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Effects Of Radiation And Heat Source On Micropolar Flow With Heat And Mass Transfer Over A Vertical Porous Moving Plate

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ABSTRACT

We present an analytical study for the unsteady, coupled, hydrodynamic, heat and mass transfer with radiation and heat source of an incompressible micropolar fluid flowing past a semi- infinite vertical moving porous plate embedded in a porous medium. The governing differential equations are solved analytically using two–term harmonic and non–harmonic functions. The dimensionless translational velocity, microrotation, temperature and mass distribution function are computed for the different thermophysical parameters controlling the flow regime. All results are shown graphically. Additionally skin friction, Nusselt number and Sherwood number, which provide an estimate of the surface shear stress, rate of cooling of the surface and rate of mass transfer coefficient, respectively, are also computed.

Key words: Micropolar fluid, heat transfer, radiation, heat source, boundary layer.

INTRODUCTION

In numerous industrial transportation processes, convective heat and mass transfer take place simultaneously. Phenomena involving stretching sheets feature widely in for example, aerospace component production metal casting Dieter [1]. Unsteady free convective flows in a porous medium have received much attention in recent time due to its wide application in geothermal and oil reservour engineering as well as other geophysical and astrophysical studies. Moreover, considerable interest has been shown in radiation interaction with convection for heat and mass transfer in fluids. This is due to the significant role of thermal radiation in the surface heat transfer when convection heat is small, espercially in free convection problems involving absorbing – emitting fluids. The unsteady fluid flow past a moving plate in the presence of free convection and radiation were studied by Monsour [2], Cogley et al. [3], Raptis and Perdikis [4]; Das et al. [5], Grief et al. [6], Ganeasan and Loganathem [7], Mbeledogo et al. [8], Makinde [9] and Satter and Kalim [10]. All these studies have been confined to unsteady fluid flow, we observe that little work were done in a non- porous medium. The effect of radiation on MHD flow and heat treansfer must be considered when high temperatures are reached. El – Hakiem [11], studied the unsteady MHD oscillatory flow on free convection – radiation through a porous medium with a vertical infinite surface that absorbs the fluid with a constant velocity. Raptis et al. [12], studied the effect of radiation on two dimensional steady MHD optically thin gray gas flow along an infinite vertical plates taking into account the induced magnetic field. Cookey et al. [13], researched the influence of viscous dissipation and radiation on unsteady MHD free – convection flow past on infinite heated vertical plate in a porous medium with time – dependent suction. Singh and Dikshit [14], investigated the hydromagnetic flow past a continously moving semi–infinite plate at large suction. Kim [15], studied unsteady MHD convective heat transfer past a semi–infinite vertical porous moving plate.

The study of heat generation or absorption effects in moving fluid is important in view of several physical problems, such as fluids undergoing exothermic or endothermic chemical reactions. Possible heat generation effects may alter the temperature distribution and consequently, the particle deposition rate in nuclear reactors, electric chips and semiconductors wafers. Seddeek [16], studied the effects of chemical reaction, thermophoresis and variable viscosity on steady hydromagnetic flow with heat and mass transfer over a flat plate in the presence of heat generation/absorption. Patil and Kulkarmi [17], studied the effects of chemical reaction on free convection flow of a polar fluid through porous medium in the presence of internal heat generation. Radiation effects on unsteady MHD convection heat and mass transfer flow past a semi–infinite vertical permeable moving plate embedded in a porous medium was studied by Ramachandroprasad et al. [18]. Recently, Pal et al. [19], studied perturbation analysis of unsteady magnetohydrodynamic convection heat and mass transfer in a boundary layer slip flow past a vertical permeable plate with thermal radiation and chemical reaction.

All the above investigations were restricted to Newtonian or non- Newtonian fluids. In many enviromental and industrial flows the classical theory of Newtonian fluids is unable to explain the microfluid mechanical characteristics observed. Micropolar fluid are fluids with microstructure belonging to a class of complex fliuds with nonsymmerical stress tensor refered to as micromorphic fluids. Physically they represent many industrially important liquids consisting of randomly – oriented particles suspended in viscous medium. The theory of micropolar fluids and its extension, the thermomicropolar fluid constitute suitable non – Newtonian hydrodynamic and thermo – hydrodynamic models which can stimulate the flow dynamics of colloidal fluids, liquid crystals, polymeric suspension, haemotological fluids etc. Hassannien and Gorla (1990), investigated the heat transfer to a micropolar fluid from nonisothermal stretching sheet with suction and blowing. Flow over a porous stretching sheet with strong suction or injection was investigated by Kelson and Farell, (2001).

The purpose of the present paper is to study the effects of radiation and heat source on micropolar flow with heat and mass transfer over a vertical porous moving plate. It is assumed that the plate is embedded in a uniform porous medium and moves with a constant velocity in the flow direction in the presence of a transverse magnetic field. The governing system of conservation equations, which are solved by using a regular perturbation method. Graphs are plotted for dimensionless translational velocity, micro – rotational (angular velocity), temperature and mass transfer function for various values of the flow parameters. We also compute dimensionless wall shear stress function, heat transfer rate and mass transfer coefficient which have been discussed.

FORMULATION OF THE PROBLEM

Two dimensional unsteady, laminar, incompressible,viscous, electrically conducting and heat/absorption micropolar fluid flowing past a semi – infinite vertical moving porous plate embedded in a uniform porous medium and subjected to a uniform transverse magnetic field in the presence of a pressure gradient. According to the coordinate system the x – axis is taken along the porous plate in the upward direction and y – axis normal to it. The simplified two – dimensional equations governing the flow in the boundary of unsteady, laminar and incompressible micropolar fluid are:

Continuity equation:

$$\frac{\partial V^*}{\partial y^*} = 0 \tag{1}$$

Momentum equation

$$\frac{\partial u^*}{\partial t^*} + v^* \frac{\partial u^*}{\partial y^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial x^*} + (\nu + \nu_r)\frac{\partial^2 u^*}{\partial y^{*2}} + g\beta_f (T - T_\infty) - \nu \frac{u^*}{k^*} - \frac{\sigma}{\rho}\beta_0^2 u^* + 2\nu_r \frac{\partial \omega^*}{\partial y^*}$$

$$-K_0^1 \frac{\partial^3 u}{\partial t^* \partial y^{*2}} + g\beta^* (C - C\infty) \tag{2}$$

Angular Momentum equation

$$\rho j^* \left[\frac{\partial \omega^*}{\partial t^*} + v^* \frac{\partial \omega^*}{\partial y^*}\right] = \gamma \frac{\partial^2 \omega^*}{\partial y^{*2}} \tag{3}$$

Energy equation

$$\frac{\partial T}{\partial t^*} + v^* \frac{\partial T}{\partial y^*} = \alpha \frac{\partial^2 T}{\partial y^{*2}} + \frac{Q}{\rho C_p}(T^* - T_\infty^*) - \frac{1}{\rho C_p}\left(\frac{\partial q_r^*}{\partial y^*}\right) \tag{4}$$

Concentration equation

$$\frac{\partial C}{\partial t^*} + v^* \frac{\partial C}{\partial y^*} = D\frac{\partial^2 C}{\partial y^{*2}} - K^*(C^* - C_\infty^*) \tag{5}$$

The third and the eighth terms on the right hand side of equation (2) denote the thermal and concentration buoyancy effects respectively. Furthermore, the last term on the right hand of the energy equation (3) represents the radiative heat flux term. where x^*, y^* *and* t^* are the dimensional distances along and perpendicular to the plate and dimensional time, respectively. u^* *and* v^* are the components of dimensional velocities along x^* *and* y^* direction, ρ is the fluid density, σ is the fluid electrical conductivity, β_0 is the magnetic induction, j^* the micro-inertia density of the component of the angular velocity vector normal to the $xy-plane$,γ the spine gradient viscosity, v is the fluid kinematic viscosity, v_r is the fluid kinematic rotational viscosity, g is the acceleration due to gravity, β_f *and* β^* are the coefficients of volume expansions for temperature and concentration, k^* the permeability of the porous medium, k_0^1 is the elastic parameter, K is the chemical reaction, ω^* is the component of angular velocity, T is the temperature, C is the concentration , α is the fluid thermal diffusivity, and q_r^* is the local radiative heat flux. The term $Q\left(T^* - T_\infty^*\right)$ is assumed to be amount of heat generated or absorbed per unit volume, Q is a constant, which may take on either positive or negative values. When the wall temperature T^* exceeds the free stream temperature T_∞^* , the source term Q is greater than zero and heat sink when Q is less than zero.

