International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 4 Issue 05, May-2015

Effects of Magnetic Field and Internal Heat Generation on the onset of Rayleigh-Bénard Convection in Nanofluid

S. Pranesh
Department of Mathematics,
Christ University
Bangalore, India

Ritu Bawa
Research Scholar, Department of Mathematics, Christ University
Bangalore, India

***Abstract* - In the present problem the effects of vertical magnetic field along with internal heat generation on the onset of Rayleigh-Bénard convection in a conducting layer of nanofluid, which is heated from below is investigated. The model that is employed for the study incorporates the effects of Brownian motion and thermophoresis as the important slip mechanism in the absence of turbulent eddies. Linear theory based on normal mode technique has been used. The eigen value of the problem is solved for different velocity boundary conditions and for isothermal temperature condition using Galerkin technique. The results obtained during the analysis have been presented graphically. It is found that magnetic field stabilizes and internal heat generation destabilizes the system.**

Keywords - Thermal instability; Nanofluid; Rayleigh-Bénard convection; Chandrashekhar number; Internal Rayleigh number.

1. INTRODUCTION

The heating and cooling techniques are required in most of the industries like transportation, power manufacturing, electronics, metallurgy, energy supply, production etc. Many technical challenges experienced by these modern industries are higher cooling performance. Therefore in order to achieve the higher heat flux densities, the development of advance fluids with improved thermal and flow characteristics are of great importance. The magnitude of thermal conductivity of solids is higher than that of fluids and therefore it has been expected that thermal conductivity of fluids will improve by the dispersions of solid particles. Several investigations are carried out using mili or micro-sized particles inside the base fluid. These particles increase the thermal conductivity of the base fluid but created other problems like settling, clogging channels, pressure drop and premature wear on components and channels. It has been observed that nanoparticles have an advantage over micro or mili-sized particles as they approach the size of the molecules in the fluid and due to this settling, clogging and wearing of channels is avoided. Choi [1] was the first person who named the fluids with dispersed solid particles as nanofluids. The size of nanoparticles that are suspended in base fluids (common fluids) are in the range of 1 to 100 nanometre and the thermal conductivity of base fluids on adding small amount of these nanoparticles increases by 10-40%. Base fluids used included ethylene glycol, water while the nanoparticle used include carbon nanotubes, Cu, Al_2O_3, TiO_2, Cuo with diameter of 1-100 nm.

The various characteristics of heat transfer behaviour and flow of nanofluid is studied by many researchers. Masuda et al. [2], Anoop et.al. [3], Das et al. [4] and Eastman et al. [5] observed the enhanced thermal conductivity of conventional fluid due to presence of nanosized particles. Unusual enhancement in thermal conductivity of nanofluid has many applications in science and engineering, for e.g. in advanced nuclear systems and nanodrug delivery as suggested by Buongiorno and Hu [6] and Kleinstreuer et al. [7]. Eastman et al. [8] gave a comprehensive review on thermal transport in nanofluids. He concluded that there is no satisfactory explanation of abnormal increase in viscosity and thermal conductivity of nanofluids. Buongiorno [9] studied convective heat transport in nanofluids. He developed a two component realistic model for convective transport of nanofluids. He explained the mechanism by which slip velocity is developed by nanoparticles with respect to base fluid and studied the effect of seven slip mechanism, Brownian diffusion, inertia, magnus effect, diffisiophoresis, fluid drainage and gravity settling. He derived the governing equations based on thermophoresis and Brownian diffusion which dominates the other slip mechanisms in the absence of turbulent eddies. Tzou [10] conducted the study based on Buongiorno [9] findings and found that critical Rayleigh number of nanofluid is lower by the order of one or two of magnitude, in comparison with that of regular fluid and concluded that regular fluids are more stable than nanofluids. Vadasz [11] studied the heat conduction in nanofluid suspensions. With the help of Buongiorno [9] equations several studies were conducted by Kim et al. [12]. Dhananjay et al. [13] studied the Rayleigh-Bénard convection in nanofluids using Galerkin method and explained the overstability of the nanofluids.

Due to enhanced thermal conductivity of nanofluids they are used as a great coolants. Many studies have been conducted in nanofluids with or without porous medium. Nield and Kuznetsov [14] studied the thermal instability in porous medium layer using Brinkman model. They concluded that value of thermal Rayleigh number depends on the nanoparticle distribution. they also observed that oscillatory convection can takes place on bottom heavy distribution of nanoparticles. Kuznetsov and Nield [15] studied double diffusive nanofluid convection in porous medium and investigated both oscillatory and non-oscillatory cases. Recently, Aggarwal and Bhadauria [16] studied the convective heat transport by longitudinal waves and derived an analogy between binary fluid convection and nanofluid convection with soret effect using linear and nonlinear analysis. Bhadauria and Shilpi [17, 18,19] studied the onset of convection in nanofluids in porous media.

The study of effect of magnetic field on the onset of convection was started several decades ago. The study of effect of magnetic field on the onset of Rayleigh-Bénard convection has wide range of applications in physics and engineering. In some of the practical applications such as magnetic field sensors, magneto-hydrodynamics generators, the cooling systems of electronic devices, geothermal reservoir's, thermal insulators, magnetic storage media which is electrically conducting through a vertical plate occurs in the presence of transverse magnetic field. Nanofluids due to its higher thermal conductivity can be used in such devices in order to improve their heat transfer performance. Heris et al. [20] studied the performance of two-phase closed thermosylon in nanofluid due to magnetic field effect and explained that thermal efficiency will be increase due to vertical magnetic field. Dhananjay [21] studied the thermal instability in nanofluids with magnetic field. He used alumina-water nanofluid for his analysis. Gupta et al. [22] also studied the thermal instability in nanofluids with magnetic field but with bottom heavy distribution of nanoparticles and explained the effect of various parameters on thermal Rayleigh number. A research on internal heat generation is much less extensive as compared to external heat generation. Bhattacharya and Jena [23]; Takashima [24]; Tasaka and Takeda [25]; Bhadauria et al [26] have studied the effect of internal heating on the onset of Rayleigh – Bénard convection under different situation.

