Abstract—Light Emitting Diode is the future of lighting. LED is a semiconductor device and its nonlinear. For driving LED we need a driver circuit. Hence LEDs are nonlinear devices and boost driver circuit is unstable, it’s difficult to achieve the desired output. So here we have taken these as a problem statement and tried to linearize and stabilize the system using a proper controller. This paper demonstrates how to stabilize the system. In this paper a controller is designed by considering LEDs are nonlinear (not the linearized form of LED is taken). The steps how to stabilize the nonlinear system is shown in detail, and it is also showing the obtained results.

Keywords – Efficient Lighting, LED, Driver Circuit, Lighting Control, Linearization, Non-linear system.

I. INTRODUCTION

Nowa day’s LEDs are replacing every lamp because of long life, good CRI, low energy consumption and free of pollution. Hence it is a semiconductor device, frequently switching does not affect the lamp life. The proposed system contains set of high power LEDs and a 12V dc supply.

The light output from LED depends on the average current flows through the LED. Because of manufacturing fault the forward voltage drop of LEDs varies, so the current flowing through each LED will differ. And due to different currents flowing through LEDs the lumen output from LEDs will vary. This leads to non-uniform distribution of light, which is not acceptable to users. Therefore the system requires a Driver circuit which will make sure that the current flowing through all LEDs are same. The design of driver circuit depends on mostly the LED forward voltage drop, available input voltage. After designing driver circuit the current through LED doesn't meet the required current, because of non-linearity. The V-I characteristic of LED is non-linear. Hence the system becomes unstable. Therefore the system needs a controller to overcome this problem.

In this paper the controller construction is discussed briefly for a non-linear dynamic system. As LED is a non-linear device, the control design process in the case of linear systems is different from non-linear systems. Feedback Linearization Method is used to construct a state-feedback control strategy. The basic idea of the feedback linearization approach is to use a control consisting of two components; one component cancels out the systems non-linearities and the other controls the resulting linear system.

II. DRIVER CIRCUIT DESIGN

The proposed system contains 36 identical LEDs, they are arranged in 6 parallel strings and each string contains 6 LEDs connected in series. The forward voltage drop of each LED is varies from 2.7V to 3.2V. The typical forward voltage drop is 2.85V. The maximum output voltage will be $3.2 \times 6 = 19.2$ V. The input voltage is varies from 10V to 14V. The minimum input voltage is 10V. When the output voltage is greater than input voltage, it requires Boost driver circuit design. In Boost circuit design the output voltage is minimum 50% greater than input voltage. The required current is 700 mA through each LED, there are six strings therefore the load current is $700 \times 6 = 4.2$ mA. The circuit shown below is Boost driver circuit.

![Boost Driver Circuit](Fig. 1. Boost Driver Circuit)

III. DRIVER CIRCUIT DESIGN

Input Voltage ($V_S$) = 10 V
Output Voltage ($V_O$) =19.2 V
Required output current ($I_{LOAD}$) = 4.2 A
Switching frequency ($F_S$) = 25 KHz

For calculating L and C value, it is required to calculate Duty Cycle (D) and Inductor current ($I_L$).

\[
\text{DutyCycle}(D) = 1 - \frac{V_{o(min)}}{V_{o(max)}} = 1 - \frac{10}{19.2} = 0.4791 \quad (1)
\]

\[
I_L = \frac{I_{Load}}{(1-D)} = \frac{4.2}{0.4791} = 8.6029A \quad (2)
\]
Inductance $L = \frac{V_{s(min)} \times D \times T}{\Delta I_L} = 1.188 \times 10^{-4} \text{H}$ (3)

Capacitance $C = \frac{I_{load} \times D \times T}{\Delta I_F \times R_d} = 1.3973 \times 10^{-5} \text{F}$ (4)

IV. CONTROLLER DESIGN

Because of non-linear devices are present in the circuit, which leads to unacceptable output current and voltage. The lumen output of LED is directly proportional to the current flow through the LED. So it is necessary to design controller which make sure that the stable desired current will flow through LEDs.

