International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 2 Issue 12, December - 2013

Effects of the Position and the Inclination of the Hole in Thin Plate on the Stress Concentration Factor

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Abstract

The stress concentration factors are widely used to predict the maximum stress value above which the mechanical structure can be destroyed. Many chart data of those factors are available in literature but they are conditioned by the structure shape and the principal geometric dimensions. This paper compares, for thin plate with eccentric hole, the stress concentration factors values calculated by classical formulas given in ulterior studies and a numerical simulation using commercial software. The effects of the relative hole position in the plate and the inclination hole axis are examined.

1. Introduction

The plates with discontinuities like circular or elliptic holes exist in all metal structures. Those areas represent dangerous zones because of the multiplication of the stresses values under the effect of the stress concentration phenomenon. These stress concentration zones are often areas of crack initiation. They can be dangerous if the loading conditions allow the brutal propagation of the cracks and than promote the rupture.

The stresses concentration phenomenon is measured by a parameter called stress concentration factor (SCF), noted Kt. This factor is the ratio of the maximum elastic stress value by the nominal stress calculated in the discontinuity area. The values of this factor are calculated using analytical approaches based on the stress and deformation distributions evaluation around the discontinuity or by numerical models or also by experimental studies using the photoelasticity method. The results of these investigations are resumed in curves according to the structure geometry dimensions.

Kotousov et al. [1,2] present a 3D analytical solution to analyze the stresses distribution around the hole for isometric plates. They indicated the plate thickness importance in the SCF calculation.

Several numerical studies using 2D models were cited by Pilkey [3]. Wu et al. [4] proposed a simple numerical method to estimate the SCF for the plates and the cylinders with centered holes and isotropic or orthotropic materials. They conclude that, for small structural dimensions, the SCF depend only on the ratio of the hole diameter to plate width. Jain et al. [5] studied the effect of the hole diameter and plate width ration upon SCF in rectangular isotropic, orthotropic and laminated composite plates with central circular hole under transversal static loading.

More complicated 3D models were devoted to evaluation of the SCF using commercial software. Amr et al. [6] developed a 3D FE model for plate with small centred notch. He found that the maximum stress and strain concentration factors occur on the mid plane of plate only for the thin plates. Darwish et al. [7] performed stress analysis around countersunk holes in orthotropic plates and developed a parametric equation fo the SFC calculation, using 3D FE analysis.

The photoelasticity method is used early and widely in experimental studies. Flynn [8] presents an approach using this technique by comparing the samples to a reference with known SCF value. In this paper, the author compares there results to many other results research using both analytical and experimental approaches.

Almost all literature studies were interesting on centred hole plates with uniform or variable thickness. She and Guo [9,10] show that the variation of SCF for elliptic hole with tensile stress is less significant in thin plate and increases with increasing the plate width. Folias and Wang11 assume that the SCF is sensitive to the ratio of hole depth to plate thickness and to Poisson’s ratio.

The objective of this work is to evaluate the stresses and displacements as well as the stresses concentration factor for perforated plate according to hole position. Fig. 1 gives the used plate and hole dimensions. The dimensions b and c represent respectively the hole position compared to the free edges of the plate along x and z axes.

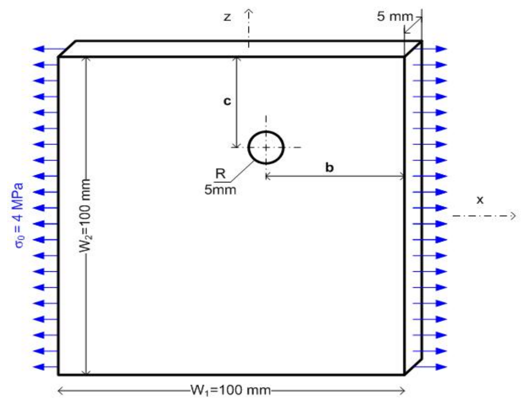

Figure 1. Plate with hole

2. Stress concentration factor calculation

The analytical models devoted to treating the thin plates with holes are based on the linear theory of elasticity. The thin plates containing holes are subdivided to two main categories, those with relatively infinite dimensions compared to the hole radius and those with finite dimensions.

The SCF is defined, as early mentioned, as the ratio of the maximum stress, σ_{max}, recorded on the plate by the nominal stress, σ_{nom}, calculated in the hole section, according to equation (1).

$$K_t = \frac{\sigma_{\max}}{\sigma_{nom}} \qquad (1)$$

For the plates of relatively infinite dimensions, the hole is supposed to be centered on the plate and the nominal stress, σ_{nom}, is the clear stress on the hole level and calculated according to Eq. (2) where σ_0 is the mean stress applied away from the hole area.

$$\sigma_{nom} = \sigma_0\left(\frac{W_2}{W_2 - 2r}\right) \qquad (2)$$

The SCF is calculated by several formulas determined by experimental data fitting. The reference [12] proposes Eq. (3) referred to Flynn's paper [8].