The above literature survey indicates that no study is done on the effects of magnetic field and internal heat generation on the onset of Rayleigh-Bénard convection in a nanofluid layer with bottom heavy distribution of nanoparticles. In the present problem combine effect of vertical magnetic field along with internal heat generation at the boundaries on the onset of Rayleigh-Bénard convection in a conducting layer of nanofluid which is heated from below is investigated using linear stability analysis.

2. MATHEMATICAL FORMULATION

Consider an infinite horizontal layer of nanofluid confined between two parallel plates separated by a distance d, heated from below and cooled from above. A uniform vertical magnetic field acts on the system as shown in the fig. (1). The viscosity, thermal conductivity, density, magnetic permeability, electrical resistivity, electrical conductivity and specific heat of nanofluids may depend on the volume fraction of the nanoparticles, for the purpose of characterization and estimates of various

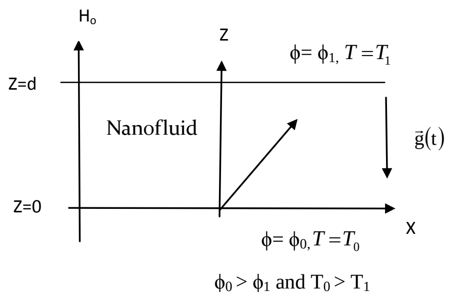


Fig. 1: Physical Configuration.

effects on the order of magnitude, all thermophysical properties of nanofluid are assumed to be constant in the analytical formulation. The temperatures at lower and upper walls are taken as T_0 and T_1, the former being greater. The nanofluid is assumed to be incompressible.

The governing equations of the problem under Boussinesq approximation are:

Conservation of mass:

$$\nabla.\vec{q} = 0\,, \tag{1}$$

Conservation of linear momentum:

$$\begin{aligned}\rho_f\left[\frac{\partial\vec{q}}{\partial t}+(\vec{q}.\nabla)\vec{q}\right] &= -\nabla p+\mu\nabla^2\vec{q}\\ &-\left[\phi\rho_p+(1-\phi)\{\rho_f(1-\beta(T-T_1))\}\right]g\,\hat{k}\\ &+\frac{\mu_m}{4\pi}(\vec{H}.\nabla)\vec{H}\,,\end{aligned}\tag{2}$$

Conservation of nanofluid particle:

$$\frac{\partial\phi}{\partial t}+(\vec{q}.\nabla)\phi = D_B\nabla^2\phi+\frac{D_T}{T_1}\nabla^2 T\,, \tag{3}$$

Conservation of energy:

$$\begin{aligned}\rho_f c_f\left[\frac{\partial T}{\partial t}+(\vec{q}.\nabla)T\right] &= k_T\nabla^2 T\\ &+\rho_p c_p\left[D_B(\nabla\phi).(\nabla T)+\frac{D_T}{T_1}(\nabla T)^2\right],\\ &+Q(T-T_1)\end{aligned}\tag{4}$$

Magnetic induction equations:

$$\frac{\partial\vec{H}}{\partial t}+(\vec{q}.\nabla)\vec{H} = (\vec{H}.\nabla)\vec{q}+\eta\nabla^2\vec{H}\,, \tag{5}$$

$$\nabla.\vec{H}=0\ , \qquad (6)$$

where, p is pressure, g is gravitational acceleration, ρ_f is fluid density, ρ_p is nanoparticle mass density, μ is viscosity of the fluid, μ_m is magnetic permeability, D_B is Brownian diffusion coefficient, D_T is thermophoretic diffusion coefficient, k_T is effective thermal conductivity, ϕ is nanoparticle volume fraction, t is time,T is temperature, H is the magnetic field,and η is the resistivity of the fluid.

The equations (1) – (6) are solved subject to the following boundary conditions. The temperature and volumetric fraction of nanoparticles is assumed to be constant at the boundaries.

Free -Free Isothermal:

$$\left.\begin{aligned} w=\frac{\partial^2 w}{\partial z^2}=0,\ T=T_0,\ \phi=\phi_0\ at\ z=0,\\ w=\frac{\partial^2 w}{\partial z^2}=0,\ T=T_1,\ \phi=\phi_1\ at\ z=1.\end{aligned}\right\} \quad (7a)$$

Rigid- Rigid Isothermal:

$$\left.\begin{aligned} w=\frac{\partial w}{\partial z}=0,\ T=T_0,\ \phi=\phi_0\ at\ z=0,\\ w=\frac{\partial w}{\partial z}=0,\ T=T_1,\ \phi=\phi_1\ at\ z=1.\end{aligned}\right\} \quad (7b)$$

Rigid-Free Isothermal:

$$\left.\begin{aligned} w=\frac{\partial w}{\partial z}=0,\ T=T_0,\ \phi=\phi_0\ at\ z=0,\\ w=\frac{\partial^2 w}{\partial z^2}=0,\ T=T_1,\ \phi=\phi_1\ at\ z=1.\end{aligned}\right\} \quad (7c)$$

