surface at this hour was found to be -333 W/m^2. This means, the sun's radiation had not reached the western-side at this hour; hence the surface looses heat. For purposes of solar heat gain, this value is accordingly set to zero. All other values of zero for other month represent the case of heat loss. At 8 am, the sun is a bit high in the atmosphere. The horizontal surface experiences an increase in value because there is an increase in the k_T value meaning the sky is becoming clearer. The west-facing surface now experiences solar radiation and gains heat. There is a gradual increase in values for the other surfaces. Higher solar heat gains at solar hours of 9 am and 10 am for the west-facing surface means that most of the sun's radiation is scattered westwards during the month of January at these hours. The atmosphere is clearer at 11 am because of high k_T value. The horizontal surface therefore has a higher heat gain compared to surfaces facing south, east and west. The east-facing surface has the lowest value at this hour because the sun's radiation is focused westwards (relative to east direction). The sun is at its highest position at 12 pm with an azimuth angle of 175° and an altitude angle of 59°. There is more scattering towards the eastern direction hence a high value of solar heat gain for the east-facing surface compared to the west-facing surface. The sun's radiation is almost perpendicular to the horizontal surface. SHGF is still high because more of the sun's radiation is focused onto the north and horizontal facing-surfaces. At 1 pm, the east-facing surface shows an increase in value than the corresponding value at 12 pm, indicating that the sun's radiation is scattered eastwards at this hour. From the hourly analysis, it can be concluded that most of the sun's radiation is focused onto the north-facing surface.

3.2 June Trend

At solar time of 7 am, the solar azimuth and solar altitude angles are 69° and 22° respectively. This means the sun rises south of east and is relatively high in the sky. All surfaces receive radiation (Figs. 5 and 6). The diffuse component of solar radiation at this hour is very high.

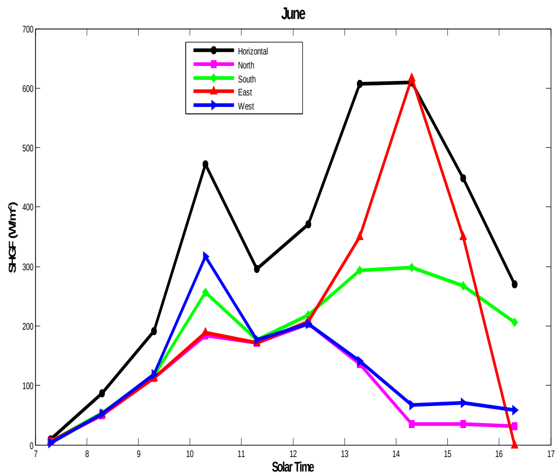

Figure 5.SHGFs for horizontal and four vertical surfaces in June

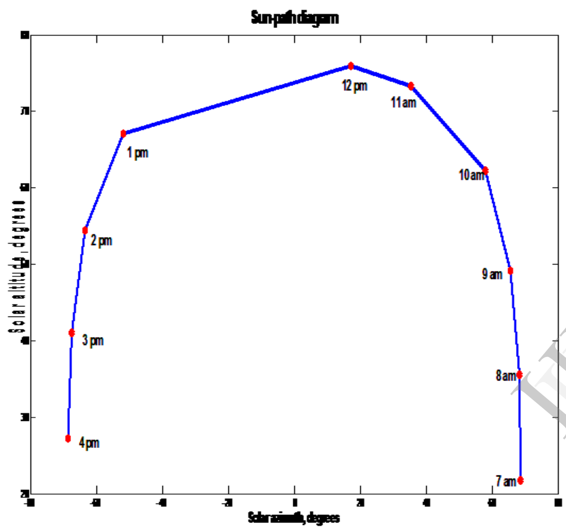

Figure 6. Sun-path diagram: 24th June

This is not surprising because the month of June is a rainy month in Ghana. The heat gains are therefore very small, with the horizontal surfaces receiving the highest heat gain. Values of SHGF indicate that there is more scattering of solar radiation onto horizontal surfaces for all hours for the month of June. Between 8 am and 9 am solar time, values increase because the sun is still rising (α_s=36° to 49°), with the southern and horizontal surfaces receiving higher heat gains. The high value of heat gain for the west-facing surface indicates scattering of the sun's radiation westwards. This trend continues to 10 am solar time and 11 am solar time. There is now scattering of the sun's radiation towards the north and continues up to 1 pm solar time. The higher heat gain values of east-facing surface and horizontal surface at 2 pm solar time indicate the fact that even though the sun is in the south-west position, most of its radiation is scattered eastwards and horizontally. At 4 pm there is no solar heat gain for the east-facing surface. The sun's intensity is low and its scattering does not reach the east-facing surface.

