Effects of Temperature Dependent Viscosity and Thermal Conductivity on Unsteady MHD Natural Convection in a Porous Medium between two long Vertical Wavy Walls

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Abstract

A numerical study of unsteady natural convection of an electrically conducting viscous incompressible fluid confined between two long vertical wavy walls in porous medium under the influence of magnetic field with variable viscosity and thermal conductivity is presented. Both the fluid viscosity and thermal conductivity are assumed to vary as inverse linear functions of temperature. The coupled non- linear system of partial differential equations are solved using Finite Difference method. The effects of variable viscosity parameter, variable thermal conductivity parameter and magnetic parameter on the velocity field and temperature field have been discussed and shown graphically.

Key words: Natural convection, MHD flow, variable viscosity, variable thermal conductivity.

1. Introduction

Lekoudis *et. al*. [5] presented a linear analysis of compressible boundary-layer flows over wavy walls using semi- analytical technique without restricting the mean flow to be linear in the disturbance sub layer. Such type of problem finds application in different areas such as transpiration cooling of re-entry vehicles and rocket booster, cross hatching of ablative surfaces and film vaporization in combustion chamber and can determine increased heat transfer due to wall roughness. It also provides the mean flow for stability analysis of flows over wavy walls. The problem of flow over wavy walls originates from the work of Miles [9] who obtained solutions for flow over small amplitude wavy surfaces. Lyne [7] discusses unsteady viscous flow over a wavy wall using the method of conformal transformation. Yao [14] concluded that total heat transfer rate for a wavy surface of any kind, in general, greater than that of flat surface.

Also it is necessary to study free convection flow through porous medium to make heat insulation of surface more effective to estimate its effect in heat transfer and because of the present and potential use of geothermal energy for power production. Vazravelu *et al.*[13] considered free convective heat transfer in a viscous incompressible fluid confined between a long vertical wavy wall and a parallel flat wall. Das and Deka[1] analyzed the same by the numerical method of superposition. Sarangi and Jose [12] studied free convection MHD flow of a viscous incompressible fluid in porous medium between long vertical periodic wavy walls when the amplitude of waviness of both walls are different and time dependent.

While this type of flow has been investigated extensively, the study of variable thermo physical properties (generally viscosity and thermal conductivity) effects has received more limited attention. Temperature dependence of these physical properties plays a significant role in fluid mechanics. A number of authors analyzed the influence of variable thermo-physical properties on the flow structure and heat transfer. Ling and Dybbs [6], Pop *et. al*[11], Kafoussias and Williams [3] established that temperature dependent viscosity has a pronounced effect on momentum and thermal transport in the boundary layer region. To accurately predict the flow and the heat transfer rate it is necessary to take into account the variation of viscosity. Problems of this type of flow have important applications in geophysics particularly geothermal energy extraction and underground storage system.

Pinarbasi *et.al.*[10] investigated the effect of variable viscosity and thermal conductivity of a non isothermal, incompressible Newtonian fluid flowing under the effect of a constant pressure gradient in plane Poiseuille flow. The viscosity and thermal conductivity of the fluid exhibit linear temperature dependence and the effect of viscous heating is included in the analysis.

The main objective of the present study is to investigate the effects of the variable viscosity and thermal conductivity on unsteady free-convective MHD flow of a viscous incompressible fluid in porous medium between two long vertical wavy walls assuming that wavelength of the wavy walls are large with different and time dependent amplitude. The fluid viscosity and thermal conductivity are taken as inverse linear functions of temperature. Numerical solutions are discussed with graphical representation.

2. Formulation of the problem

We consider the unsteady free-convective flow of a viscous incompressible and electrically conducting fluid in porous medium confined between two long non-conducting wavy walls in presence of uniform transverse magnetic field.

The surfaces are assumed wavy with the form [12]

$$y^* = \varepsilon^* (\cos \lambda^* x^* + \omega^* t^*)\ ,$$

$$y^* = d\,(1 + h\,\varepsilon^* \cos(\lambda^* x^* + \omega^* t^*))$$

where ε^* and $dh\varepsilon^*$ are amplitude of the respective walls and maintained at constant but different temperature with λ^* and ω^* as perturbation parameter .