2.1. Non- Dimentionalisation

Equations (1) to (6) are non-dimensionalized using the following definition:

$$\left.\begin{aligned} &(x^*,y^*,z^*)=\left(\frac{x}{d},\frac{y}{d},\frac{z}{d}\right),\ \vec{q}^*=\frac{d}{k_T}\vec{q},\\ &t^*=\frac{k_T}{d^2}t,\ p^*=\frac{d^2}{\mu k_T}p,\ \phi^*=\frac{\phi-\phi_1}{\phi_0-\phi_1},\\ &T^*=\frac{T-T_1}{T_0-T_1},\vec{H}^*=\frac{\vec{H}}{H_0}.\end{aligned}\right\} \quad (8)$$

Substituting equation (8) into equations (1)-(6), we get the following non -dimensionalized equation after dropping asterisks:

$$\nabla.\vec{q}=0, \qquad (9)$$

$$\frac{1}{P_r}\left[\frac{\partial\vec{q}}{\partial t}+(\vec{q}.\nabla)\vec{q}\right]=-\nabla p+\nabla^2\vec{q} -\{R_n\phi-R_aT+R_m\}\hat{k}+\frac{Q\,Pr}{Pr_M}(\vec{H}.\nabla)\vec{H}, \qquad (10)$$

$$\frac{\partial T}{\partial t}+(\vec{q}.\nabla)T=\nabla^2T+\frac{N_B}{L_e}(\nabla\phi.\nabla T) +\frac{N_AN_B}{Le}\nabla T.\nabla T+R_iT, \qquad (11)$$

$$\frac{\partial\phi}{\partial t}+(\vec{q}.\nabla)\phi=\frac{1}{Le}\nabla^2\phi+\frac{N_A}{Le}\nabla^2T\ , \qquad (12)$$

$$\frac{\partial\vec{H}}{\partial t}+(\vec{q}.\nabla)\vec{H}=(\vec{H}.\nabla)\vec{q}+\frac{Pr}{Pr_M}\nabla^2\vec{H}\ , \qquad (13)$$

$$\nabla.\vec{H}=0\ , \qquad (14)$$

The non-dimensional parameters in the equations (9) - (14) are:

$Pr=\dfrac{\mu}{\rho_f k_T}$, (Prandtl number)

$R_n=\dfrac{(\rho_p-\rho)(\phi_0-\phi_1)gd^3}{\mu k_T}$, (Concentration Rayleigh number)

$R_m=\dfrac{[\rho_p\phi_1+\rho(1-\phi_1)]gd^3}{\mu k_T}$, (Basic density Rayleigh number)

$R_a=\dfrac{\rho g\beta d^3(T_0-T_1)}{\mu k_T}$, (Rayleigh number)

$$Q=\frac{\mu_m H_0^{\,2} d^2}{4\pi\rho_f \nu\eta},$$ (Chandrashekhar number)

$$Pr_M=\frac{\mu}{\rho_f\,\eta},$$ (Magnetic Prandtl number)

$$Le=\frac{k_T}{D_B},$$ (Lewis number)

$$N_A=\frac{D_T\left(T_0-T_1\right)}{D_B T_1\left(\phi_0-\phi_1\right)},$$ (Modified diffusivity ratio)

$$N_B=\frac{\rho_p c_p\left(\phi_0-\phi_1\right)}{\rho_f c_f},$$ (Modified particle density increment)

and

$$R_i=\frac{Q d^2}{k_T}.$$ (Internal Rayleigh number)

In dimensionless form boundary condition equation (7a)-(7c) can be written as:

Free-Free Isothermal:

$$\left.\begin{array}{l} w=D^2w=0, T=0,\ \phi=1\ at\ z=0,\\ w=D^2w=0, T=0,\ \phi=0\ at\ z=1.\end{array}\right\} \quad (15a)$$

Rigid-Rigid Isotherma

$$\left.\begin{array}{l} w=Dw=0, T=0,\ \phi=1\ at\ z=0,\\ w=Dw=0, T=0,\ \phi=0\ at\ z=1.\end{array}\right\} \quad (15b)$$

Rigid-Free Isothermal:

$$\left.\begin{array}{l} w=Dw=0, T=0,\ \phi=1\ at\ z=0,\\ w=D^2w=0, T=0, \phi=0\ at\ z=1.\end{array}\right\} \quad (15c)$$

2.2. Basic state

The basic state of the nanofluid is being quiescent and is given by

$$\left.\begin{array}{c} \vec{q}=0,\ T=T_b(z),\ \phi=\phi_b(z),\ p=p_b(z),\\ \vec{H}=H_0\,\hat{e}_z\,. \end{array}\right\}$$

(16)

Substituting equation (16) in equations (10)-(14) we get:

$$\left.\begin{array}{l} -\dfrac{dp_b}{dz}+\left\{-R_n\phi_b(z)+R_aT_b(z)-R_m\right\}\\ \quad +\dfrac{Q\,Pr}{Pr_M}\left(H_0\hat{e}_z.\nabla\right)H_0\hat{e}_z=0, \end{array}\right\} (17)$$

$$\left.\begin{array}{l} \dfrac{d^2T_b}{dz^2}+\dfrac{N_B}{Le}\left(\dfrac{d\phi_b}{dz}.\dfrac{dT_b}{dz}\right)\\ +\dfrac{N_AN_B}{Le}\left(\dfrac{dT_b}{dz}.\dfrac{dT_b}{dz}\right)+RiT_b(z)=0 \end{array}\right\}, \quad (18)$$

$$\frac{1}{Le}\frac{d^2\phi_b}{dz^2}+\frac{N_A}{Le}\frac{d^2T_b}{dz^2}=0. \quad (19)$$