$$K_t = 3 - 3.13\left(\frac{2r}{W_2}\right) + 3.66\left(\frac{2r}{W_2}\right)^2 - 1.53\left(\frac{2r}{W_2}\right)^3 \qquad (3)$$

Heywood [6] proposes for SCF the equation (4).

$$K_t = \frac{2 + \left(1 - \frac{r}{W_2}\right)^3}{1 - \frac{r}{W_2}} \qquad (4)$$

For the plates with decentred hole compared to the applied external load direction, the nominal stress depends on the distance c indicated in Fig. 1. The reference [13] proposes the Eqs. (5) and (6) to calculate the nominal stress and the SCF for decentred hole case compared to the applied external stress direction.

$$\sigma_{nom} = \sigma_0\left(\frac{1 - \frac{c}{W_2}}{1 - \frac{r}{c}}\right)\frac{\sqrt{1 - \left(\frac{r}{c}\right)^2}}{1 - \left(\frac{c}{W_2}\right)\left[2 - \sqrt{1 - \left(\frac{r}{c}\right)^2}\right]} \qquad (5)$$

$$K_t = 3 - 3.13\left(\frac{r}{c}\right) + 3.66\left(\frac{r}{c}\right)^2 - 1.53\left(\frac{r}{c}\right)^3 \qquad (6)$$

The equations presented in this paragraph are widely used in the literature. However, it does not take into account all dimensions of the plate, as indicated in Fig. 1, like the length w_1, the thickness t and distance b. In the finite elements model, presented in the following section, all mentioned dimensions are specified and consequently the numerical simulations results are influenced.

3. Finite Element Model

The finite element analysis allowed the production of stresses and displacements distributions in all plate directions. Using the analytical definition for the SCF, presented by Eq. (1), the effects of the hole position and orientation and also the plate thickness are expected. The plate is modeled by a 3D brick with six nodes. The geometrics parameters b and c, which determine the hole position on the plate, take 50-40-30-20 and 10 mm as values. The orientation of the hole is indicated by the angle between the hole axis and the plate normal. For this simulation, this angle is varied from 0 to 20 degrees in order to take into account the small inclinations.

The software used to carry out the finite elements simulation is ANSYS [14]. The developed finite element model is composed of the meshed geometry, the boundary conditions and the loads. Since SCF is independent of mechanical characteristics of the plate, the material chosen for the simulation is the ordinary steel with the conventional elastic mechanical characteristics (E=210 GPa, ν□= 0.3). The mesh is refined to carry out at the same time the convergence and the optimization of the data-processing resources in memory and simulation times.

All previous studies indicate that the SCF is independent of the applied stress on the structure and only the geometric characteristics are involved in the calculations. For the same reason, a single value of the mean stress chosen arbitrarily without greatly deform the plate is used. As indicated in Fig. 1, σ_0=4 MPa is applied to the plate.

The study proposes to examine the influence of the hole position, the plate thickness and the inclination of the hole axis compared to the normal direction in the front plate face on the SCF.

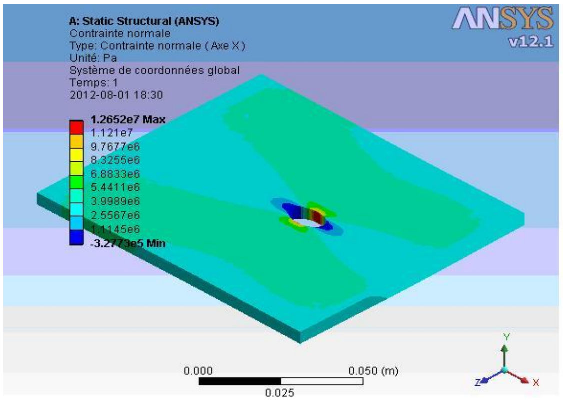

Figure 2. Normal stress distribution along the x-axis

4. Results and Discussion

The stress concentration factor (SCF) is calculated from the maximum stress, which can be determined from the finite element simulation results, and the nominal stress, calculated by the equations (2) and (5).

Fig. 2 presents the distribution of the normal stress in the applied external load direction. It is clear that the normal stress is not uniform in the surrounding of the hole. The maximum value of this stress is localised on the bore perpendicular to the applied load direction. Fig. 3 gives the displacement along the plan plate axes. The values of these displacements depend on the applied pressure. For displacement along the x axis, the iso-displacements are lines parallel to the z-axis.

The equations (3) and (4) do not take into account the hole position on the plate. Indeed they utilize only the ratio of hole radii, r, by the plate width, w2. However, the equation (6) introduce the parameter c, which expresses the hole position relative to the z-axis which is perpendicular to the external plate load.