The boundary conditions for the velocity, temperature and concentration fields are:

$$\left.\begin{aligned}
&u^* = u_p^*, v^* = -v_0\left(1+\varepsilon e^{n^* t^*}\right), T = T_\omega + \varepsilon\left(T_\omega - T_\infty\right)e^{n^* t^*},\\
&\frac{\partial \omega^*}{\partial y^*} = -\frac{\partial^2 u}{\partial y^{*2}}, C = C_\omega + \varepsilon\left(C_\omega - C_\infty\right)e^{n^* t*} \quad at\ y = 0\\
&u^* \to u_\infty^* = u_0\left(1+\varepsilon e^{n^* t*}\right), T \to T_\infty, C \to C_\infty,\\
&\omega^* \to 0 \quad as \quad y^* \to \infty
\end{aligned}\right\} \tag{6}$$

Where u_p^* is the velocity of the moving plate, T_ω^* *and* C_ω^* are the temperature and the concentration respectively, U_∞^* is the free stream velocity, and U_0 *and* n^* are constants. From equation (2), it is obvious that the suction velocity at the plate is either a constant or a function of time. Hence the suction velocity normal to the plate is assumed in the form

$$v^* = -V_0\left(1+\varepsilon A e^{n^* t^*}\right) \tag{7}$$

Where A is a real positive constant, ε *and* εA is small values less than unity, and V_0 is the scale of suction velocity which is non-zero positive constant. The negative sign indicates that the suction is towards the plate.

Outside the boundary layer, equation (2) gives

$$-\frac{1}{\rho}\frac{\partial p^*}{\partial x^*}=\frac{dU_\infty^*}{dt^*}+\frac{v^*}{k^*}U_\infty^*+\frac{\sigma}{\rho}\beta_0^2U_\infty^* \tag{8}$$

Eliminating $\frac{\partial p^*}{\partial x^*}$ between equation (2) and equation (8), we obtain

$$\frac{\partial u^*}{\partial t^*}+v^*\frac{\partial u^*}{\partial y^*}=\frac{dU_\infty^*}{dt^*}+\frac{v}{k}U_\infty^*+\frac{\sigma}{\rho}\beta_0^2U_\infty^*+\left(v+v_r\right)\frac{\partial^2u^*}{\partial y^{*2}}+g\beta_f\left(T-T_\infty\right)-$$

$$v\frac{u^*}{k_0^{*1}}-\frac{\sigma}{\rho}\beta_0^2u^*+2v_r\frac{\partial\omega^*}{\partial y^*}-K_0^1\frac{\partial^3u^*}{\partial t^*\partial^{*^2}}+g\beta^*\left(C-C^*\right) \tag{9}$$

Where $v=\frac{\mu}{\rho}$ is the coefficient of the kinematic viscosity. The fifth term on the RHS of the momentum equation (8) denote body force due to nonuniform temperature, the last term denote body force due to nonuniform concentration.

The governing boundary layer equation with temperature dependent heat generation/absorption, constant magnetic field and radiation is given by equation (4), by using Rosseland approximation the radiation heat flux is given by

$$q_r^*=-\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \tag{10}$$

Where σ^* *and* k^* respectively, the stephan – Boltzmann constant and mean absorption coefficient. We assume that the temperature difference within the flow is such that T^4 may be expand in a Taylor series. Hence, expanding T^4 about T_∞ and neglecting higher order terms we get

$$T^4\cong 4T_\infty^3T-3T_\infty^4 \tag{11}$$

Now using equations (10) and (11) , equation (4) becomes

$$\frac{\partial T}{\partial t^*}+V\frac{\partial T}{\partial y^*}=\alpha\frac{\partial^2T}{\partial y^{*2}}+\frac{Q}{\rho C_p}\left(T^*-T_\infty^*\right)+\frac{16\sigma^*T_\infty^*}{3k^*}\frac{\partial^2T}{\partial y^2} \tag{12}$$

To write the governing equations and the boundary conditions in dimensionless form, the following non – dimensional quantities are introduced

$$\left.\begin{array}{l} u=\dfrac{u^*}{U_0}, v=\dfrac{v^*}{V_0}, v=\dfrac{V_0 y^*}{v}, U_\infty=\dfrac{U_\infty{}^*}{U_0}, U_p=\dfrac{u_p{}^*}{U_0}, v=\dfrac{v}{U_0 V_0}\omega^*, v=\dfrac{t^* V_0^2}{V_0}, \\ \theta=\dfrac{T-T_\infty}{T_\omega-T_\infty}, C=\dfrac{C-C_\infty}{C_\omega-C_\infty}, \theta=\dfrac{T-T_\infty}{T_\omega-T_\infty}, n=\dfrac{n^* v}{V_0^2}, k_0^1=\dfrac{k_0{}^{*1} V_0^2}{V_0^2}, j=\dfrac{V_0^2 j^*}{V_0^2}, \\ n=\dfrac{v}{\alpha}, M=\dfrac{\sigma\beta_0^2 v}{\rho V_0^2}, Gr=\dfrac{v\beta_f g\left(T_\omega-T_\infty\right)}{U_0 V_0^2}, Gc=\dfrac{v\beta^* g\left(T_\omega-T_\infty\right)}{U_0 V_0^2}, N=M+\dfrac{1}{k} \\ \eta=\dfrac{\mu j^*}{\gamma}=\dfrac{2}{2+\beta}, K_r=\dfrac{K^* v}{V_0^2} \end{array}\right\} \tag{13}$$

Then substituting from equation (13) into equations (9), (3) , (12) and (5) and taking into account equation (7) we obtain

$$\frac{\partial u}{\partial t}-\left(1+\varepsilon Ae^{nt}\right)\frac{\partial u}{\partial y}=\frac{dU_\infty}{dt}+\left(1+\beta\right)\frac{\partial^2 u}{\partial y^2}+Gr\theta+N\left(U_\infty-u\right)+$$

$$2\beta\frac{\partial\omega}{\partial y}-K\frac{\partial^3 u}{\partial t\partial y^2}+GcC \tag{14}$$

$$\frac{\partial\omega}{\partial t}-\left(1+\varepsilon Ae^{nt}\right)\frac{\partial\omega}{\partial y}=\frac{1}{\eta}\frac{\partial^2\omega}{\partial y^2}$$
(15)

$$\frac{\partial\theta}{\partial t}-\left(1+\varepsilon Ae^{nt}\right)\frac{\partial\theta}{\partial y}=z\frac{\partial^2\theta}{\partial y^2}+S\theta \tag{16}$$

$$\frac{\partial C}{\partial t}-\left(1+\varepsilon Ae^{nt}\right)\frac{\partial C}{\partial y}=\frac{1}{Sc}\frac{\partial^2 C}{\partial y^2}-KrC \tag{17}$$

Where $Gr=\dfrac{v\beta_f g\left(T_\omega-T_\infty\right)}{U_0 V_0^2}$ is the thermal Grashof number, $Gc=\dfrac{v\beta^* g\left(C_\omega-C_\infty\right)}{U_0 V_0^2}$ is the solutal Grashof number, $M=\dfrac{\sigma\beta_0^2 v}{\rho V_0^2}$ is the magnetic field parameter, $S=\dfrac{vQ}{\rho V_0^2 C_p}$ is the dimensionless heat generation/absorption coefficient, $Pr=\dfrac{v\rho C_p}{\alpha}$ is the Prandtl number, $R=\dfrac{4\sigma^* T_\infty^{*3}}{k^*\alpha}$ is the thermal radiation parameter, $K_r = \frac{k^* v}{V_0^2}$ is the chemical reaction parameter, $Gr = \frac{v}{D}$ is the Schmidt number and $N = M + \frac{1}{k}$.