Following Lai and Kulacki [4] the fluid viscosity is assumed to be inverse linear function of temperature as

$$\frac{1}{\mu} = \frac{1}{\mu_s}[1 + \gamma(T - T_s)] = a(T - T_r) \tag{1}$$

Where

$$a = \frac{\gamma}{\mu_s} \quad \text{and} \quad T_r = T_s - \frac{1}{\gamma} \tag{2}$$

μ is the coefficient of dynamic viscosity, μ_s is the coefficient of viscosity in static condition. a and T_r are constants and their values depend on the reference state and thermal property of the fluid. In general $a > 0$ for liquids and $a < 0$ for gases . γ is a constant based on the thermal property of the fluid.

Also the variation of thermal conductivity is considered as follows (Hazarika and Lahkar [2])

$$\frac{1}{k} = \frac{1}{k_s}[1 + \xi(T - T_s)] = c(T - T_k) \tag{3}$$

where

$$c = \frac{\xi}{k_s} \quad , T_k = T_s - \frac{1}{\xi} \tag{4}$$

k is the thermal conductivity of the fluid , k_s is the thermal conductivity of the fluid in static condition, c and T_k are constants and their values depend on the reference state and thermal property of the fluid ξ , a constant based on thermal property of the fluid . $c>0$ for liquids and $c<0$ for gases.

x^*- axis is taken in the flow direction opposite to the direction of gravity and y^*- axis perpendicular to it. The flow is permeated by a uniform magnetic field of strength B_0 applied transverse to the direction of the flow. The induced magnetic field in comparison to the applied magnetic field and the viscous dissipation term in the energy equation are neglected. The partial Boussinesqs approximation (density variation in the body force term of the vertical momentum equation only) is used to analyze the influence of variable viscosity and variable thermal conductivity on the unsteady free convective hydromagnetic flow in porous medium confined between two long non-conducting wavy walls.

The governing equations are

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0 \tag{5}$$

$$\frac{\partial u^*}{\partial t^*} + u^*\frac{\partial u^*}{\partial x^*} + v^*\frac{\partial u^*}{\partial y^*} = -\frac{1}{\rho^*}\frac{\partial p^*}{\partial x^*} + g\beta(T^* - T_s) - \frac{\vartheta^* u^*}{K} - \frac{\sigma B_0^2}{\rho^*}u^* + \frac{1}{\rho^*}\left\{\frac{\partial}{\partial x^*}\left(\mu_*\frac{\partial u^*}{\partial x^*}\right) + \frac{\partial}{\partial y^*}\left(\mu^*\frac{\partial u^*}{\partial y^*}\right)\right\} \tag{6}$$

$$\frac{\partial v^*}{\partial t^*}+u^*\frac{\partial v^*}{\partial x^*}+v^*\frac{\partial v^*}{\partial y^*}=-\frac{1}{\rho^*}\frac{\partial p^*}{\partial y^*}-\frac{\vartheta^* v^*}{K}$$
$$-\frac{\sigma B_0^{\,2}}{\rho^*}v^*+\frac{1}{\rho^*}\left\{\begin{array}{l}\frac{\partial}{\partial x^*}\left(\mu^*\frac{\partial v^*}{\partial x^*}\right)\\+\frac{\partial}{\partial y^*}\left(\mu^*\frac{\partial v^*}{\partial y^*}\right)\end{array}\right\} \tag{7}$$

$$\rho^* c_p\left(\frac{\partial T^*}{\partial t^*}+u^*\frac{\partial T^*}{\partial x^*}+v^*\frac{\partial T^*}{\partial y^*}\right)=\frac{\partial}{\partial x^*}\left(k^*\frac{\partial T^*}{\partial x^*}\right)+\frac{\partial}{\partial y^*}\left(k^*\frac{\partial T^*}{\partial y^*}\right) \tag{8}$$

where u^* and v^* are the velocity components in the directions of X^*-and Y^*-axis, p^* is the pressure, g is the acceleration due to the gravity , β is the coefficient of the volume expansion, ρ^* is the density , σ is the electrical conductivity, K is the permeability parameter, ϑ^* is the kinematic viscosity , c_p is the specific heat.

