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Eigen Frequency Vibration Analysis for Thin Plate

Kashmira Ajay Puranik[1] ,
Civil Engineering Department
DIEMS
Aurangabad, India

Surbhi R. Tharewal[2], Monica S. Mhetre[3],
Civil Engineering Department
DIEMS
Aurangabad, India

***Abstract*— Experimental Eigen frequency values for Square and Rectangular flat Plate were measured using Comsole analysis and various mode shapes were recorded experimentally with considerations of various parameters like acceleration, velocity, maximum and minimum surface area, Reaction force and time duration. It was found that these mode shapes agree totally with the theoretical considerations and assumptions. It suggest that plate and membrane should have very similar vibration behaviour and by adding several waves on boundary approximate eigen frequency equations are derived.**

Keywords*— Rectangular Plate,Square Plate Eigen Frequency, Modes,Vibrations.

I. INTRODUCTION

The vibration of beams and plates is important in many applications pertaining to Mechanical, Civil and Aerospace Engineering. Beams and plates used in real practice may have appreciable thickness where the transverse shear and the rotary inertia are not negligible as assumed in the classical theories. The usual first step in performing dynamic analysis is determining natural frequencies and mode shapes of structures with damping neglected. These results characterize the basic dynamic behavior of the structure and are an indication to how the structure will respond to dynamic loading. The natural frequencies of the structure are frequencies at which the structure naturally tends to vibrate.

Many times, we use the terms vibration and oscillation without knowing the difference between them. The term oscillation refers strictly to the repeating motion of a point mass or that of a rigid body while the term vibration refers to the repeating motion or deformations of an elastic structure. Thus any oscillation is a term used only in cases like the motion of a pendulum or that of a ship as a rigid body moving on the wavy seas while vibration is a term used for phenomenon exhibited by structures such as rotating fans or motors etc. Vibration involves deformation by definition while an oscillating structure does not deform. In Structural and Mechanical engineering context, oscillation and vibration are differentiated based on the "restoring force" present in the system. For both oscillation and vibration, the mass of a system at rest (static equilibrium) need to be disturbed from its rest or equilibrium position. The mass gets excited and initiate oscillation or vibration; but only with the presence of a restoring force. If the disturbance is from an unstable or neutral equilibrium, possibility of any restoring force is lacking and the mass eventually moves away and occupies a new position. In the case of (i) stable equilibrium of a mass or (ii) mass being attached to a spring (or an elastic member),a restoring force comes into being and the mass is tended back to the original equilibrium position and as a result, periodic motion starts. In case (i), generally, component of gravity (g) operated upon the associated mass component serves as restoring force. Thus the system undergoes no "elongations or strains" than rigid body motion, as in the case of a simple pendulum. Such "no-strain" periodic motions are referred to as oscillation. In case (ii), spring force developed on the attached spring or elastic member serves as the restoring force. Thus the attached member undergoes "periodic elongation or strain". Thus the mass as well as attached member undergoes periodic motion. Such "strained" periodic motions are referred to as vibrations. In the expression for natural frequency, ω, s'g' is a parameter in the case of oscillations and 'k' in the case of vibrations. For the oscillation of simple pendulum, ω =sqrt (g/l), whereas for the vibration of spring-mass system, ω = sqrt(k/m); with usual notations.

II. DYNAMIC LOAD

The very nature of the load means that the response of the structure i.e the displacement, stress, reactions etc. also varies with time. Dynamic load is the load which varies. The variation may be with respect to time (Ex: reciprocating engine) or with respect to space (Ex : Moving vehicle on bridge). For example in a Dam the water level changes throughout the year then should we consider it as a dynamic load for designing the dam? NO, because though the load is changing with respect to time its inertial effect is very less for practical consideration. To make it more clear, a Dynamic load (externally applied) is not only a function of time, say F(t), but also one which excites the mass of the system causing inertial effect. The physicist who deals with the problem has to decide whether the force, F(t) is a dynamic one or not based on the significance of the excitation. It is also interesting to note that a dynamic load is not essential to produce vibration or oscillation. An initial displacement or velocity will trigger free vibration in an appropriate system (Ex: a spring-mass system like shock absorber of automobile)

Research on the vibration beams and plates can be divided into three categories. Firstly, there exist exact solutions only for a very restricted number of simple cases .Secondly, studies of semi-analytic solutions, including the differential quadrature method and the boundary characteristic orthogonal polynomials. Finally, there are the most widely used discretization methods such as the finite element method and the finite difference method. As it is more useful to have analytical results than to resort to numerical methods.