The dimensionless form of the boundary conditions (6) becomes

$$\left.\begin{aligned} & u = U_p, v = -\left(1+\varepsilon A e^{nt}\right), \theta = 1+\varepsilon A e^{nt}, C = 1+\varepsilon A e^{nt}, \\ & \qquad \frac{\partial \omega}{\partial y} = -\frac{\partial^2 u}{\partial y^2} \quad at \;\; y = 0 \\ & u \to U_\infty, \theta \to 0, \omega \to 0, C \to 0, \qquad as \quad y \to \infty \end{aligned}\right\} \tag{18}$$

ANALYTICAL SOLUTIONS

In order to reduce the above system of partial differential equations to a system of ordinary equations in dimentionless form, we may represent the velocity, angular velocity, temperature and concentration as

$$u = u_0\left(y\right) + \varepsilon e^{nt} u_1\left(y\right) + 0\left(\varepsilon^2\right), \tag{19}$$

$$\omega = \omega_0\left(y\right) + \varepsilon e^{nt} \omega_1\left(y\right) + 0\left(\varepsilon^2\right), \tag{20}$$

$$\theta = \theta_0\left(y\right) + \varepsilon e^{nt} \theta_1\left(y\right) + 0\left(\varepsilon^2\right), \tag{21}$$

$$C = C_0\left(y\right) + \varepsilon e^{nt} C_1\left(y\right) + 0\left(\varepsilon^2\right), \tag{22}$$

By substituting the above equations (19)–(22) into equations (14)–(17) and equating the harmonic and non–harmonic terms and neglecting the higher–order terms of $0\left(\varepsilon^2\right)$. We obtain the following pairs of equations for $\left(u_0, \omega_0, \theta_0, C_0\right)$ and $\left(u_1, \omega_1, \theta_1, C_1\right)$.

$$\left(1+\beta\right) u_0'' + u_0' - N u_0 = -N - Gr\theta_0 - GcC_0 - 2\beta\omega_0' \tag{23}$$

$$E u_1'' + u_1' - \left(N+n\right) u_1 = -\left(N+n\right) - A u_0' - Gr\theta_1 - GcC_1 - 2\beta\omega_1' \tag{24}$$

$$\omega_0'' + \eta\omega_0' = 0 \tag{25}$$

$$\omega_1'' + \eta\omega_1' - n\eta\omega_1 = -A\eta\omega_0' \tag{26}$$

$$Z\theta_1'' + \theta_1' - \left(n-S\right)\theta_1 = -A\theta_0' \tag{27}$$

$$Z\theta_1''+\theta_1'-\left(n-S\right)\theta_1=-A\theta_0' \tag{28}$$

$$C_0''+ScC_0'-k_rScC_0=0 \tag{29}$$

$$C_1''+ScC_1'-\left(n+k_r\right)ScC_1=-AScC_0' \tag{30}$$

Where primes denotes ordinary differentiation with respect to y

The corresponding boundary conditions are

$$\left.\begin{array}{l} u_0=u_p, u_1=0, \omega_0=-u'', \omega_1=-u_1'', \theta_0=1, \theta_1=1, C_0=1, C_1=1 \ \ at \ \ y=0 \\ u_0\to 1, u_1\to 1, \omega_0\to 0, \omega_1\to 0, \theta_0\to 0, \theta_1\to 0, C_0\to 0, C_1\to 0 \ \ as \ \ y\to\infty \end{array}\right\} \tag{31}$$

The analytical solutions of equations (23)–(30) stisfying boundary conditions (31) are given by

$$u\left(y,t\right)=1+B_4e^{-m_2y}+B_6e^{-Pry}+B_7e^{-m_1y}+B_8e^{-\eta y}+ \varepsilon e^{nt}\left\{\begin{array}{l}1+B_{15}e^{-m_7y}+B_{18}e^{-Pry}+B_{21}e^{-m_5y}+B_{23}e^{-m_6y}+ \\ \qquad B_{25}e^{-m_4y}+B_{27}e^{-m_2y}+B_{28}e^{-m_1y}+B_{29}e^{-\eta y}\end{array}\right\} \tag{32}$$

$$\omega\left(y,t\right)=B_1e^{-\eta y}+\varepsilon e^{nt}\left(B_9e^{-m_4y}+B_{10}e^{-\eta y}\right) \tag{33}$$

$$\theta\left(y,t\right)=B_2e^{-m_1y}+\varepsilon e^{nt}\left(B_{11}e^{-m_5y}+B_{12}e^{-m_1y}\right) \tag{34}$$

$$C\left(y,t\right)=B_3e^{-m_2y}+\varepsilon e^{nt}\left(B_{13}e^{-m_6y}+B_{14}e^{-m_2y}\right) \tag{35}$$

Where

$$m_1=\frac{1}{2Z}\left(1+\sqrt{1-4SV}\right), \qquad m_2=\frac{1}{2}\left(Sc+\sqrt{S^2c+4ScK_r}\right),$$

$$m_3=\frac{1}{2(1+\beta)}\left(1+\sqrt{1+4N\left(1+\beta\right)}\right),$$

$$m_4=\frac{\eta}{2}\left(1+\sqrt{1+\frac{4n}{\eta}}\right), \qquad m_5=\frac{1}{2Z}\left(1+\sqrt{1+4Z\left(n-S\right)}\right), \qquad m_6=\frac{Sc}{2}\left(1+\sqrt{1+\frac{4\left(n+K_r\right)}{Sc}}\right),$$

$$m_7=\frac{1}{2E}\left(1+\sqrt{1+4\left(N+n\right)E}\right), \quad B_1=-U_p, \quad B_2=1, \quad B_5=1, \quad B_6=\frac{-Gr}{\left(1+\beta\right)P^2r-Pr-N},$$

$$B_7 = \frac{-Gc}{(1+\beta)m_1^2 - m_1 - N}, \qquad B_8 = \frac{2B_1\beta\eta}{(1+\beta)\eta^2 - \eta - N}, \qquad B_4 = U_p - (1 + B_6 + B_7 + B_8),$$

$$B_{10} = \frac{A\eta B_1}{n}, \quad B_9 = -(u_1 + B_{10}), \qquad B_{12} = \frac{Am_1}{Zm_1^2 - m_1 - (n-S)}, \qquad B_{11} = 1 - B_{12},$$

$$B_{14} = \frac{Am_2Sc}{m_2^2 - m_2Sc - (n+K_r)Sc}, \quad B_{13} = 1 - B_{14}, \quad B_{16} = 1, \quad B_{17} = \frac{AB_4m_2}{Em_2^2 - m_2 - (n-S)},$$

$$B_{18} = \frac{AB_6Pr}{EP^2r - Pr - (N+n)}, \qquad B_{19} = \frac{AB_7m_1}{Em_1^2 - m_1 - (N+n)}, \qquad B_{20} = \frac{AB_8\eta}{E\eta^2 - \eta - (N+n)},$$

$$B_{21} = \frac{-GcB_{11}}{Em_5^2 - m_5 - (N+n)}, \qquad B_{22} = \frac{-GrB_{12}}{Em_1^2 - m_1 - (N+n)}, \qquad B_{23} = \frac{-GcB_{13}}{Em_6^2 - m_6 - (N+n)},$$

$$B_{24} = \frac{-GcB_{14}}{Em_2^2 - m_2 - (N+n)}, \qquad B_{25} = \frac{2B_9\beta m_4}{Em_4^2 - m_4 - (N+n)}, \qquad B_{26} = \frac{2B_{10}\beta\eta}{E\eta^2 - \eta - (N+n)},$$

$$B_{15} = -(1 + B_{17} + B_{18} + B_{19} + B_{20} + B_{21} + B_{22} + B_{23} + B_{24} + B_{25} + B_{26}), \qquad B_{27} = B_{17} + B_{24},$$

$$B_{28} = B_{19} + B_{22}, \qquad B_{29} = B_{20} + B_{26}.$$

Skin friction

Using equation (32), the skin friction at the wall is given by

$$C_f = -\{m_2B_4 + \Pr B_6 + m_1B_7 + \eta B_8 + \varepsilon e^{nt}[m_7B_{15} + \Pr B_{18} + m_5B_{21} + m_6B_{23} + m_4B_{25} + m_2B_{27} + m_1B_{28} + \eta B_{29}]\} \tag{36}$$