Using an order of magnitude analysis the second and the third terms in the equation(18) are small and can be neglected (Tzou [10]), thus we have:

$$\frac{d^2T_b}{dz^2}+RiT_b=0. \quad (20)$$

from equation(19) on using (20) we get:

$$\frac{d^2\phi_b}{d\,z^2}-N_ARiT_b=0, \quad (21)$$

integrating equation (20) and (21) and using boundary condition (7) we get:

$$T_b=\frac{Sin\sqrt{Ri}\,(1-z)}{Sin\sqrt{Ri}}, \quad (22)$$

$$\phi_b=\frac{-N_ASin\sqrt{Ri}\,(1-z)}{Sin\sqrt{Ri}}+(1+N_A)(1-z)\ ,(23)$$

2.3. Stability Analysis

We now superimpose infinitesimal perturbations on the basic state as given below:

$$\left.\begin{array}{l} \vec{q}=\vec{q}_b+\vec{q}',\ p=p_b+p',\\ \rho=\rho_b+\rho', T=T_b+T',\\ \phi=\phi_b+\phi',\ \vec{H}=\hat{e}_z+\vec{H}'. \end{array}\right\} \quad (24)$$

where, the primes indicate that the quantities are infinitesimal perturbations and subscript b indicates basic state value.

Substituting equation (24) into equations (10)-(14) and using the basic state solutions, we get linearized equations governing the infinitesimal perturbations in the form:

$$\left.\begin{aligned}\frac{1}{Pr}\frac{\partial \vec{q}'}{\partial t} &= -\nabla p' + \nabla^2 \vec{q}' - Rn\phi' \\ &+ RaT' + \frac{Q\,Pr}{Pr_M}\left\{\left(\vec{H}'.\nabla\right)\hat{e}_z + \left(\hat{e}_z.\nabla\right)\vec{H}'\right\}\end{aligned}\right\}, \tag{25}$$

$$\frac{\partial T'}{\partial t} = \nabla^2 T' + f(z)w' + RiT', \tag{26}$$

$$\left.\begin{aligned}\frac{\partial \phi'}{\partial t} + \{N_A f(z) - 1 - N_A\}w' &= \\ \frac{1}{Le}\nabla^2\phi' + \frac{N_A}{Le}\nabla^2 T',\end{aligned}\right\} \tag{27}$$

$$\frac{\partial \vec{H}'}{\partial t} = \frac{\partial w'}{\partial z} + \frac{Pr}{Pr_M}\nabla^2 H', \tag{28}$$

$$\nabla.\vec{H}' = 0, \tag{29}$$

where, $f(z) = \dfrac{\sqrt{R_i}\cos\left[\sqrt{R_i}(1-z)\right]}{\sin\left[\sqrt{R_i}\right]}$.

Operating curl twice on equation (25) to eliminate pressure, on using equation (28) and writing only the z component, we get:

$$\left.\begin{aligned}\left(\nabla^2 - \frac{1}{Pr}\frac{\partial}{\partial t}\right)\nabla^2 w' + Ra\nabla_1^2 T' \\ - Rn\nabla_1^2\phi' - Q\frac{\partial^2 w'}{\partial z^2} = 0\end{aligned}\right\}, \tag{30}$$

where ∇_1^2 is the two dimensional laplacian operator. Using normal mode analysis we seek the solution of the unknown fields w', T', ϕ' in the form

$$\begin{pmatrix} w' \\ T' \\ \phi' \end{pmatrix} = \begin{pmatrix} W(z) \\ T(z) \\ \phi(z) \end{pmatrix} exp\left(ilx + imy\right), \tag{31}$$

where l and m are horizontal wave number in x and y direction.

using equation (31) in equations (26), (27) and (30) we get:

$$\left.\begin{aligned}\left(D^2 - a^2\right)^2 W - a^2 RaT \\ + a^2 Rn\phi - Q\,D^2 W = 0\end{aligned}\right\}, \tag{32}$$

$$\left(D^2 - a^2\right)T + f(z)W + RiT = 0, \tag{33}$$

$$\left.\begin{aligned}\frac{1}{Le}\left(D^2 - a^2\right)\phi + \frac{N_A}{Le}\left(D^2 - a^2\right)T \\ - \{N_A f(z) - 1 - N_A\}W = 0\end{aligned}\right\}, \tag{34}$$

where,

$D \equiv \dfrac{d}{dz}$ and $a^2 = l^2 + m^2$ is the dimensionless wave number.

The set of differential equations (32)-(34) are solved using Galerkin technique. Multiplying equation (32) by W, equation (33) by θ and equation (34) by Φ, integrating the resulting equations by parts with respect to z from 0 to 1 and taking $W = Aw_1$, $T = BT_1$ and $\phi = C\phi_1$ where A,B and C are constants and w_1, T_1 and ϕ_1 are trial functions which satisfies the boundary conditions (15a)-(15c). This procedure yields the following equation for the Rayleigh number R_a:

$$R_a = \left(\frac{\left\langle T_1\left(D^2 - a^2\right)T_1\right\rangle + Ri\left\langle T_1^2\right\rangle}{a^2\left\langle w_1 T_1\right\rangle\left\langle T_1 f(z) w_1\right\rangle}\right)\left(X_1 - X_2 - X_3\right) \tag{35}$$

where,

$$X_1 = Q\left\langle w_1 D^2 w_1\right\rangle,$$

$$X_2 = \left\langle w_1\left(D^2 - a^2\right)^2 w_1\right\rangle,$$

$$X_3 = \frac{a^2 Rn\left\langle w_1\phi_1\right\rangle Le Y_1}{\left\langle \phi_1\left(D^2 - a^2\right)\phi_1\right\rangle},$$

$$Y_1 = N_A\left\langle \phi_1 f(z) w_1\right\rangle + \left(-1 - N_A\right)\left\langle \phi_1 w_1\right\rangle + \frac{N_A\left\langle \phi_1\left(D^2 - a^2\right)T_1\right\rangle\left\langle T_1 f(z) w_1\right\rangle}{Le\left(\left\langle T_1\left(D^2 - a^2\right)T_1\right\rangle + Ri\left\langle T_1^2\right\rangle\right)}$$

The value of critical Rayleigh number depends on the boundaries. The following boundary combinations are considered to evaluate equation (35).