1. LINEARIZATION OF DIFFERENTIAL EQUATIONS OF NON-LINEAR SYSTEM

The replacement of a nonlinear system model by its linear approximation is called linearization. The need for linearization is that the dynamical behavior of many nonlinear system models can be well approximated within some range of variables by linear system models. The derivation of dynamic system model by nonlinear differential equation;

When Switch is closed; According to Kirchhoff’s Voltage and Current laws following equations can be obtained.

$$V_S = V_L + I_L R_L + I_L R_S$$ (5)

Where $R_L$, $R_S$ and $C_L$ are the internal resistances of Inductor, Switch and Capacitor respectively and whose values are 0.1, 0.001 and 0.01 ohm respectively.

$$V_L = V_S - I_L(R_L + R_S)$$ (6)

$$\frac{di_L}{dt} = \frac{1}{L}(V_S - I_L(R_L + R_S))$$ (7)

$$I_C = -I_O$$ (8)

$$C \frac{dV_C}{dt} = -I_S + \frac{V_C}{V_T}$$ (9)

$$\frac{dV_C}{dt} = -\frac{I_S}{C}e^{\frac{V_C}{V_T}}$$ (10)

The output current $I_o = I_S \times e^{\frac{V_C}{V_T}}$ (11)

The output voltage $V_o = V_C$ (12)

The above set of equations is continuously differentiable. Using Taylor’s series expansion method the state space matrix can be written in the following way

$$\dot{X} = \left[ \begin{array}{cc} -\frac{R_L + R_S}{L} & \frac{1}{C} \frac{V_C}{V_T} \\ 0 & -\frac{1}{C} \frac{V_C}{V_T} \end{array} \right] \left[ \begin{array}{c} X_1 \\ X_2 \end{array} \right] + \left[ \begin{array}{c} \frac{1}{L} \\ 0 \end{array} \right] U$$

$$Y = \left[ \begin{array}{cc} \frac{1}{L} & \frac{1}{C} \frac{V_C}{V_T} \end{array} \right] \left[ \begin{array}{c} X_1 \\ X_2 \end{array} \right] + \left[ \begin{array}{c} 0 \\ 0 \end{array} \right] U$$

At equilibrium point (0, 0) the state space matrix will become,

$$\dot{X} = \left[ \begin{array}{cc} -\frac{R_L + R_S}{L} \times 0 & \frac{1}{C} \frac{V_C}{V_T} \times 0 \\ 0 & -\frac{1}{C} \frac{V_C}{V_T} \times 0 \end{array} \right] \left[ \begin{array}{c} X_1 \\ X_2 \end{array} \right] + \left[ \begin{array}{c} \frac{1}{L} \times 0 \\ 0 \end{array} \right] U$$

Where,

$$A_{ON} = \left[ \begin{array}{cc} -\frac{R_L + R_S}{L} \times 0 & \frac{1}{C} \frac{V_C}{V_T} \times 0 \\ 0 & -\frac{1}{C} \frac{V_C}{V_T} \times 0 \end{array} \right], B_{ON} = \left[ \begin{array}{c} \frac{1}{L} \times 0 \\ 0 \end{array} \right]$$

$$C_{ON} = \left[ \begin{array}{c} 0 \\ 1 \end{array} \right], D_{ON} = \left[ \begin{array}{c} 0 \\ 0 \end{array} \right]$$

When switch is opened; According to Kirchhoff’s laws the following equation can be written

$$V_S = V_L + V_C + I_L R_L + I_C R_C$$ (13)

$$V_L = V_S - V_C - I_L R_L - (I_L - I_O) R_C$$ (14)

$$\frac{di_l}{dt} = \frac{1}{L}(V_S - V_C - I_L(R_L + R_C) + R_C I_S e^{\frac{V_C}{V_T}})$$ (15)

$$I_L = I_C + I_O, I_C = I_L - I_O$$ (16)

$$\frac{dV_C}{dt} = \frac{1}{C}(I_L - I_S e^{\frac{V_C}{V_T}})$$ (17)