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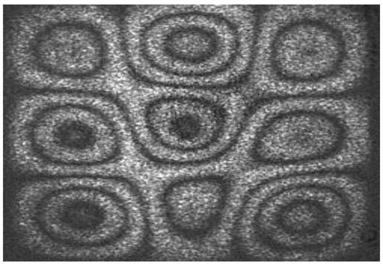

Fig 1 Vibration mode of a clamped rectangular plate

III. EXPERIMENTAL ANALYSIS

Consider a rectangular plate which has dimensions (a*b) in the (x_1, x_2) plane and thickness 2h in x_3 direction. We seek to find out the free modes of vibration

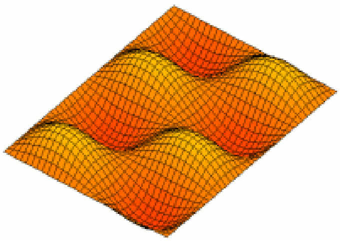

Fig 2 Vibration mode of a rectangular plate

Assume a displacement field of the form,

$\acute{\omega}$ (x_1, x_2, t)= W(x_1, x_2) F(t)

$$\nabla^2\nabla^2 w = w_{1111} + 2w_{1212} + w_{2222} = [\frac{\delta_4 w}{\delta x_4} + \frac{2\delta_4 w}{\delta x_1{}^2 \delta x_2{}^2} + \frac{\delta_4 w}{\delta x_4}] f(t) \tag{1}$$

And

$$\omega = W(x_1, x_2)\frac{\delta^2 f}{\delta t^2} \tag{2}$$

Plugging these into the governing equation gives,

$$\frac{D}{2\rho h w}[\frac{\delta_4 w}{\delta x_4} + \frac{2\delta_4 w}{\delta x_1{}^2 \delta x_2{}^2} + \frac{\delta_4 w}{\delta x_4}] = -\frac{1}{f}\frac{\delta_2 f}{\delta t^2} = w^2 \tag{3}$$

where w^2 is a constant because the left hand side is independent of *t* while the right hand side is independent of x_1, x_2. From the right hand side, we then have

$$f(t) = A^{iwt} + B^{-iwt} \tag{4}$$

Altogether there are 21 combinations of simple boundary conditions i.e., either clamped (C), simply supported (SS), or free (F) for rectangular plates. Frequency parameters are expressed in the terms if $\omega\alpha^2\sqrt{(\frac{p}{D})}$, where α is a length dimension, and do not depend upon Poisson's ratio unless atleast one edges of the plate is free. However, because D contains v, the frequencies themselves depend upon v for all the cases.

Warburton presented the first comprehensive collection of solutions for rectangular plates. He used Rayleigh method with deflection functions as the product of the beam functions; that is,

$$W(x,y) = X(x)\,Y(y) \tag{5}$$

Where X(x) and Y(y) are fundamental mode shapes having boundary conditions of the plate. These function, exactly satisfies the boundary conditions for the plate, except in the case of free edge where shear condition is approximately satisfied. The six possible distinct sets of boundary conditions along the edges x=0 and x=a are satisfied by the following mode shapes.

a) Simply supported at x=0 and x=a:

$$X(x) = \frac{\sin(m-1)\Pi x}{a} \tag{6}$$

b) Clamped at x=0 and x=a:

$$X(x) = \cos\gamma_1(\frac{x}{a} - \frac{1}{2}) + \frac{\sin(\gamma_1/2)}{\sinh(\gamma_1/2)}\cosh\gamma_1(\frac{x}{a} - \frac{1}{2}) \tag{7}$$