The corresponding boundary conditions are

$$\left.\begin{array}{l}\text{at } y^*=\varepsilon\cos(\lambda^* x^*+\omega^* t^*),\\ \quad u^*=0=v^*,\ T^*=T_1\ ;\\ \text{at } y^*=d(1+h\varepsilon^*\cos(\lambda^*+\omega^* t^*)),\\ \quad u^*=0=v^*,\ T^*=T_2\end{array}\right\} \tag{9}$$

Dimensionless quantities are introduced as follows [12]

$$x=\frac{x^*}{d},\quad y=\frac{y^*}{d},\quad u=\frac{ud^*}{\vartheta_s},$$
$$v=\frac{v^* d}{\vartheta_s},\quad \mu=\frac{\mu^*}{\mu_s},\ k=\frac{k^*}{k_s},$$
$$\rho=\frac{\rho^*}{\rho_s},\ \vartheta=\frac{\vartheta^*}{\vartheta_s},\ p=\frac{p^*}{\rho^*(\frac{\vartheta_s}{d})^2}, \tag{10}$$
$$\theta=\frac{T^*-T_s}{T_1-T_s},\ t=\frac{t^*\vartheta_s}{d^2},\ \varepsilon=\frac{\varepsilon^*}{d}$$

With the help of above transformation (10), equations (5) to (8) reduces to

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{11}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\frac{\theta_r}{(\theta_r-\theta)^2}\left(\frac{\partial u}{\partial x}\frac{\partial \theta}{\partial x}+\frac{\partial u}{\partial y}\frac{\partial \theta}{\partial y}\right)+\frac{\theta_r}{\theta_r-\theta}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+G_r\theta-u\left(\alpha^2+M^2\right) \tag{12}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{\partial p}{\partial y}+\frac{\theta_r}{(\theta_r-\theta)^2}\left(\frac{\partial v}{\partial x}\frac{\partial \theta}{\partial x}+\frac{\partial v}{\partial y}\frac{\partial \theta}{\partial y}\right)+\frac{\theta_r}{\theta_r-\theta}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-v\left(\alpha^2+M^2\right) \tag{13}$$

$$P_r\left\{\frac{\partial \theta}{\partial t}+u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}\right\}=\frac{\theta_k}{(\theta_k-\theta)^2}\left\{\left(\frac{\partial \theta}{\partial x}\right)^2+\left(\frac{\partial \theta}{\partial y}\right)^2\right\}+\frac{\theta_k}{\theta_k-\theta}\left(\frac{\partial^2 \theta}{\partial x^2}+\frac{\partial^2 \theta}{\partial y^2}\right) \tag{14}$$

The transformed boundary conditions are

$$\begin{array}{l}\text{at } y=\varepsilon\cos(\lambda x+\omega t),\quad u=0, v=0, \theta=1,\\ \text{at } y=1+\alpha_1\varepsilon\cos(\lambda x+\omega t),\\ \qquad u=0,\ v=0,\ \theta=m\end{array} \tag{15}$$

where $Gr = \frac{g\beta d^3 (T_1 - T_s)}{\vartheta^{*2}}$ is the Grashoff number, $\Pr = \frac{\mu^* C_p}{k^*}$ is the Prandtl Number, $M^2 = \frac{\sigma B_0^2 d^2}{\rho^* \vartheta^*}$ is the Hartmann Number, $m = \frac{T_2 - T_s}{T_1 - T_s}$ is the ratio of wall temperature difference, $\alpha = \frac{d}{K}$ is the porosity parameter, $\alpha_1 = hd$ is the amplitude parameter, $\lambda = \lambda^* d$, $\omega = \frac{\omega^* d^2}{\vartheta^*}$ are the modified frequency parameters,

$\theta_r = \frac{T_r - T_s}{T_1 - T_s} = -\frac{1}{\gamma(T_1 - T_s)}$ is a viscosity measuring parameter ranging from -10 to +10 which is positive for gases and negative for liquids when T_1-T_s is positive, and

$$\theta_k = \frac{T_k - T_s}{T_1 - T_s} = -\frac{1}{\xi(T_1 - T_s)}$$

is the thermal conductivity variation parameter.