Nusselt number

Similarly, Nusselt number from (34) is given by

$$N_u = -m_1B_2 - \varepsilon e^{nt}(m_5B_{11} + m_1B_{12}) \tag{37}$$

Sherwood number

Also the sherwood number from (35) gives

$$S_h = -m_2B_3 - \varepsilon e^{nt}(m_6B_{13} + m_2B_{14}) \tag{38}$$

RESULTS AND DISCUSSION

The problem of effects of radiation and heat source on micropolar flow with heat and mass transfer over a vertical porous moving plate in the presence of variable suction has been formulated, analysed and solved by using multi – parameter perturbation technique. Approximate solutions has been derived for the mean velocity, angular velocity, mean temperature and mean concentration. The effects of the flow parameters such as Hartmann number (M), Schmidt number (Sc), Prandtl number (Pr), Grashof number for heat and mass transfer (Gr, Gc), heat source (S), porosity parameter (K_0), chemical reaction parameter (K_r), and radiation parameter (R) on the mean velocity, angular velocity, mean temperature and mean concentration profiles of the flow field are presented with help of mean velocity profiles (Figures 1–12), angular velocity profiles (figures 13–15), mean temperature profiles (Figures 16–18) and concentration profiles (Figures 19–20).

Figures 1–3 represent the mean velocity profiles due to variations in M (magnetic parameter), Sc (Schmidt number) and Pr (Prandtl number) . It is observed that the mean velocity decreases with increase of Magnetic parameter. It is also observed that the increases in Schmidt number and Prandtl number causes the decrease in mean velocity.

Figures 4–9 represent the mean velocity profiles due to variations in Gr (Thermal Grashoff number), Gc (solutal Grashoff number), S (heat source parameter), K_0 (permeability parameter), K_r (chemical reaction parameter) and R (radiation parameter). It is obsreved that the mean velocity increases with increase of thermal Grashoff number, solutal Grashoff number, heat source parameter and permeability parameter whereas it decreases with increase in chemical reaction parameter and radiation parameter.

Figures 10–15 reveals the mean velocity profiles due to variations in U_p (velocity of the moving porous plate), ε (epsilon) and t (time). It is noticed that whenever the velocity of the moving porous plate, epsilon and time increases causes the increase in mean velocity.

Figures 13–15 are graphed to see the influence of $U_p, \varepsilon \text{ and } t$ on micro – rotation velocity (angular velocity) ω. It is noticed that whenever $U_p, \varepsilon \text{ and } t$ increases causes the decrease in micro – rotation velocity.

Figures 16–18 reveals the mean temperature profiles due to variations in Pr (Prandtl number), R (radiation parameter) and S (heat source parameter). It is noticed that whenever Prandtl number and radiation parameter increases the mean temperature decreases whereas the mean temperature increases with increase in heat source parameter.

Figures 19 and 20 elucidate the effects of Sc (Schmidt number) and K_r (chemical reaction parameter) on the mean concentration profiles of the flow field. It is noticed that the increases in Schmidt number and chemical reaction parameter cause the decrease in mean concentration.

Table 1: The effects of the Schmidt number Sc, Prandtl number Pr, and radiation parameter R, on the skin–friction coefficients C_f, Nusselt number N_u and the Sherwood number Sh are given in table (2). It is seen from the table that as Sc increases the skin–friction coefficients decreases and Sherwood number increases whereas the Nusselt number remains unchanged. However, as the Prandtl number effects increase, the skin–friction coefficients, Nusselt number and Sherwood number increases. Also, increases in the heat generation effects result in increase in skin–friction coefficient, Nusselt number and Shrewood number.

Table 2: The effects of heat source parameter S, chemical reaction parameter K_r and permeability parameter K_0, on the skin–friction coefficient C_f, Nusselt number N_u and the Sherwood number Sh, are given in table 2. It is observed from this table that as S increases, the skin–friction coefficients, the Nusselt number and the Sherwood number decrease. However, as the chemical reaction parameter effects increase the skin–friction coefficient decreases and the Sherwood number increases whereas the Nusselt number remains unchanged. Increases in the permeability effects increase the skin–friction coefficient whereas the Nusselt number and the Sherwood number remains unaltered.

Table 3: The numerical values of the skin–friction coefficients C_f, Nusselt number N_u and the Sherwood number Sh for different values of the thermal Grashof number Gr, solutal Grashof number Gc and magnetic field parameter M are shown in table 3. It can be noticed from this table that an increasing in Gr *and* Gc lead to an increasing in the value of the skin–friction coefficients, whereas the Nusselt number and Sherwood number remains unchanged. However, as M increases, the skin–friction coefficient decreases whereas the Nusselt number and Sherwood number remains unchanged.

Table 1: Shows the effects of Schmidt number Sc, Prandtl number Pr and radiationparameter R on skin - friction coefficient C_f , Nusselt number N_u and Sherwood number Sh

Sc	Pr	R	Cf	Nu	Sh
0.2	0.71	0.1	0.2891	0.8139	0.8017
0.3	0.71	0.1	0.1362	0.8139	0.8049
0.4	0.71	0.1	0.0136	0.8139	0.8079
0.2	0.81	0.1	0.3996	0.9282	0.9111
0.2	0.91	0.1	0.5199	1.0431	1.0198
0.2	1.01	0.1	0.6402	1.1484	1.1173
0.2	0.71	0.2	0.3240	0.9093	0.8931
0.2	0.71	0.3	0.3619	1.0051	0.9841

0.2	0.71	0.4	0.4041	1.1019	1.0747

Table 2: Shows the effects of heat source parameter S, chemical reaction parameter K_r and permeability parameter R on skin - -friction coefficient C_f , Nusselt number N_u and Sherwood number Sh

S	K_r	K_0	C_f	N_u	Sh
0.02	0.10	0.50	0.2891	0.8139	0.8017
0.03	0.10	0.50	0.2867	0.8071	0.7950
0.04	0.10	0.50	0.2843	0.8002	0.7881
0.02	0.20	0.50	0.2161	0.8139	0.8021
0.02	0.30	0.50	0.1567	0.8139	0.8025
0.02	0.40	0.50	0.1054	0.8139	0.8028
0.02	0.10	0.60	0.3418	0.8139	0.8017
0.02	0.10	0.70	0.3877	0.8139	0.8017
0.02	0.10	0.80	0.4280	0.8139	0.8017

Table 3: Shows the effects of thermal Grashof number Gr, solutal Grashof number Gc and magnetic field parameter M on skin - friction coefficient C_f , Nusselt number N_u and Sherwood number Sh

Gr	Gc	M	C_f	N_u	Sh
2.0	1.0	1.0	0.2891	0.8139	0.8017
3.0	1.0	1.0	0.4269	0.8139	0.8017
4.0	1.0	1.0	0.5647	0.8139	0.8017
2.0	2.0	1.0	0.4519	0.8139	0.8017
2.0	3.0	1.0	0.6147	0.8139	0.8017
2.0	4.0	1.0	0.7775	0.8139	0.8017
2.0	1.0	2.0	0.1808	0.8139	0.8017
2.0	1.0	3.0	0.1141	0.8139	0.8017
2.0	1.0	4.0	0.0686	0.8139	0.8017