(i) When both boundaries are free, isothermal and iso-nano concentration:

The boundary conditions are:

$$\left.\begin{aligned} w = D^2 w = 0, T = 0, \phi = 0 \\ at \quad z = 0, z = 1\end{aligned}\right\} \tag{36}$$

Trial functions satisfying the boundary conditions (36) are:

$$\left.\begin{aligned} w_1 &= \sin \pi z , \\ T_1 &= z - z^2 , \\ \phi_1 &= z - z^2 \end{aligned}\right\} . \quad (37)$$

(ii)When both boundaries are rigid, isothermal and iso-nano concentration:

The boundary conditions are:

$$\left.\begin{aligned} & w = Dw = 0,\ T = 0,\ \phi = 0 \\ & at \quad z = 0,\ z = 1 \end{aligned}\right\} . \quad (38)$$

Trial functions satisfying the boundary conditions (38) are

$$\left.\begin{aligned} w_1 &= z^2(1-z)^2 , \\ T_1 &= z - z^2 , \\ \phi_1 &= z - z^2 \end{aligned}\right\} . \quad (39)$$

(iii) When upper boundary is free, isothermal and iso-nano concentration and lower is rigid, isothermal and iso-nano concentration:

The Boundary conditions are :

$$\left.\begin{aligned} & w = Dw = 0, T = 0, \phi = 0, at\ z = 0 \\ & w = D^2 w = 0, T = 0,\ \phi = 0, at\ z = 1 \end{aligned}\right\} ,(40)$$

Trial functions satisfying the boundary conditions (40) are

$$\left.\begin{aligned} w_1 &= z^2(1-z)(3-2z), \\ T_1 &= z - z^2 , \\ \phi_1 &= z - z^2 \end{aligned}\right\} . \quad (41)$$

Substituting trial functions (37), (39) and (41) in equation (35) .we obtain the critical Rayleigh number R_{a_c} which attains the minimum when $a = a_c$ for different boundary combinations.

3. RESULTS AND DISCUSSIONS

The effect of magnetic field and internal heat generation on the onset of Rayleigh- Bénard convection in a nanofluid using linear stability analysisis studied in this paper. The eigenvalue of the problem is obtained using Galerkin method as a function of density Rayleigh number Rn, Lewis number Le, modified diffusivity ratio N_A , Chandrashekhar number Q and internal Rayleigh number R_i for free-free, rigid – rigid, rigid-free velocity isothermal and iso-nano concentration boundaries are considered, so that temperature and nano concentration vanishes at the boundaries. The linear stability theory expresses the criteria of stability in terms of critical Rayleigh number R_{a_c} below which the system is stable and if $R_a > R_{a_c}$ system is unstable. The results obtained in this paper are depicted in the figures (2)-(4).

Fig (2) is the plot of Rayleigh number R_a versus the wave number a in the case of free-free, isothermal and iso-nano concentration boundaries for different values of (a) Rn, (b) Le, (c) Q (d) N_A and (e) R_i. From figure 2(a) it is observed that increase in the values of Rn, increases the R_a thereby stabilizing the system indicating the delay of onset of convection. The positive value of Rn is taken which implies that the particle density decreases upwards. The stabilizing effect of Rn is due to increase in the concentration of nanoparticles and temperature difference between the plates there is a transfer of energy between the fluid and nanoparticles and hence delays the onset of convection. In figure 2(b) the value of Le are taken large because it is inversely proportional to brownian diffusion coefficient D_B which is small thus the decrease in the brownian diffusion coefficient increases R_a, thereby stabilizing the system. From figure 2(c), we observe that when Q increases, i.e. the strength of the applied magnetic field increases, it induces the viscosity into the fluid and hence magnetic field lines are distorted by convection. Then these magnetic lines hinder the growth of disturbance, lending to delay in the onset of convection. From the figure 2(d) when N_A increases, R_a decreases since N_A is directly proportional to thermophoresis diffusion coefficient D_T, the increase in N_A increase D_T, thereby decreases R_a thus the increase in thermophoresis diffusion coefficient, N_A leads to destabilization of the system. In figure 2(e) we observe that the increase in the internal Rayleigh number Ri increases the heat transport in the system thereby advancing the onset of convection. Thus, increase in internal Rayleigh number R_i destabilizes the system.

Figure (3) and (4) are the plot of critical Rayleigh number Ra versus wave number a for rigid-rigid and rigid –free isothermal and iso-nano concentration boundaries respectively. The results obtained in this case are qualitatively similar to the that obtained in the case of free – free, isothermal and iso-nano concentration case except that

$$R_{a_c}^{FF} < R_{a_c}^{RF} < R_{a_c}^{RR} \quad \text{and} \quad a_c^{FF} < a_c^{RF} < a_c^{RR}$$

where, superscript represents different boundary combinations.

4 CONCLUSIONS

In this article, the effects of magnetic field and internal heat generation on the onset of Rayleigh – Bénard convection in a horizontal layer of nanofluid heated from below is investigated using Galerkin technique. The following conclusions are drawn from the study:

(i) The presence of magnetic field is to reduce the intensity of Rayleigh – Bénard convection and hence leads to more stable system.