Knowing the velocity field, the expression for the Skin-friction components at the surface in the x^*-direction (i.e. main flow direction) is given by

$$C_f = \frac{d^2 \tau_w}{\rho_s \vartheta_s^2} = \frac{\theta_r}{\theta_r - 1} \left.\frac{\partial u}{\partial y}\right|_{y=\varepsilon\cos(\lambda x + \omega t)} \qquad (16)$$

where

$$\tau_w = \mu \left(\frac{\partial u^*}{\partial y^*} \right)_{y^* = \varepsilon \cos(\lambda^* x^* + \omega^* t^*)}$$

is the wall shear stress.

The rate of heat transfer in terms of the Nusselt number at the surface is given by

$$Nu = \frac{qd}{k_s (T_1 - T_s)} = -\frac{\theta_k}{\theta_k - 1} \left.\frac{\partial \theta}{\partial y}\right|_{y=\varepsilon\cos(\lambda x + \omega t)} \qquad (17)$$

where

$$q = -k \left.\frac{\partial T}{\partial y^*}\right|_{y^* = \varepsilon \cos(\lambda^* x^* + \omega^* t^*)}$$

is the heat flux at the wall.

3. Results and Discussions

The system of differential equations (11)-(14) governed by the boundary conditions (15) is solved numerically .A finite difference solution is straight forward since the computational grids are fitted to shape of the wavy wall. The central difference is used for the diffusion terms and the forward difference scheme is used for the convection terms. In the present analysis the results are limited to P_r=0.7 fluid with Grashof number G_r = 5, porosity parameter α=2 and amplitude parameter α_1=2, modified frequency parameters $\omega = \pi/4$, $\lambda = \pi/100$, $\varepsilon = 0.2$, x=1, t=1 and m=2.

From Table 1. and Table 2. it is observed that the values of Skin-friction coefficient C_f decreases for increasing values of viscosity variation parameter θ_r and Hartmann number M respectively. Table 3. shows that the values of Skin-friction coefficient C_f and the Nusselt number Nu increases with values of θ_k.

Figure1- Figure3 depict the effects of the viscosity variation parameter θ_r on the velocity, temperature and cross flow velocity for fixed values of thermal conductivity variation parameter θ_k=-15, Hartman

number M=0.5.This parameter has marked effect on these profiles. It is observed that due to an increase in the magnitude of θ_r within -15 to -12 (not precisely determined) both velocity and cross flow velocity increase. But if the magnitude of θ_r increased beyond the limit -12 (possibly), the velocity profiles show a decreasing effect. This is due to the fact that for large values of θ_r the viscosity is very small and hence the resistive effect of the viscosity is diminished. Temperature profiles increase with the increase of θ_r.

The velocity profiles for values of the thermal conductivity variation parameter θ_k=-15,-10,-5 for a fluid P_r=0.7and viscosity variation parameter θ_r=-15 are depicted in Figure.4. It is observed that an increase in the values of θ_k first leads to increase in the velocity and then decrease. The temperature profiles for θ_k=-15,-12,-9,-5 for a fluid P_r=0.7and viscosity variation parameter θ_r=-15 are depicted in Figure 5. From this figures it can be seen that temperature profiles increase with the increase of the θ_k.

Figure 6. and Figure 7. show the velocity and cross flow velocity profiles for various Hartman number M=0,1,3,5 with constant P_r =0.7, the viscosity and thermal conductivity of the fluid being uniform θ_k=-15, θ_r =-15.These velocity curves show that the rate of transport is reduced with the increase of M. It clearly indicates that the transverse magnetic field opposes the transport phenomena. Figure 8. depicts the effect of the magnetic field on temperature profiles. It is observed that the fluid temperature decreases in the presence of magnetic field.

Table 1. Values of Skin-friction coefficient C_f for different values of θ_r and for θ_k= -15, P_r=0.7, M =0.5, G_r =5.

θ_r	C_f
-15	1.240667
-13	1.235668
-11	1.228964
-9	1.219501
-7	1.205127
-5	1.180658
-3	1.129496
-1	0.948924

Table2. Values of Skin-friction coefficient C_f for different values of M and for θ_k= -15, P_r=0.7, θ_r= -15, G_r =5.