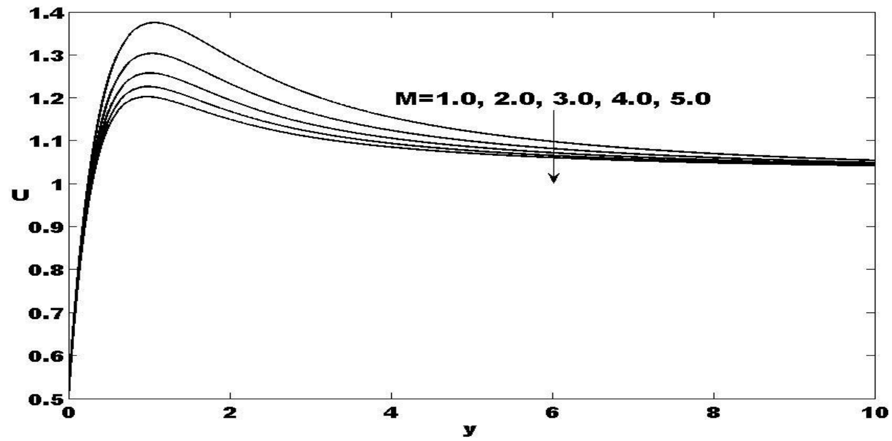

Fig 1: Effects of M on velocity profile

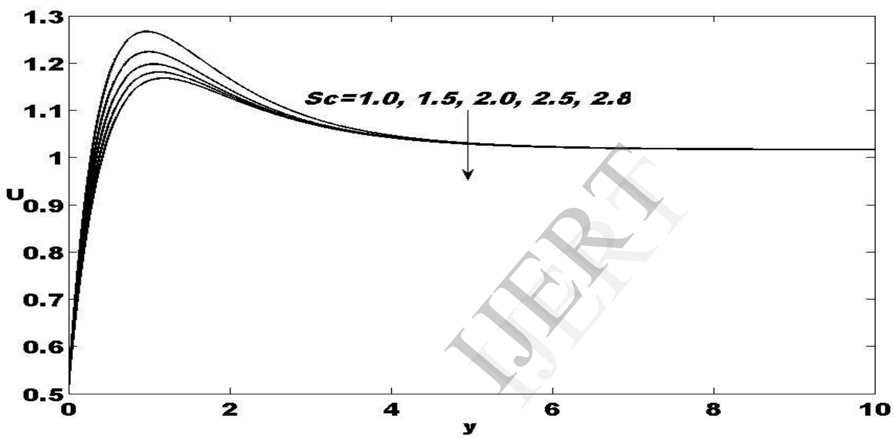

Fig 2: Effects of Sc on velocity profile

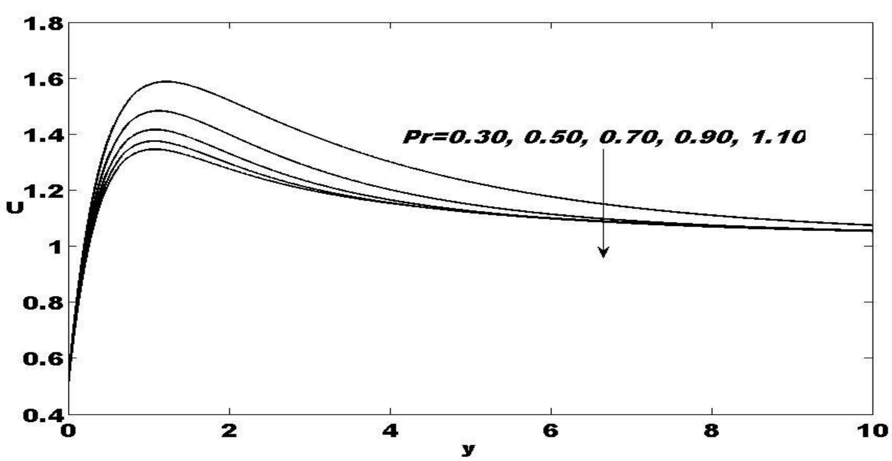

Fig 3: Effects of Pr on velocity profile

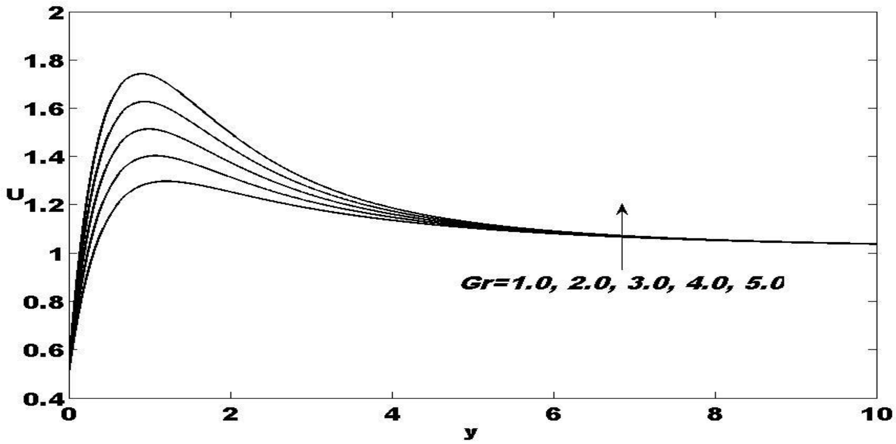

Fig 4: Effects of *Gr* on velocity profile

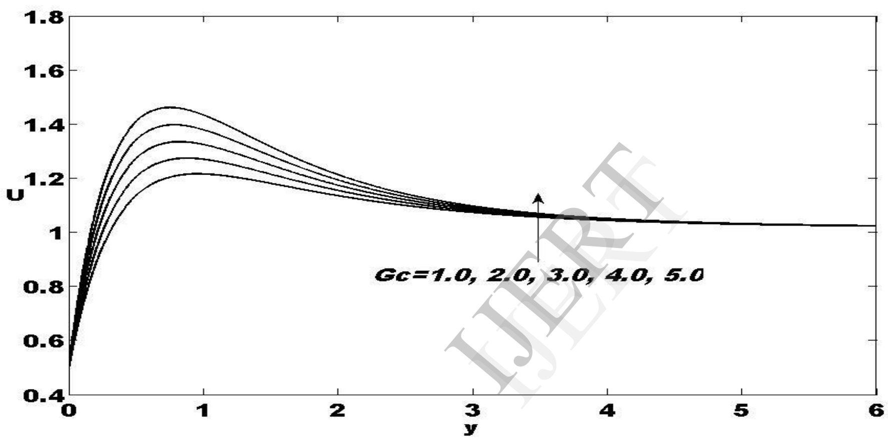

Fig 5: Effects of *Gc* on velocity profile

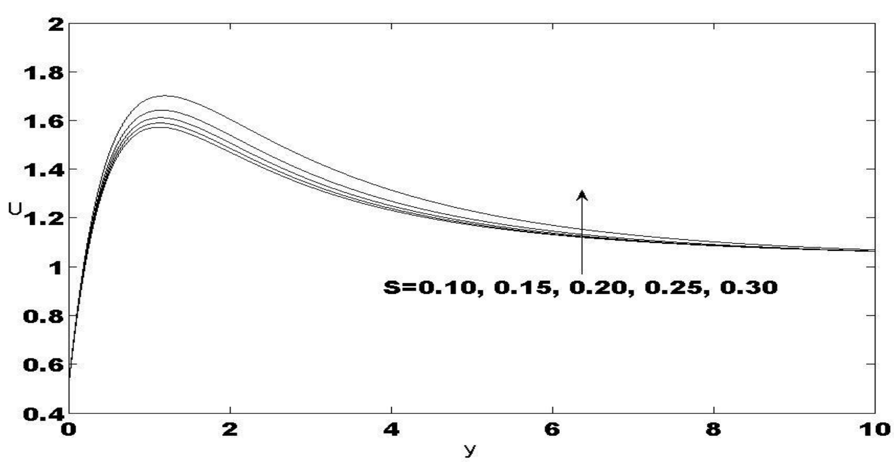

Fig 6: Effects of *S* on velocity profile

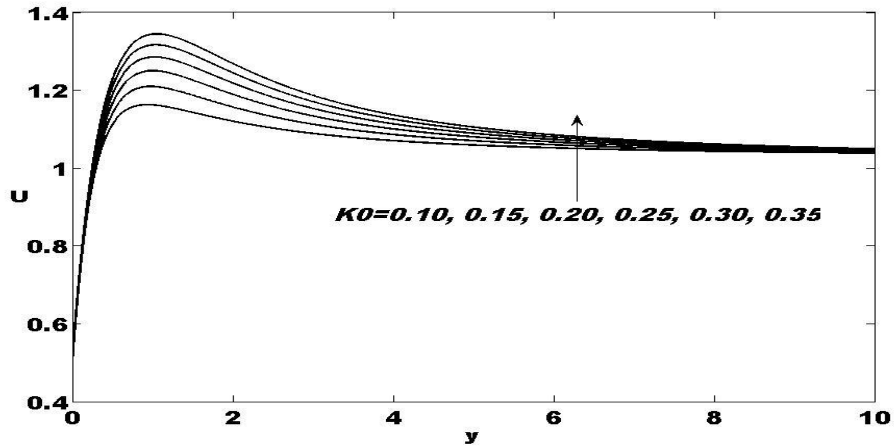

Fig 7: Effects of K_0 on velocity profile

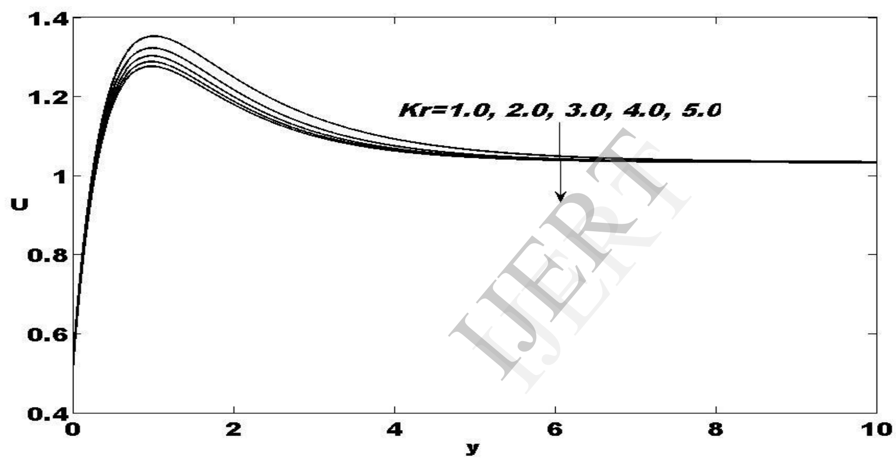

Fig 8: Effects of K_r on velocity profile

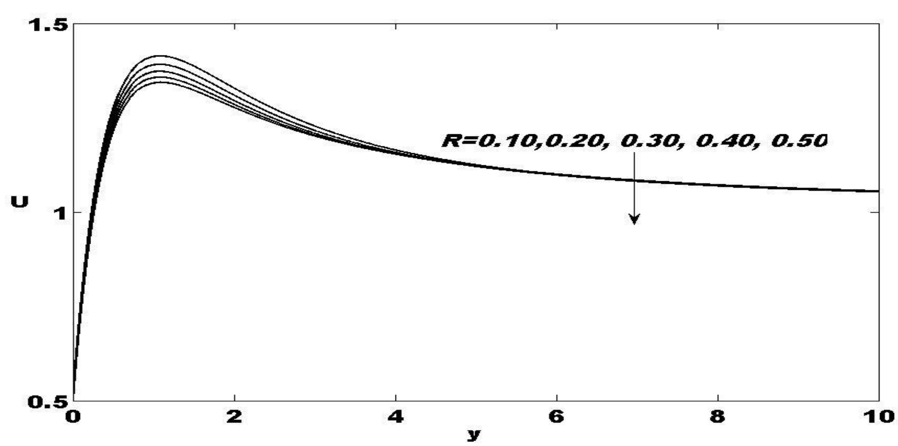

Fig 9: Effects of R on velocity profile

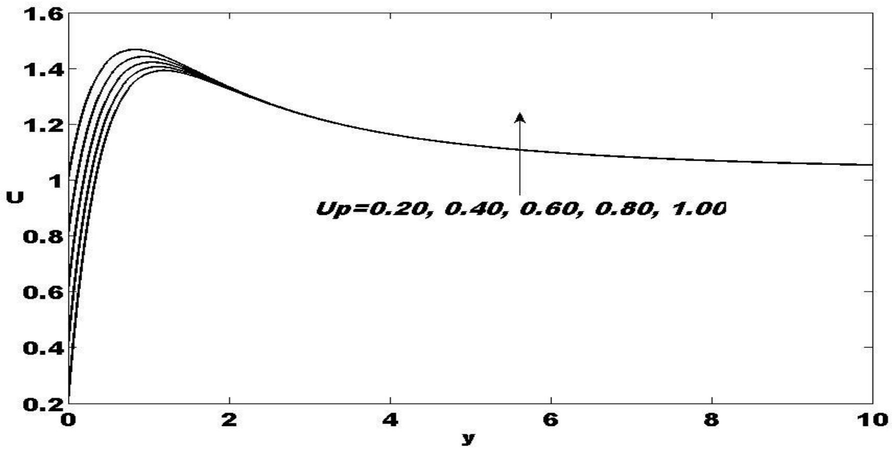

Fig 10: Effects of U_p on velocity profile

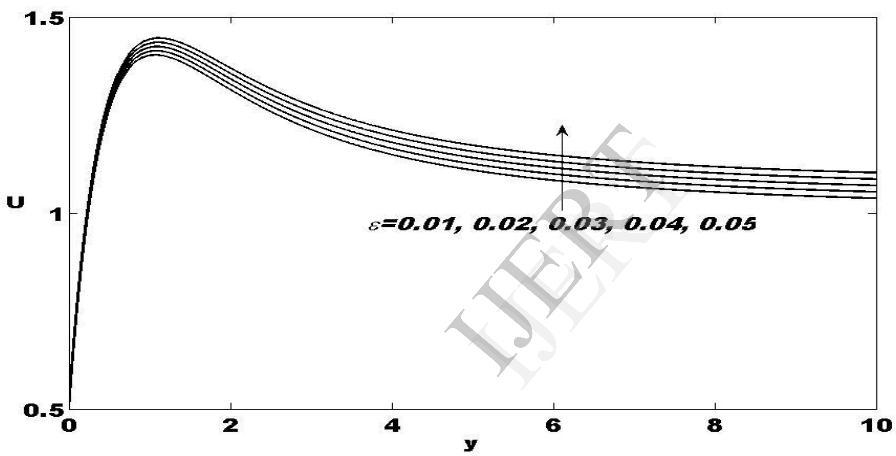

Fig 11: Effects of ε on velocity profile

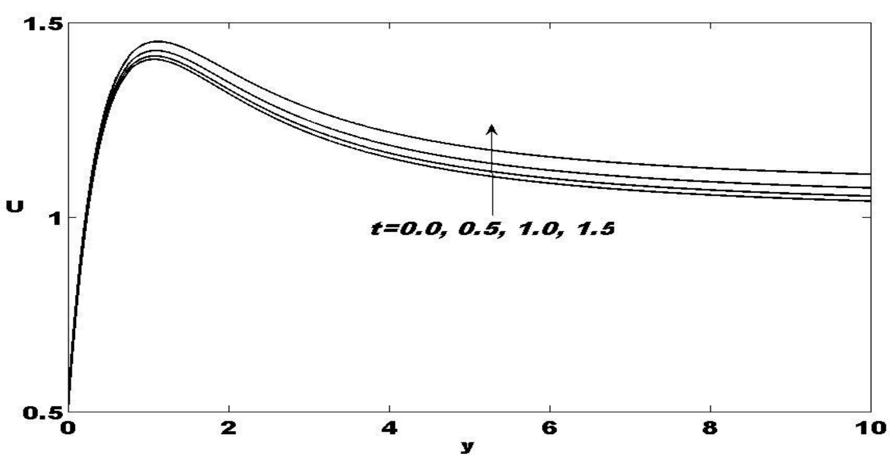

Fig 12: Effects of t on velocity profile

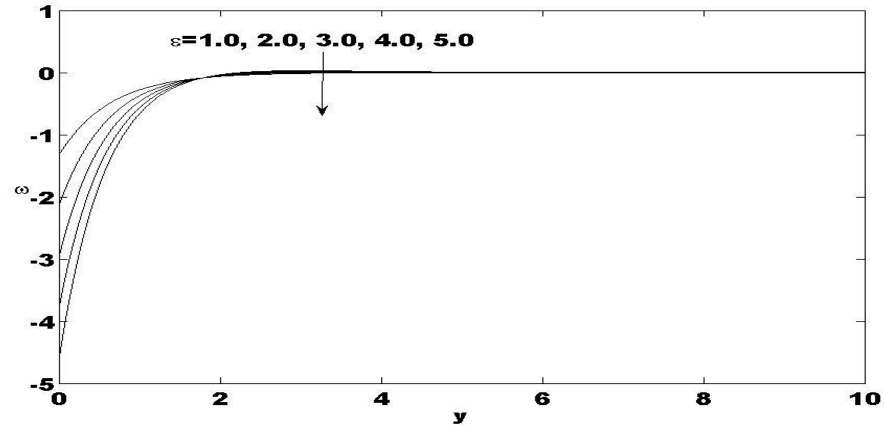

Fig 13: Effects of ε on angular velocity profile

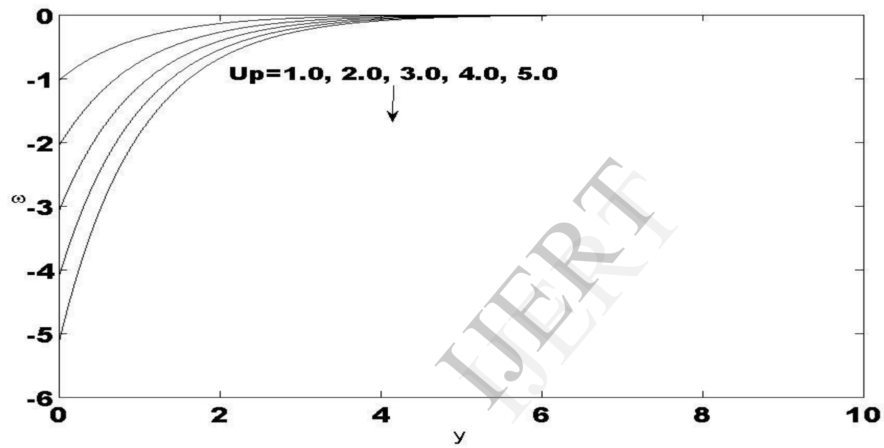

Fig 14: Effects of U_p on angular velocity profile

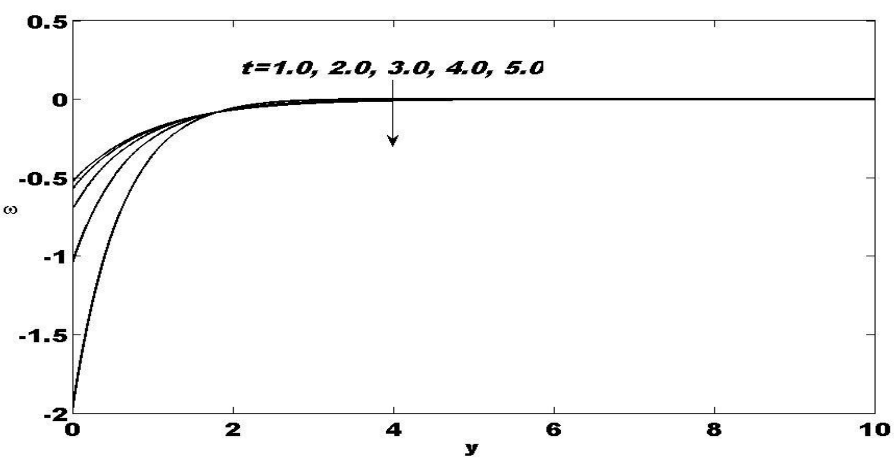

Fig 15: Effects of t onangular velocity profile

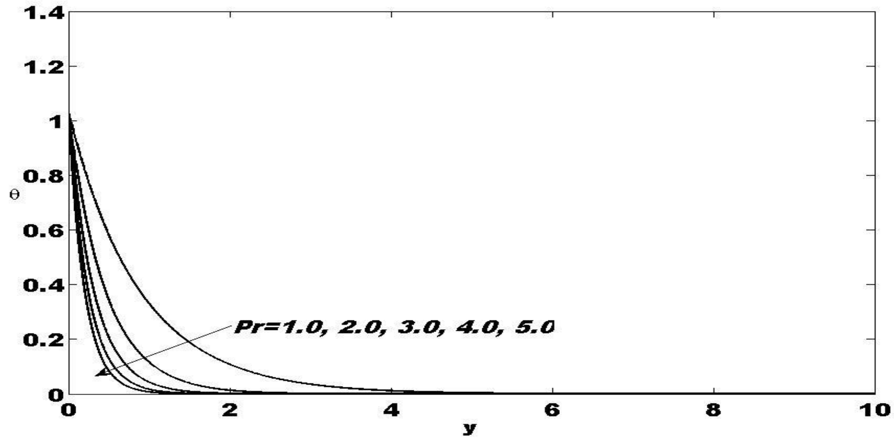

Fig 16: Effects of *Pr* ontemperature profile

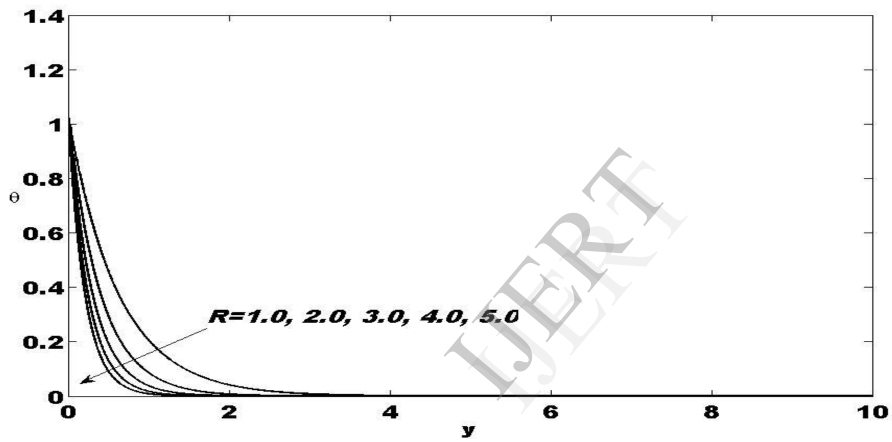

Fig 17: Effects of *R* on temperature profile

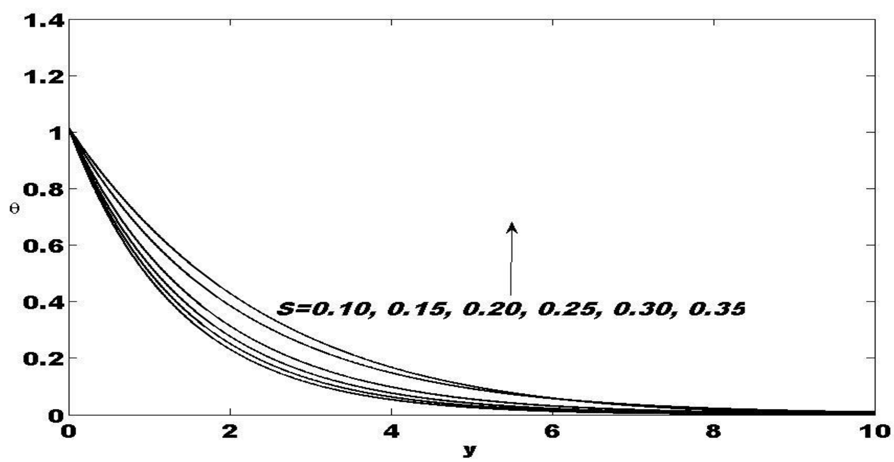

Fig 18: Effects of *S* on temperature profile

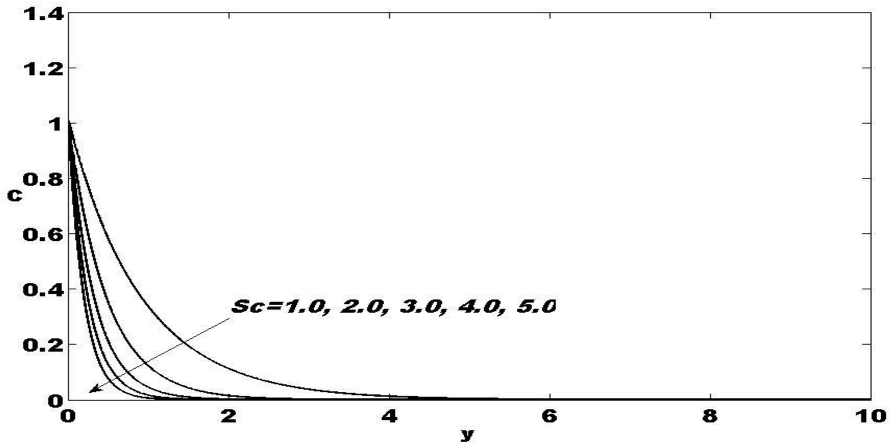

Fig 19: Effects of Sc on concentration profile

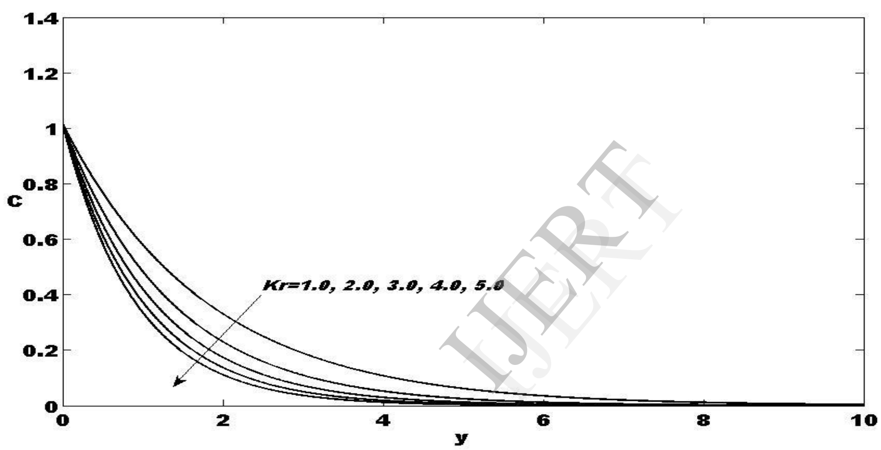