Where the values of γ_1 are obtained as roots of

$$\tan(\gamma_1/2) + \tanh(\gamma_1/2) = 0$$

and

$$X(x) = \sin\gamma_2(\frac{x}{a} - \frac{1}{2}) - \frac{\sin(\gamma_2/2)}{\sinh(\gamma_2/2)}\sinh\gamma_2(\frac{x}{a} - \frac{1}{2}) \tag{8}$$

Where the values of γ_2 are obtained as roots of

$$\tan(\gamma_2/2) + \tanh(\gamma_2/2) = 0$$

a) Free at x=0 and x=a:

$$X(x) = 1$$

$$X(x) = 1 - \frac{2x}{a}$$

$$X(x) = \cos\gamma_1(\frac{x}{a} - \frac{1}{2}) - \frac{\sin(\gamma_1/2)}{\sinh(\gamma_1/2)}\cosh\gamma_1(\frac{x}{a} - \frac{1}{2})$$

$$X(x) = \sin\gamma_2(\frac{x}{a} - \frac{1}{2}) - \frac{\sin(\gamma_1/2)}{\sinh(\gamma_1/2)}\sinh\gamma_2(\frac{x}{a} - \frac{1}{2}) \tag{9}$$

With γ_1 and γ_2 are defined in above equations

b) Clamped at x=0 and free at x=a:

$$X(x) = \cos\frac{\gamma_3 x}{a} - \cosh\frac{\gamma_3 x}{a} + (\frac{\sin\gamma_3 - \sinh\gamma_3}{\cos\gamma_3 - \cosh\gamma_3})(\sin\frac{\gamma_3 x}{a} - \sinh\frac{\gamma_3 x}{a}) \tag{10}$$

Where $\cos\gamma_3 \cosh\gamma_3 = -1$

c) Clamped at x=0 and simply supported at x=a:

$$X(x) = \sin\gamma_2(\frac{x}{2a} - \frac{1}{2}) - \frac{\sin(\gamma_2/2)}{\sinh(\gamma_2/2)}\sinh\gamma_2(\frac{x}{2a} - \frac{1}{2}) \tag{11}$$

With γ_2 is defined in above equation

d) Free at x=0 and simply supported at x=a:

$$X(x) = \sin\gamma_2(\frac{x}{2a} - \frac{1}{2}) - \frac{\sin(\gamma_2/2)}{\sinh(\gamma_2/2)}\sinh\gamma_2(\frac{x}{2a} - \frac{1}{2}) \tag{12}$$

With γ_2 is defined in above equation

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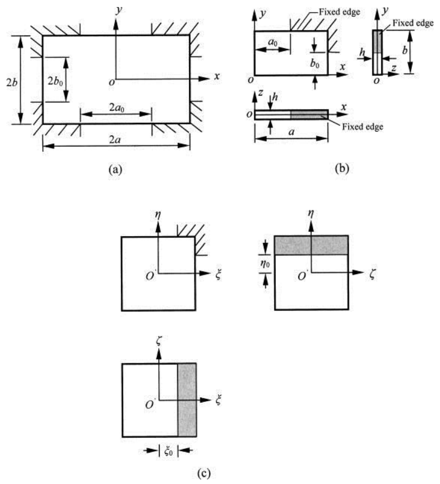

Fig 3 Boundary Conditions of rectangular plate

IV Boundary Conditions for SS-SS-SS-SS

The plates with all sides SS is the most simple to solve for rectangular plate.

The boundary conditions are

For x=(0,a),

$$W_{mn} = A_{mn} \sin\frac{m\Pi x}{a} \sin\frac{n\Pi y}{b} \tag{13}$$

$$\omega = \sqrt{\frac{D}{\rho}}[(\frac{m\Pi}{a})^2 + (\frac{n\Pi}{b})^2]$$

For x=(0,b),

$$w = 0, M_V = 0$$

The equation $W_{mn} = A_{mn} \sin\frac{m\Pi x}{a} \sin\frac{n\Pi y}{b}$ satisfies the boundary conditions, where A_{mn} is an amplitude coefficient determined from the initial conditions, m and n are integers.