(ii) The effect of internal heat generation has significant influence on the Rayleigh – Bénard convection and is clearly a destabilizing factor to make the system more unstable.

(iii) For three cases considered, rigid-free, rigid- rigid and free-free surfaces, it is found that the critical values of the Rayleigh number in rigid-rigid surfaces are the highest. This show that the used of rigid-rigid surfaces can delay the onset of convection.

(iv) It is also observed that, the effects of increasing Rn, and Le is to delay the onset of convection, while increase in N_A and Ri is to advances the onset of convection.

(v) The results obtained in this study will favour the research workers and industrialists to identify the convective modes in order to control the quality of production.

(vi) The results of this study indicate that the onset of convection in nanfluids is always delayed when compared to fluid without nanoparticles.

5 ACKNOWLEDGMENT

Authors would like to acknowledge management of Christ University for their support in completing the work and also to Dr Pradeep G Siddheshwar, Professor, Department of Mathematics Bangalore University, Bangalore for his valuable suggestions during the completion of their work which increased the quality of the paper.

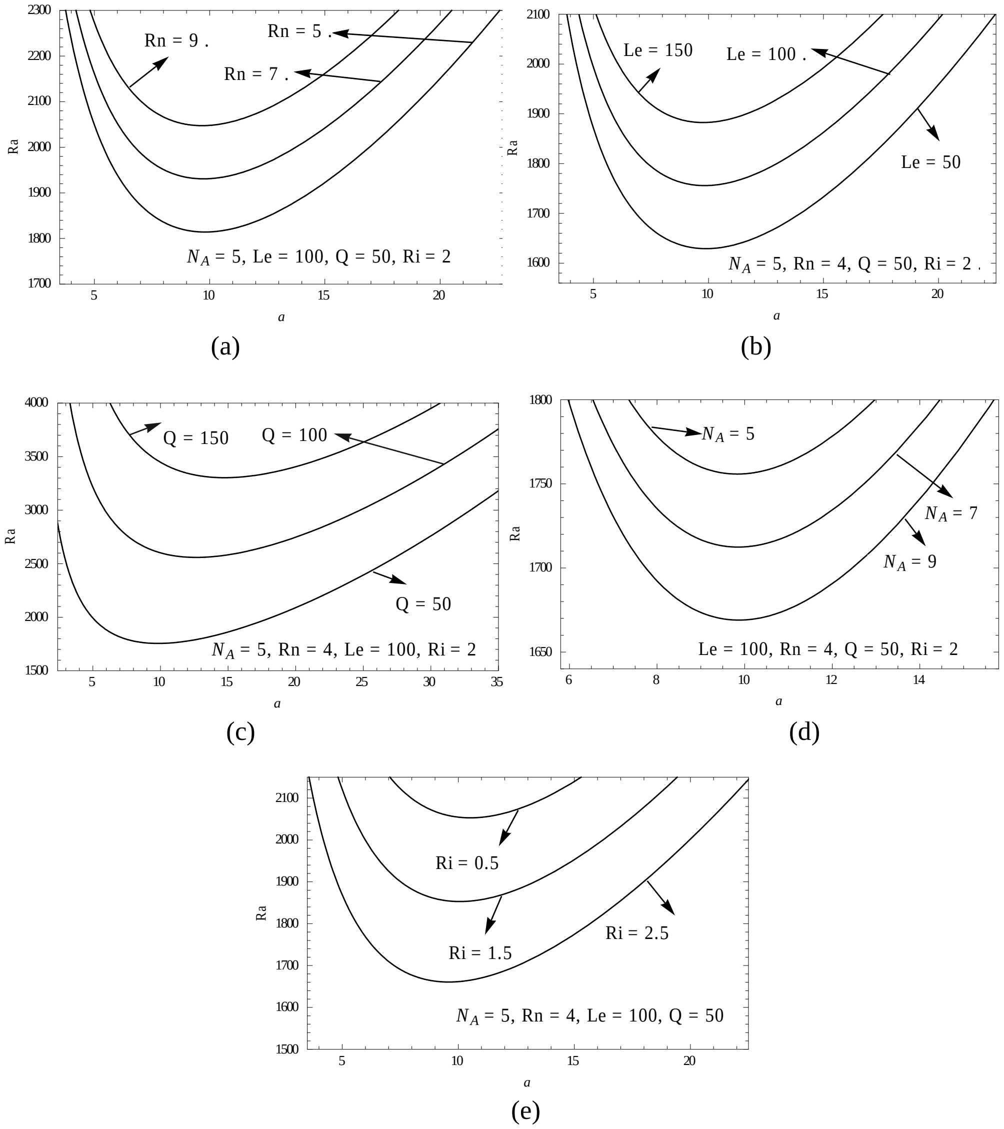
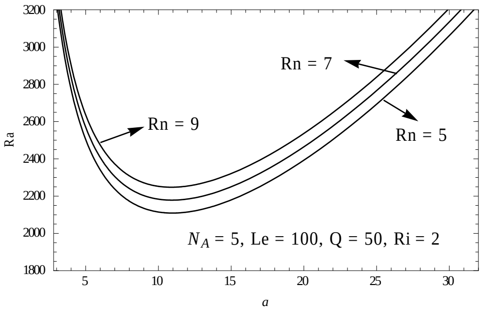
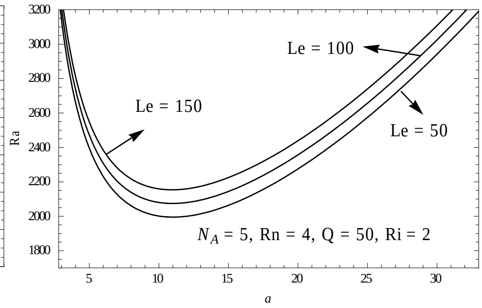