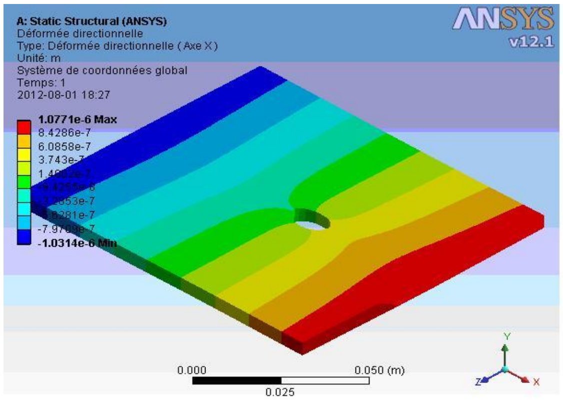

Figure 3. Displacement along the x-axis

Fig. 4 compares the values of the SCF given by the equations (3), (4) and (6) and those resulting from finite elements analysis, according to the parameter c for several values of b. It is noticed that for any value of c, the four approaches give largely different values. Indeed, for example, when the hole is centered on the plate (b=c=50 mm), the finite element model gives for the SCF 2.839, the Eq. (3) gives 2.696, the Eq. (4) gives 3.008 and the Eq. (6) gives 2.722. On other hand, for every value of the parameter b, the SCF increases when the hole approaches the edge of the plate. The effect of the parameter b is not expressed in the SCF calculation equations. In the same time, the numerical simulation allows to estimate the importance of this parameter when the evaluation of the SCF is done. This effect is presented in Fig. 5. It's showed that the SCF values increase if the hole is close to the plate edges. This makes it possible to evaluate the minimal threshold of the distance between the hole center and the ends of the plate. It is clear that this threshold should not be smaller than the hole radii. A more advanced study is necessary to evaluate this value exactly.

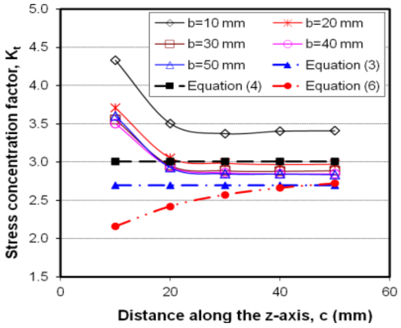

Figure 4. SCF variation according to c

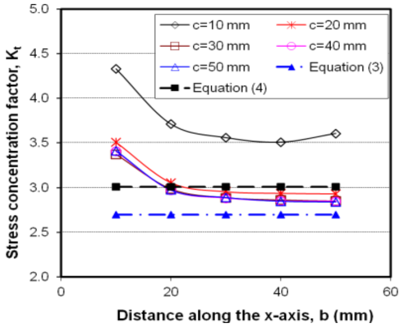

Figure 5. Effect of distance b on the SCF

Fig. 6 presents the effect the plate thickness, t, on the SCF values. Clearly, the plate thickness does not influence the values of Kt and mainly when the hole is far from the free edges of the plate.

The results presented in Fig. 2 to 6 are established when the axis of the hole is perfectly normal to the plate face and it is the case when the hole manufacture is carried out without defect. Under the usual manufacture conditions, the angle between the hole axis and the plan-face of the plate is not certainly null. In order to evaluate the importance of the precision in the hole machining, Fig. 7 presents the variation of the SCF according to angle of the hole axis inclination compared to the normal of the plate plan-face. When this angle is more significant, the SCF increases considerably. Indeed, for example for b = 30 mm, when the angle passes from 10 to 15 then to 20 degrees, the SCF increase from 2.948 to 3.011 then to 3.127. The SCF increase rate according to the inclination angle is practically independent of the hole position in the plate.

The different cases presented in this study show clearly that the parameters guiding the choice of SCF for plate with eccentric circular hole is not limited to the overall dimensions of the plate and the hole. It include also the relative position of the hole on the plate, the plate thickness and the quality of the hole machining.

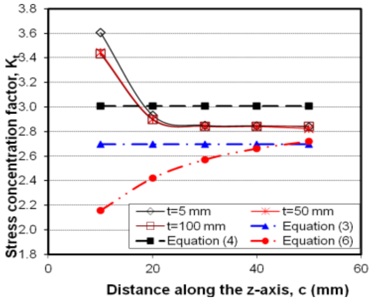

Figure 6. Effect of plate thickness on the SCF

More detailed simulations are necessary to establish new charts which take into account all the geometrical parameters. Further studies will present the results of these simulations supported by appropriates analytical models.

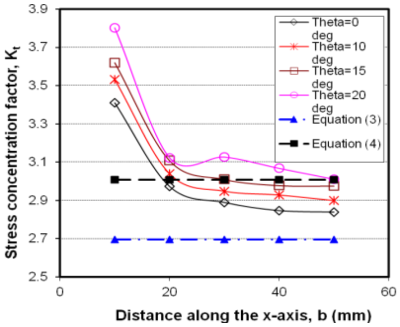

Figure 7. Effect of hole axis inclination on the SCF

5. Conclusion

The calculation of the SCF is largely treated in the literature, but the obtained results remain valid for the treated cases and cannot be always generalized. This study shows that the conventional literature formulas do not give the same values obtained by the finite element analysis using 3D model. It is recommended to use the preset curves of the SCF with precaution and to check always in several sources the validity of these data. The best way remind the experimental tests using specially the Non-destructive techniques in order to establish more validated SCF charts.

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