In order to obtain data that would be helpful for building engineers to assess the energy performance of buildings during the initial design stage, average hourly SHGFs were computed and are in Table 5.

Table 5. Average SHGFs for Wa

Month/Period	Average SHGF (W/m²)				
	H	N	S	E	W
January	342	436	230	254	281
February	403	438	227	296	291
March	505	563	207	385	364
April	335	118	155	203	142
May	566	42	246	208	143
June	336	96	189	205	121
July	390	65	189	190	118
August	338	116	168	234	143
September	347	282	109	452	259
October	491	542	192	669	458
November	481	1111	353	614	523
December	478	1183	392	596	538
Annual	418	416	222	359	282
6-month Rainy Season	385	120	176	249	154
6-month Dry Season	450	712	267	469	409

The maximum average SHGF for the dry season months occurs on the north-facing surface. Values range from 436 W/m^2 in January to 1183 W/m^2 in December. The maximum mean SHGF for the rainy-season months occurs on the horizontal surface. Values range from 335 W/m^2 in April to 566 W/m^2 in May. The maximum horizontal average SHGF occurs in May. This indicates more clear days in May and more scattering of solar radiation onto the horizontal surface. For the vertical surfaces, the average SHGF ranges from 42 W/m^2 in May for the north-facing surface to 1183 W/m^2 in December still for the north-facing surface. The low value of SHGF in May indicates less solar intensity onto the north-facing surface. The high SHGF in December indicates diffuse sky conditions (e.g. due to the Harmattan) with more scattering of solar radiation onto the north-facing surface. Values of the SHGFs for the east-facing surface and west-facing surface are not symmetrical due to the same reasons outlined above. The 6-month dry season has higher SHGFs for all orientations compared to the 6-month rainy season. This means that due to the sun-path in Wa, all surfaces (i.e. H,N,S,E,W) receive more direct solar radiation than the diffuse component of solar radiation. Generally, electricity consumption due to air-conditioning equipment in the dry-season will be higher than that for the rainy-season.

The maximum annual average SHGF occurs on the horizontal and north-facing surfaces followed by the east-facing surface, the west-facing surface and finally the south-facing surface. Values range from 222 W/m^2 for the south-facing surface to 418 W/m^2 for the horizontal surface. These have design implications. The very high values of SHGF for the horizontal surface imply that roof design with skylight is not desirable in Wa. More windows should be placed in the southern and western directions because of the relatively low values of SHGFs. For example, a building having 6 windows (1m^2 area) distributed as follows; north=2, south=1, east=2,west=1, will have a total SHGF of 2172 W/m^2. If the distribution is done as follows; south=3,west=2,east=1, the total SHGF will be 1589 W/m^2. An extra 583 W/m^2 is avoided and the air-conditioner's consumption due to solar heat gain by fenestration is reduced. Calculations of this nature are required so as to give the arrangement with the least SHGF, hence the need to have SHGFs data.

4. Conclusion

SHGFs data is valuable for air-conditioning equipment sizing. In this paper, SHGFs based on empirical correlations have been determined for the horizontal surface and for surfaces facing the four main cardinal points: north, south, east and west. It was found out that SHGFs are dependent on the sun's position in the sky and the direction of scattering of its radiation. For the dry season (October-March), it was found out that most of the sun's radiation is focused onto the north-facing surface. Most of the sun's radiation, however, is focused onto the horizontal surface in the rainy season (April-September). Annual average of SHGFs also showed that the horizontal and north-facing surfaces receive almost equal amount of solar heat gain. For this reason, roof design with skylights is not recommended for residential and commercial buildings in Wa; except for commercial building application where lighting levels cannot be compromised. This is not surprising because most buildings in Wa do not have skylights. Buildings should be designed with more windows facing the southern and western surfaces based on the SHGF obtained. For already existing buildings with more windows facing the north, shading designs (e.g. using overhangs, blinds, frosted louver blades, etc.) should be provided. Further works, however, will be carried out to find the optimum angle and dimensions of overhangs for correct lighting levels in rooms.

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