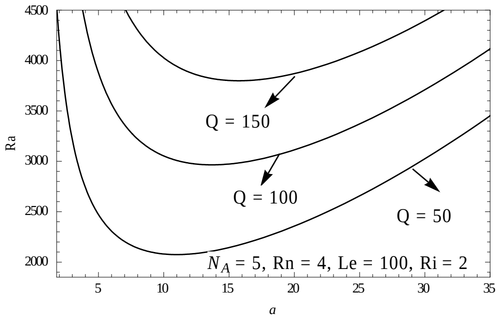
Fig 2 a-e-Plot of Rayleighnumber Ra versus wave number a for free- free isothermal and iso-nano concentration boundaries for different values of (a) concentration Rayleigh number, Rn, (b) Lewis number, Le, (c) Chandrashekhar number, Q, (d) modified diffusivity ratio, N_A and (e) internal Rayleigh number, Ri.



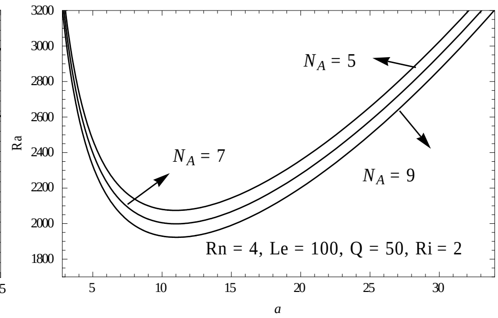
(a)



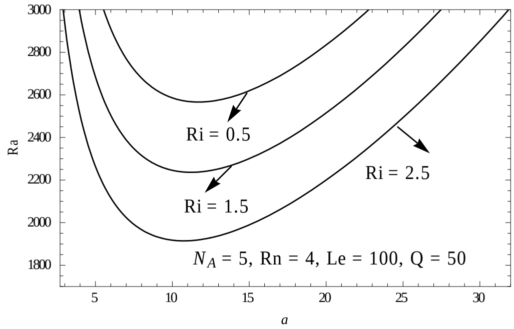
(b)



(c)

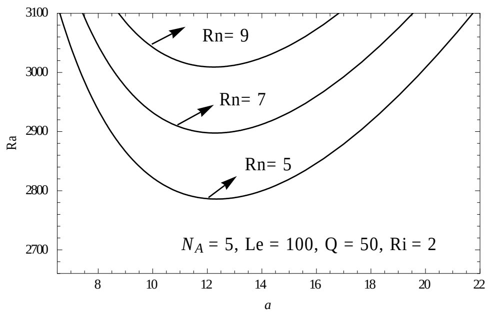


(d)

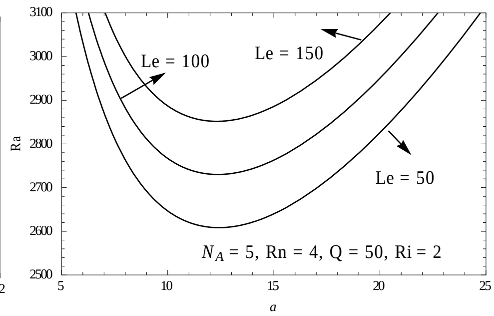


(e)

Fig 3 a-e Variation of Ra with respect to wave number a for rigid-free isothermal and iso-nano concentration boundaries for different values of (a) concentration Rayleigh number, Rn (b) Lewis number, Le, (c) Chandrashekhar number, Q (d) modified diffusivity ratio, N_A, and (e) internal Rayleigh number, Ri.

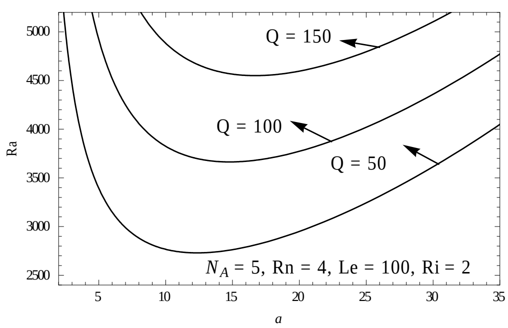


(a)

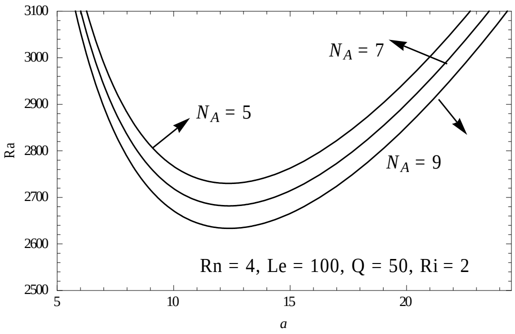


(b)

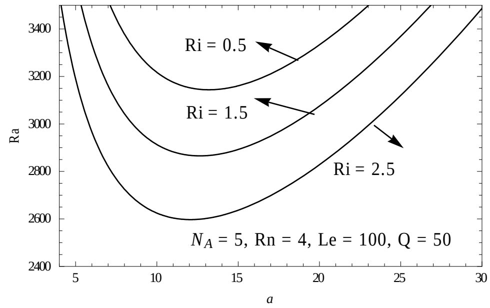
International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 4 Issue 05, May-2015



(c)



(d)



(e)

Fig 4 a-e- Variation of Ra with respect to wave number a for rigid-rigid isothermal and iso-nano concentration boundaries for different values of (a) concentration Rayleigh number, Rn, (b) Lewis number, Le, (c) Chandrashekhar number, Q, (d) modified diffusivity ratio, N_A and (e) internal Rayleigh number, Ri.