At equilibrium point (0, 0) the state space matrix

$$\dot{X} = \left[ \begin{array}{cc} -\frac{R_L + R_S}{L} \times \frac{1}{L} & \frac{1}{L} \times \frac{L_R + R_C}{V_T} \\ -\frac{1}{L} \times \frac{L_R + R_C}{V_T} & -\frac{1}{L} \times \frac{L_R + R_C}{V_T} \end{array} \right] \left[ \begin{array}{c} X_1 \\ X_2 \end{array} \right] + \left[ \begin{array}{c} \frac{1}{L} \times 0 \\ 0 \end{array} \right] U$$

$$Y = \left[ \begin{array}{cc} \frac{1}{L} \times \frac{1}{L} & \frac{1}{L} \times \frac{L_R + R_C}{V_T} \\ 0 & 0 \end{array} \right] \left[ \begin{array}{c} X_1 \\ X_2 \end{array} \right] + \left[ \begin{array}{c} 0 \\ 0 \end{array} \right] U$$

Where,

$$A_{OFF} = \left[ \begin{array}{cc} -\frac{R_L + R_S}{L} \times \frac{1}{L} & \frac{1}{L} \times \frac{L_R + R_C}{V_T} \\ -\frac{1}{L} \times \frac{L_R + R_C}{V_T} & -\frac{1}{L} \times \frac{L_R + R_C}{V_T} \end{array} \right], B_{OFF} = \left[ \begin{array}{c} \frac{1}{L} \times 0 \\ 0 \end{array} \right]$$

Fig. 2. Equivalent Boost Converter circuit for the switch closed

Fig. 3. Equivalent circuit for switch opened
By substituting all the values the state space matrix of the system can be obtained as shown in below.

\[ A = \begin{bmatrix}
-889.630 & -4047.392 \\
37.277 \times 10^3 & -55.0484 \times 10^4
\end{bmatrix}, B = \begin{bmatrix}
8417.508 \\
0
\end{bmatrix}, \\
C = \begin{bmatrix}
0 & 7.6923 \\
0 & 1
\end{bmatrix} \text{ and } D = \begin{bmatrix}
0 & 0
\end{bmatrix}
\]

2. CONTROLLER FORM:

The objective of this is to constructing state-feedback stabilizing controller for dynamic system model. To construct such a controller, it is advantageous to work with an equivalent system model rather than with the original nonlinear one. Thus the nonlinear system model reduced to linear system model.

The Controllability matrix \( Q_C = [B : AB] \)

\[ Q_C = \begin{bmatrix}
0.0001 \times 10^8 & -0.0749 \times 10^8 \\
0 & 3.1378 \times 10^8
\end{bmatrix}
\]

\( T_1 \) the last row of Inverse of the Controllability matrix

\[ Q_C^{-1} = \begin{bmatrix}
0.1188 \times 10^{-3} & 0.0028 \times 10^{-3} \\
0 & 0.3187 \times 10^{-8}
\end{bmatrix}
\]

\[ T_1 = \begin{bmatrix}
0 & 0.3187 \times 10^8
\end{bmatrix} \begin{bmatrix}
X_1 \\
X_2
\end{bmatrix}
\]

\( T_1 = (0.3187 \times 10^{-8})X_2 \) The goal is now construct a state variable transformation \( Z = T(X) \) for which there is a inverse \( X = T^{-1}(Z) \).