Frequency= $\omega = \sqrt{\frac{D}{\rho}}[(\frac{m\Pi}{a})^2 + (\frac{n\Pi}{b})^2]$ (14)

The node lines for general rectangle are simply straight lines parallel to the edges. For square plates however two mode shapes may have the same frequency and exist simultaneously, their for a frequency are shown in the figure.

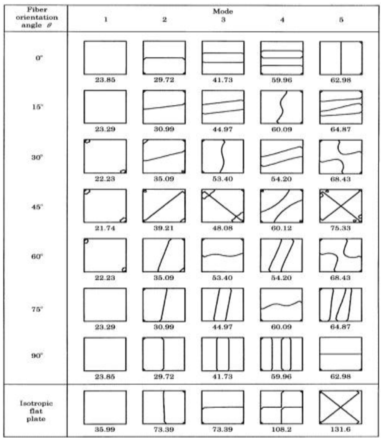

Fig 4 Angle orientation with modes

V CALCULATIONS

% Model: Simply Supported Plate.mph
% Version: COMSOL 5.2.1.262
% Dimension: 3
% Nodes: 1047
% Expressions: 11
% Description: % Length unit: metre

TABLE I

Time (secs)	Reaction force, z component			
	%X	*%Y*	*%Z*	*Solid.RFz (N)*
0	0	0	0	0.007369
0.1	0	0	0	33.4412
0.2	0	0	0	47.2360
0.3	0	0	0	96.1057
0.4	0	0	0	98.05461
0.5	0	0	0	98.05461

% Model: Simply Supported Plate.mph
% Version: COMSOL 5.2.1.262
% Dimension: 3
% Nodes: 1047
% Expressions: 11
% Description: Total acceleration, Z component

TABLE II

Time (secs)	Total acceleration, z component			
	%X	%Y	%Z	solid.accZ (m/s^2)
0	0.08333	0	0.075	-0.006533
0.1	0.08333	0	0.075	-0.006533
0.2	0.08333	0	0.075	0.01410
0.3	0.08333	0	0.075	0.00294

Time (secs)	Total acceleration, z component			
	%X	%Y	%Z	solid.accZ (m/s^2)
0.4	0.08333	0	0.075	-0.01571
0.5	0.083333	0	0.075	0.00818

% Model: Simply Supported Plate.mph

% Version: COMSOL 5.2.1.262

% Dimension: 3

% Nodes: 1047

% Expressions: 11

% Description: Reaction force, y component

% Length unit: m

TABLE III

Time (secs)	Reaction force, y component			
	%X	%Y	%Z	Solid.RFz (N)
0	0.109246	0.12593	0.15	0
0.1	0.109246	0.12593	0.15	0
0.2	0.109246	0.12593	0.15	0
0.3	0.109246	0.12593	0.15	0
0.4	0.109246	0.12593	0.15	0
0.5	0.109246	0.12593	0.15	0

% Model: Simply Supported Plate.mph

% Version: COMSOL 5.2.1.262

% Table: Maximum and minimum values - Max/min surface

TABLE IV

Time (secs)	Maximum and minimum values - Max/min surface			
	%X	*%Y*	*%Z*	*Total displace ment (m)*
0	3	0.416666	0.075	2.891E-5
0.1	1.49999	0	0.0749998	9.047E-5

% Model: Simply Supported Plate.mph

% Version: COMSOL 5.2.1.262

% Dimension: 3

% Nodes: 1047

% Expressions: 11

% Description: Time

TABLE V

Time (secs)	Time			
	%X	%Y	%Z	Time (seconds)
0	0	0.0833	0.075	0
0.1	0	0.0833	0.075	0.09999
0.2	0.0833	0	0.075	0.1999
0.3	0.0833	0	0.075	0.3000
0.4	0.0833	0	0.075	0.39999
0.5	0.0833	0	0.075	0.5000