M	C_f
0	-3.97036
1	-4.10652
2	-4.14375
3	-4.20579
4	-4.29265
5	-4.40433

Table 3 Values of Skin-friction coefficient C_f and the Nusselt number Nu for different values of θ_k and for M =0.5, P_r=0.7, θ_r= -15, G_r =5.

θ_k	C_f	Nu
-15	1.240667	-1.67777
-13	1.241276	-1.59257
-11	1.242124	-1.47724
-9	1.243385	-1.31203
-7	1.245464	-1.05435
-5	1.249585	-0.58857
-3	1.262601	0.635911
-2	1.319054	5.287658

4. Conclusion

An analysis is performed to study the flow and heat transfer characteristics for the case of an unsteady free-convective MHD flow of a viscous incompressible fluid in porous medium between two long vertical wavy walls assuming that wavelength of the wavy walls are large with different and time dependent amplitude under the assumption of the temperature dependent viscosity and thermal conductivity. The result pertaining to the present study indicates that fluid viscosity and thermal conductivity (hence thermal diffusivity) play an important role in the flow characteristics of laminar boundary layer problems. Fluid properties are significantly affected by the variation of the temperature. The effects of Lorentz force or the usual resistive effect of the magnetic field on the velocity profiles is apparent.

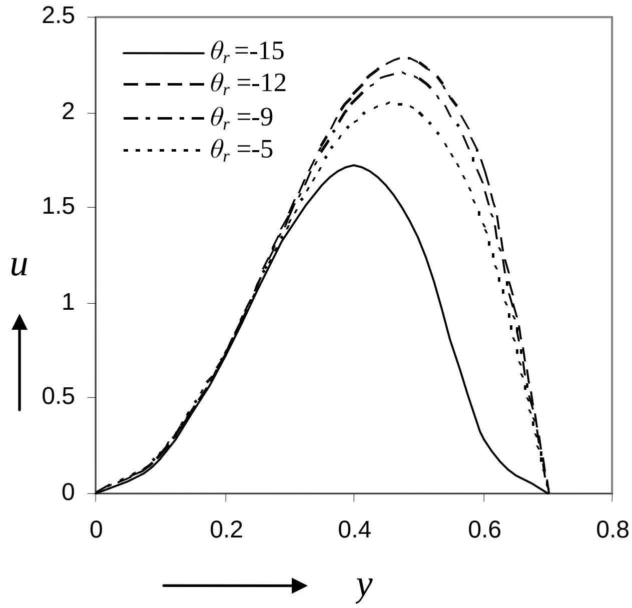

Figure 1. Velocity Profiles for θ_k=-15, P_r =0.7 M=0.5, G_r =5

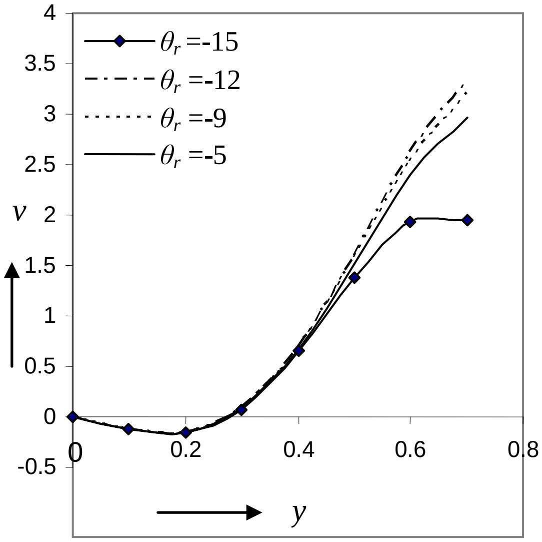

Figure 3. Cross flow Velocity Profiles for θ_k=-15, P_r=0.7, M=0.5,

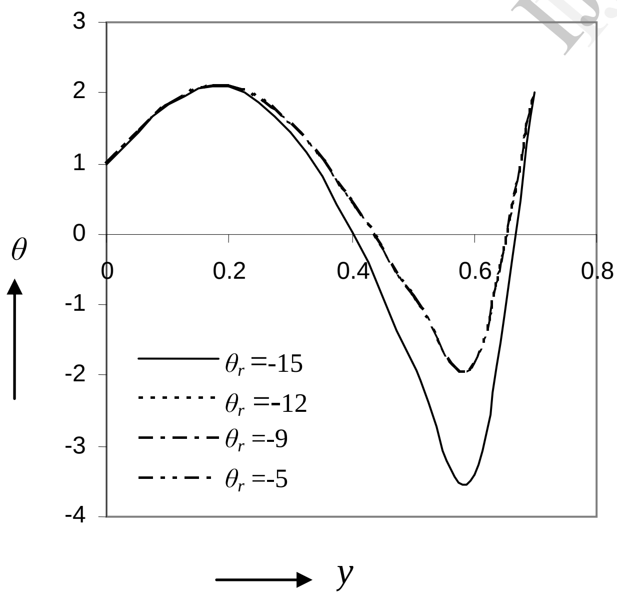

Figure 2. Temperature Profiles for θ_k=-15, P_r =0.7, M=0.5, G_r=5

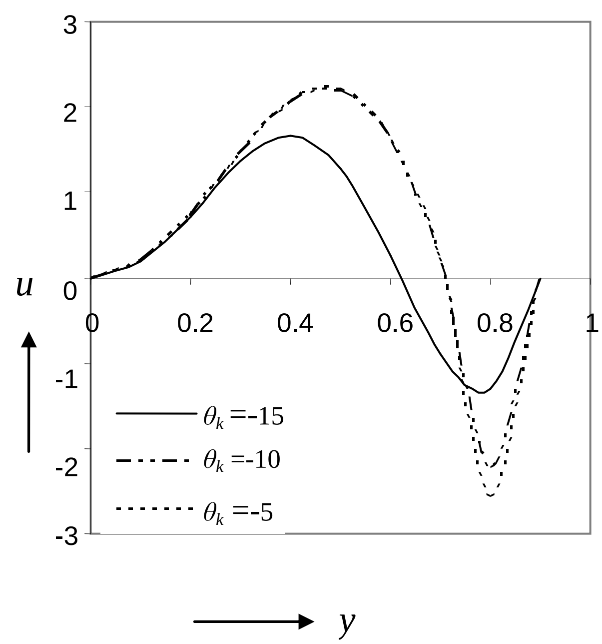

Figure 4. Velocity Profiles for θ_r =-15, P_r=0.7, M=0.5, G_r=5.

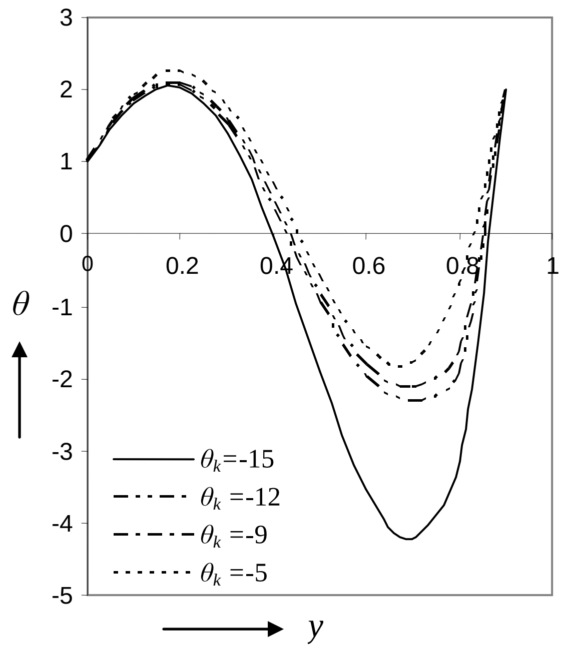

Figure 5. Temperature Profiles for θ_r=-15, P_r=0.7, M=0.5, G_r=5.