Fig 20: Effects of K_r on concentration profile

CONCLUSION: We have examined the governing equations for effects of radiation and heat source on micropolar flow with heat and mass transfer over a vertical porous moving plate. Analytical solution for the governing momentum, micro–rotation, energy and diffusion equations was obtained which allows the computation of the flow and heat transfer characteristics and their dependence on the material parameters. It is observed that the velocity decreases with an increasing magnetic field parameter, Schmidt number, Prandtl number, chemical reaction parameter and radiation parameter, however, an increase in the thermal Grashof number, solutal Grashof number, heat source parameter and permeability parameter results to an increase in velocity. The micro-rotation velocity decreases with an increasing epsilon, velocity of the moving porous plate and time respectively. The temperature field decreases with an increasing Prandtl number and radiation parameter whreas it increases with an increasing heat source parameter. Finally, the concentration profile decreases with an incresing Schmidt number and chemical reaction parameter respectively.

REFERENCES

[1] Dieter, G.E. (1986), *"Mechanical Metallurgy*", New York, McCraw-Hill

[2] Mansour M.A. (1990), " Radiation and Free Convection effects on the Oscillating flow past a vertical plate", *Astrophys. Space Sci.* 166, Pp. 269-275.

[3] Cogley, A.C., Vincenti, W.C. and Gilles, S.E. (1968), " Differential approximation for radiation transfer in a non-gray gas near equilibrium", *Am. Inst. Aeronout, Astronout. J.* 6, Pp. 551-555.