\[ T(X) = \begin{bmatrix}
T_1 \\
L_fT_1
\end{bmatrix}
\]

\[ L_fT_1 = \frac{\partial T_1}{\partial T_1}f_1 + \frac{\partial T_1}{\partial x_2}f_2 = (0.3187 \times 10^{-8}).f_2 = (0.3187 \times 10^{-8}).(37277.1706X_1 - 550484.6742X_2) = (1.188 \times 10^{-4})X_1 - (1.7543 \times 10^{-3})X_2 
\]

\[ T(X) = \begin{bmatrix}
(0.3187 \times 10^{-8})X_2 \\
(1.188 \times 10^{-4})X_1 - (1.7543 \times 10^{-3})X_2
\end{bmatrix}
\]

Let \( (0.3187 \times 10^{-8})X_2 = Z_1 \) and \( (1.188 \times 10^{-4})X_1 - (1.7543 \times 10^{-3})X_2 = Z_2 \)

Then, \( X_2 = 313774709.8Z_1 \)

\[ (1.188 \times 10^{-4})X_1 - (1.7543 \times 10^{-3})X_2 = Z_2 
\]

\[ X_1 = 8417.508Z_2 + 4633459372Z_1 
\]

\[ \begin{bmatrix}
X_1 \\
X_2
\end{bmatrix} = \begin{bmatrix}
(8417.508)Z_2 + (4633459372)Z_1 \\
313774709.8Z_1
\end{bmatrix}
\]

Differentiate \( Z(t) \) with respect to time gives

\[ \dot{Z} = \frac{\partial T}{\partial x}F(x) + \frac{\partial T}{\partial x}G(x)\dot{u} = T^{-1}(Z)
\]

\[ \dot{Z}_1 = (0).f_1 + (0.3187 \times 10^{-8}).f_2 = (0.3187 \times 10^{-8}).(37277.17X_1 - 550484.6742X_2) = Z_2 
\]

\[ \dot{Z}_2 = (1.188 \times 10^{-4}).f_1 - (1.7543 \times 10^{-3}).f_2 = (0.3187 \times 10^{-8}).(-889.6304X_1 - 4047.397X_2) - (1.7543 \times 10^{-3}).(37277.17X_1 - 550484.6742X_2) = 965.715X_2 - 65.5521X_1 = (3.0765 \times 1011)Z_1 - (551785.3262)Z_2
\]

\[ \frac{\partial T}{\partial x}G(x) = (1.188 \times 10^{-4}) \times (8417.508)u = u
\]

\[ \dot{Z} = \begin{bmatrix}
Z_2 \\
(3.0765 \times 1011)Z_1 - (551785.3262)Z_2
\end{bmatrix} + \begin{bmatrix}
0 \\
u
\end{bmatrix}
\]

Let \( v = ((3.0765 \times 1011)Z_1 - (551785.3262)Z_2) + u \)

3. LINEAR STATE-FEEDBACK CONTROL:

The linear state-feedback control law, for a system modeled by \( \ddot{X} = AX + Bu \), is the feedback of a linear combination of all the state variables and has the form \( u = -KX \), where \( K \) is a constant matrix.

The closed loop system then is \( \ddot{X} = (A - BK)X \)

The poles of the closed loop system are the roots of the characteristic equation

\[ |SI - (A - BK)| = 0 \]

The roots of the closed-loop characteristic equation \( |SI - (A - BK)| = 0 \) are in desirable locations in the complex plane.

The Transfer Function of linearized system is

\[ G(S) = \frac{2.414 \times 10^9}{S^2 + (5.514 \times 10^5)S + (6.406 \times 10^8)} \]

![Fig. 4. Closed loop System](attachment:image.png)