REFERENCES

[1]. S. Choi, "Enhancing Thermal Conductivity of Fluids with Nanoparticles in: D. A. Siginer and H. P. Wang (Eds), Developments and Applications of Non-Newtonian Flows,"ASME FED, vol. 231, pp. 99-105, 1995.

[2]. H. Masuda, A. Ebata, K. Teramae and N. Hishinuma, "Alteration of thermal conductivity and viscosity of liquid by dispersing ultra-fine particles," NetsuBussei, vol. **7,** pp. 227– 233, 1993.

[3]. K.B Anoop, S. Kabelac, T. Sundararajan and S.K. Das, "Rheological and flow characteristics of nanofluids," influence of electroviscous effects and particle agglomeration, J. Appl. Phys, vol.106, pp. 034909,2009.

[4]. S.K. Das, N. Putra, P. Thiesen and W. Roetzel, "Temperature dependence of thermal conductivity enhancement for nanofluids," ASME J. Heat Transf, vol. 125, pp. 567– 574, 2003a.

[5]. J.A. Eastman, U.S. Choi, W. Yu and L.J. Thompson, "Anomalously increased effective thermal conductivities of ethylene glycol-based nanofluids containing copper nanoparticles," Appl. Phys. Lett, vol.78, pp. 718–720, 2001.

[6]. J. Buongiorno and W. Hu, "Nanofluid coolant for advanced nuclear power plants," Proceedings of ICAPP'05, Seoul, Paper No. 5705 , pp. 15-19, 2005.

[7]. C. Kleinstreuer, J. Li and J. Koo, "Microfluidics of nano-drug delivery," Int. J. Heat Mass Transf, vol.51, pp. 5590– 5597, 2008.

[8]. J.A. Eastman, U.S. Choi, W. Yu and L.J. Thompson, "Thermal transport in Nanofluids," Annu. Rev. Mater. Res., vol.34, pp. 219–246,2004.

[9]. J.Buongiorno,"Convective Transport in Nanofluids," ASME Journal of Heat Transfer, vol. 128, pp. 240–250,2006.

[10]. D.Y. Tzou, "Thermal instability of nanofluids in natural convection," Int. J. Heat Mass Transf, vol. 51, pp. 2967–2979, 2008.

[11]. Vadasz, Heat conduction in nanofluid suspensions, ASME J. Heat Transfer, vol. 128, pp.465-477,2006.

[12]. J. Kim, Y.T. Kang and Choi, "Analysis of convective instability and heat transfer characteristics of nanofluids," Physics of Fluid, vol.16 (7) , pp. 2395-2401,2004.

[13]. G.S. Dhananjay, Agarwal and R. Bhargava, "Rayleigh Benard convection in nanofluid,"Int. J. of appl.Math and Mech, vol.7(2), pp. 61-76, 2011.

[14]. D.A. Nield and A.V. Kuznetsov, "Thermal instability in a porous medium layer saturated by nanofluid," Int. J. Heat Mass Transf, vol. 52, pp. 5796–5801, 2009.

[15]. D. A. Nield and A. V. Kuznetsov,"Onset of Convection with Internal Heating in a Porous Medium Saturated by a Nanofluid," Transp Porous Med, vol. 99, pp. 73–83,2013.

[16]. S. Agarwal and B. S. Bhadauria, "Convectional heat transport by longitudinal rolls in dilute nanoliquids." Journal of nanofluids, vol. 11, pp. 380-390, 2014.

[17]. B.S. Bhadauria, S. Agarwal and A. Kumar,"Non-linear two-dimensional convection in a nanofluid saturated porous medium," Transp. Porous Media, vol.90(2), pp. 605–625, 2011b.

[18]. B.S. Bhadauria and S. Agarwal, "Natural convection in nanofluid saturated rotating porous layer: A nonlinear study", Transp Porous Med. Vol. 87, pp. 585 – 602, 2011.

[19]. B. S. Bhadauria, S. Agarwal and P. G. Siddheshwar, "Thermal instability of a nanofluid saturating a rotating anisotropic porous medium," Spec. Top. Rev. Porous Media, vol. 2, pp. 5- 64, 2011a.

[20]. S.Z. Heris, H. Salehi and S.H. Noie, "The effect of magnetic field and nanofuid on thermal performance of two phase closed thermosyphon (TPCT)," Int J Phys Sci, vol.7 pp. 534- 543, 2012.

[21]. Dhananjay Yadav, R. Bhargava and G.S. Agarwal, "Thermal instability in nanofluid layer with a vertical magnetic field," J. Eng. Math, vol. 80, pp. 147-164,2013.

[22]. Urvashi Gupta, Jyoti Ahuja and R.K. wanchoo, "Magneto convection in a nanofluid layer," Int J of heat and mass transfer, vol.64, pp.1163-1171, 2013.

[23]. S.P. Bhattacharya and S.K. Jena, "Thermal instability of a horizontal layer of micropolar fluid with heat source," Proc. Indian Acad. Sci., vol. 93(1), pp. 13–26,1984.

[24]. M. Takashima, "The stability of natural convection in an inclined fluid layer with internal heat generation," Journal of the Physical Society of Japan, vol.58, pp. 4431–4440, 1989.

[25]. Yuji Tasaka and Yasushi Takeda, "Effects of heat source distribution on natural Convection induced by internal heating," International Journal of Heat and Mass Transfer,vol.48, pp.1164–1174,2005.

[26]. B.S. Bhadauria et al.,"Natural convection in a rotating anisotropic porous layer with internal heat generation," Transp Porous Med, vol. 90, pp.687–705,2011a.