% Model: simply supported plate.mph

% Version: COMSOL 5.2.1.262

% Dimension: 3

% Nodes: 1047

% Expressions: 11

% Description: First principal strain

% Length unit: m

TABLE VI

Time (secs)	First principal strain			
	%X	*%Y*	*%Z*	*solid.ep1*
0	0	0	0	1.21689
0.1	0	0	0	7.8797
0.2	0	0	0	1.0659
0.3	0	0	0	2.0651
0.4	0	0	0	2.1072
0.5	0	0	0	1.3505

% Model: simply supported plate.mph

% Version: COMSOL 5.2.1.262

% Dimension: 3

% Nodes: 1047

% Expressions: 11

% Description: Velocity magnitude

% Length unit: m

TABLE VII

Time (secs)	Velocity magnitude			
	%X	%Y	%Z	solid.vel (m/s)
0	0	0	0	3.88178
0.1	0	0	0	5.9605
0.2	0	0	0	6.4712
0.3	0	0	0	9.0645
0.4	0	0	0	4.7696
0.5	0	0	0	2.4333

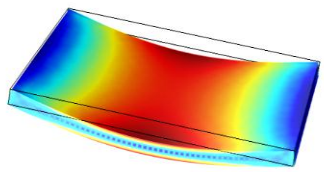

Fig 7 i) Rectangular Plate after applied load for different values of time "t"

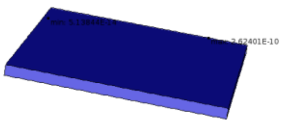

Fig 7 ii) Rectangular Plate after applied load for minimum and maximum surface

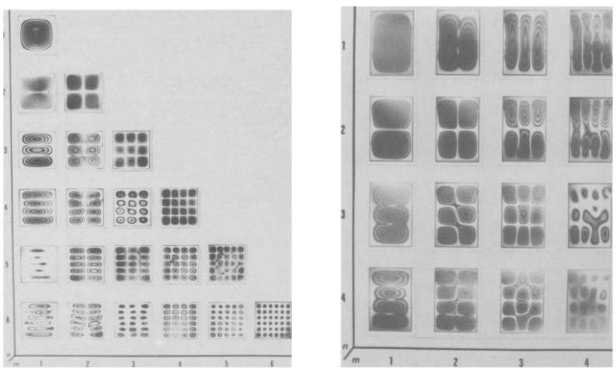

Fig 8 Photographic record of time averaged holographic interferograms of rectangular plate

VI CONCLUDING POINTS

The vibration frequency for functionally graded plates under in-plane hydrostatic pressure and resting on, have readily been given in terms of the eigenvalue of the membrane with the shape of the plate, and clamped at the edges. Therefore, the exact correspondence between the buckling and vibration eigenvalues of the third order plate theory, the first order plate theory and the classical plate theory for functionally graded polygonal plates with simply supported rectilinear edges and the vibration eigenvalue of the corresponding membrane has been established. Some available analogies between single-layer homogeneous plates, symmetric sandwich plates and laminated plates and membranes are special cases of the present results. The present results also apply to a transversely isotropic plate because we have not required the shear modulus to satisfy $\mu' = E/2(1+v)$. For a transversely isotropic material with the plane of isotropy parallel to the mid-plane of the plate, E and l are respectively Young's modulus and the Poisson ratio in the plane of isotropy, and μ' is the shear modulus in the transverse direction. A typical example is laminated composite plate with transversely isotropic lamina, which is widely used in missiles and re-entry vehicles due to its special thermo-mechanical properties suited for the thermal protection and its high flexibility in transverse shear.

The Experimental Eigen values for square and rectangular plate agree favorably with several predictions and theoretical considerations and agree very well with several considerations like Rayleigh Ritz method for mode shapes.

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