[4] Raptis, A., Perdkis, C. and Leontitsis, A., (2003), " Effects of radiation in an optically thin gray gas flowing past a vertical infinite plate in the presence of a magnetic field", *Heat Mass Transfer,* 39, Pp. 771-773.

[5] Das, U.N., Deka, R., and Soundalgekar, V.M., (1966), " Radiation effect on flow past an impulsively started vertical plate-an exact solutions", *Theo. Appl. Fluid Mech.* 1(2), Pp. 111-115.

[6] Grief, R., Habib, I.S. and Lin J.C., (1971) " Laminar convection of radiating gas in a vertical channel", *J. Fluid Mech.* 46, Pp. 513-520.

[7] Ganesan, P. And Loganathan, P. (2002), " Radiation and mass transfer effects on flow of an imcompressible viscous fluid past a moving vertical cylinder", *Int. J. of Heat and Mass Transfer,* **45,** Pp. 4281-4288.

[8] Mbeledogu I.U, Anakiri A.R.C and Ogulu A. (2007), " Unsteady MHD free convection flow of a compressible fluid past a moving vertical plate in the presence of radiative heat transfer",*Int. J. of Heat and Mass Transfer* 50, Pp. 1668–1674.

[9] Makinde O.D. (2005), Free-convective flow with thermal radiation and mass transfer past a moving vertical porous plate, *Int. Comm Heat Mass Transfer,* 32, pp. 1411-1419.

[10] Abdas-Sattar M.D. and Hamid Kalim M.D. (1996), " Unsteady Free–Convection radiation with thermal radiation in a boundary layre flow past a vertical porous plate", *J. Math. Phys. Sci.* 30, Pp.25-37.

[11] El-Hakiem M.A. (2000), " MHD oscillatory flow on free-convection radiation through a porous medium with constant velocity", *J. Magn. And Magnetic Mater,* 220(23), Pp. 271-276.

[12] Singh, A.K. and Dikshit, C.K., (1988), " Hydromagnetic flow past a continuously moving semi-infinite plate at large suction", *Astrophys. Space Sci.,* 148, Pp. 249-256.