The characteristics equation of closed loop is \( 1 + KG(S) = 0 \)

\[ S^2 + (5.51410^5)S + 6.406 \times 10^8 + K(2.414 \times 10^9) = 0 \]
Routh’s stability criterion can be used to find the range of $K$ at which the system remain stable. For the value of $K$ greater than -0.2653, the system is stable. By tuning the $K$ value in simulation the desired output can obtained. Here the $K$ value is 0.7351 at which the output current meets the desired current output i.e; 4.2 A. Let the desired Eigen values be $\lambda_1, \lambda_2$ then

$$(S - \lambda_1)(S - \lambda_2) = 1 + KG(S)$$

$$S^2 + S(\lambda_1 + \lambda_2) + \lambda_1\lambda_2 = S^2 + S(5.514 \times 105) + (6.406 \times 108) + K(2.414 \times 10^9)$$

Substitute $K = 0.7351$;

$$S^2 + S(\lambda_1 + \lambda_2) + \lambda_1\lambda_2 = S^2 + S(5.514 \times 105) + 2415131400$$

By comparing co-efficients $\lambda_1, \lambda_2$ values can be found

$$\lambda_1 = -4415.355$$

$$\lambda_2 = -546984.645$$

$$|SI - (A - BK)|;$$

$$(A - BK) = \begin{bmatrix} -889.630 & -4047.392 \\ 37277.17 & -550484.6742 \\ 8417.508K_1 & 8417.508K_2 \\ 0 & 0 \end{bmatrix}$$

$$|SI - (A - BK)| = \begin{bmatrix} -889.63 - 8417.508K_1 & -4047.392 - 8417.508K_2 \\ 37277.17 & -550484.6742 \end{bmatrix}$$

$$= \begin{bmatrix} 551374.3046 + 8417.508K_1 & 4047.392 + 8417.508K_2 \\ -37277.17 & S + 550484.6742 \end{bmatrix}$$

$$= (S + 889.63 + 8417.508K_1)(S + 550484.6742) - (4047.392 + 8417.508K_2)(-37277.17)$$

Compare the above polynomial with the closed loop system characteristic equation

$$551374.3046 + 8417.508K_1 = 5.514 \times 10^5$$

$$K_1 = 3.0526 \times 10^{-3}K_2 = 5.61023$$

V. RESULT:

The Simulation Model of nonlinear System:

$$\frac{di_L}{dt} = \frac{-(R_l + R_c)D - (R_l + R_c)(1 - D)}{L}i_L$$

$$+ \frac{i_sR_c}{CV}e^{\frac{V_c}{V}} - \frac{1}{L}(1 - D)V_c + \frac{1}{L}U$$

$$\frac{dV_c}{dt} = \frac{1}{L}(1 - D)i_L + \frac{I_s}{CV}e^{\frac{V_c}{V}}V_c$$

Substitute all the values, then the equations are

$$\frac{di_L}{dt} = -889.63i_L - 1515.6639V_C + 8417.508U$$

$$\frac{dV_C}{dt} = 37277.17i_L - 550780.6117V_C$$

The output of the system can be observed clearly as unstable.
is shown below.

\[ V = (3.0765 \times 10^{11})Z_1 - (551785.3262)Z_2 + U \]

Where \[Z_1 = (0.3187 \times 10^{-8})X_2\],

\[ Z_2 = (1.188 \times 10^{-4})X_1(1.7543 \times 10^{-3})X_2 \]

\[ v = K_1.i_L - K_2.V_C \]

\[ U = (65.5552)i_L - 1954.08773V_C \]

Here it can be clearly observed that the system became stable even though there are non-linear elements in system using State Feedback Linearization Control Method.

VI. CONCLUSION:

To reduce energy consumption and to improve efficiency, other conventional lamps can be replaced by LEDs. Number of LEDs are used in a fixture to provide high light output. If the LED voltage is greater than the input available voltage then a Boost Driver circuit is required to drive current through LEDs. But the boost converter is an unstable converter, so that eventually the system becomes unstable and nonlinear. The unstable nonlinear system can be stabilized using Linearizing State-Feedback Controller. In this paper the unstable current through LED is stabilized using Linearizing State-Feedback Control technique. Simulation result and mathematical results indicates the same. Further this can be taken forward for practical implementation in lighting industry.

REFERENCES