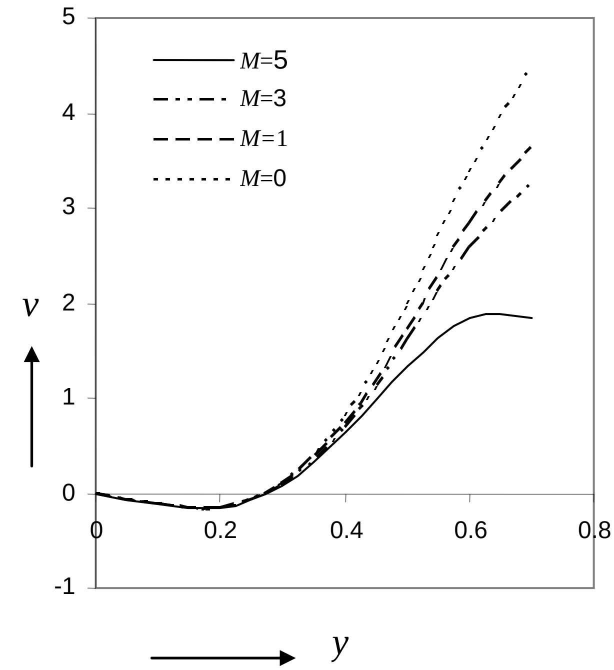

Figure 7. Cross flow Velocity Profiles θ_r=-15, θ_k=-15, P_r =0.7, G_r=5.

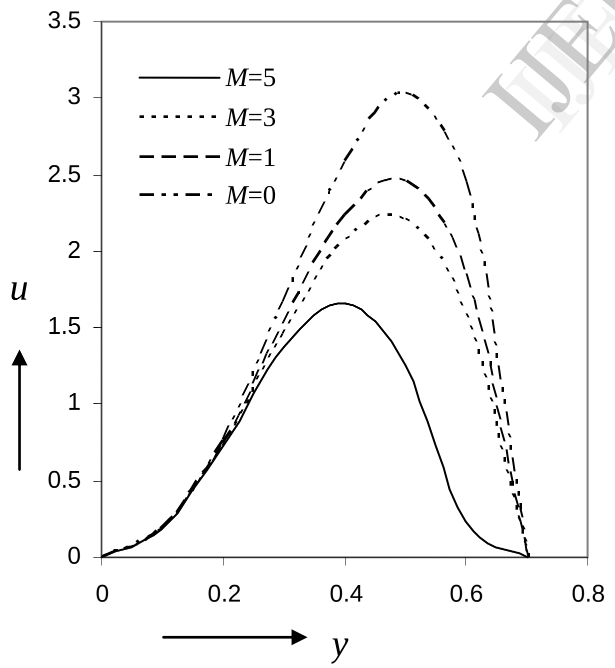

Figure 6. Velocity Profiles for θ_r=-15, P_r=0.7, M=0.5, G_r=5.

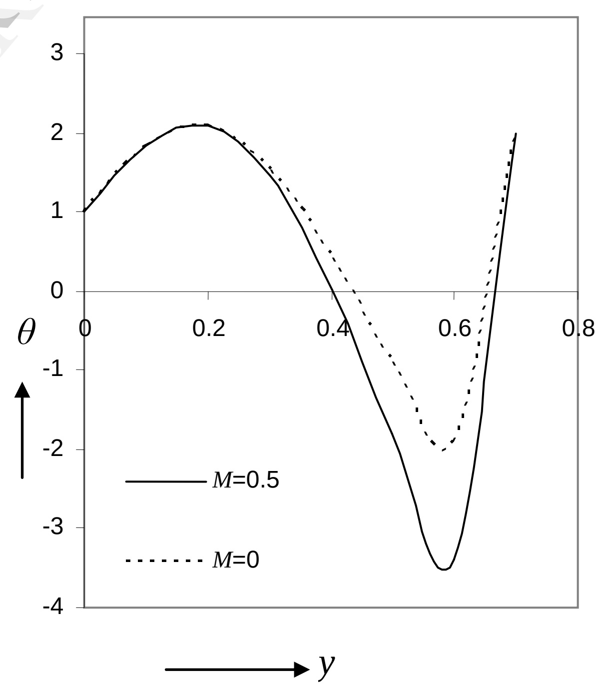

Figure 8. Temperature Profiles for θ_r=-15, θ_k=-15, P_r =0.7, G_r=5.

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