[13] Kim, Y.J., (2000), " Unsteady MHD convective heat transfer past a semi-infinite vertical porous moving plate with variable suction", *Int. J. Eng. Sci.* 38, Pp.833-845.

[14] Seddek, M.A. (2005), " Finite-element method for the effects of chemical reaction, variable viscosity, thermophoresis and heat generation/absorption on a boundary layer hydromagnetic flow with heat and mass transfer over a heat surface", *ACTA Mech.*, 177, Pp. 1-18.

[15] Patil, P. M. and Kulkani, P. S. (2008), " Effects of chemical reaction on free convective flow of a polar fluid through a porous medium in the presence of internal heat generation", *Int. J. Therm. Sci.* 4, Pp. 1043-1054.

[16] Ramachandra Prasad, V. and Bhaskar Reddy, N. (2008), "Radiation effects on an unsteady MHD convective heat and mass transfer flow past a semi-infinite vertical permeable moving plate embedded in a porous medium", *Journals of Energy Heat and mass transfer* 30, Pp. 57-68 .

[17] Dulal Pal and Babulal Talukdar (2010), "Perturbation analysis of unsteady magnetodynamic convective heat and mass transfer in a boundary layer slip flow past a vertical permeable plate with thermal radiation and chemical reaction", *Communication in Nonlinear science and Numerical Simulation*, Pp.1813-1830.

[18**]** Hassanien I.A. and Gorla R.S.R. (1990), "Mixed convection boundary layer flow of a micropolar fliud near a stagnation point on a horizontal cylinder", *Acta Mech.*, **84,** Pp. 191-199.

[19] Kelson N.A. and Farell T.W. (2001), " Micropolar flow over a porous stretching sheet with strong suction or injection", *Int. Comm. Heat Mass Transfer*, **28**, Pp.479-488.