

Effectiveness of Tuned Liquid Dampers on High Rise Buildings in Kerala

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Abstract— The need to build flexible and slender buildings, that have relatively low damping properties has attracted engineers attention to look for efficient and economical techniques to control the vibrational response of structures. Recently, Tuned Liquid Dampers (TLD) has increased in popularity due to its low installation and maintenance costs. TLD is a passive damping device that consists of rigid tanks filled with liquid to suppress the horizontal vibration of structures. The tank is designed so that the liquid surface wave has frequency tuned to the fundamental frequency of the building. The main objective of this work was to propose TLD tanks for a 40 storeyed building in Kerala. The structure was first modelled and then its fundamental natural frequency was found out by carrying out free vibration analysis. TLD is then modelled into the structure and changes in natural frequencies were monitored. The structure was subjected to an earthquake loading (El-Centro Earthquake) and its frequency response was compared without TLD's and with TLD's. The optimum mass ratio was obtained at 0.8 % and corresponding reduction in displacement was found to be 28.73 %. Based on the optimum mass ratio obtained, number of TLD tanks, its dimensions and required water depth for the structure to control vibrations was proposed.

Keywords—Tuned Liquid Dampers; Eigen Value Analysis; Fluid Structure Interaction; Response Spectrum Analysis;

I. INTRODUCTION

Nowadays, there is an increasing trend to construct tall structures, to minimise the increasing space problems in urban areas. These structures are often comparatively light and flexible, possessing low damping, which make them more vibration prone. Thus to ensure functional performance of tall buildings, against wind and earthquake forces, it is important to keep the frequency of structural motion below threshold. The various techniques to achieve this can be classified broadly into 2 categories i.e. Active Damping Techniques and Passive Damping Techniques. Active dampers use a power source to create an additional force between the damper and the structure. This type of supplying energy to the system is also known as negative damping. Passive damping refers to energy dissipation within the structure. The force that is exerted on a building due to external loads is channeled through the passive system and is dissipated through the damping device. The advantage is that there is no power source required in the operation of the system and so is environmental friendly. The best example is the addition of an auxiliary mass system to increase the level of damping (Eg. TMD, TLD).

The use of Tuned Liquid Dampers as a method of controlling structural vibrations was studied in this work.

TLD's can be broadly classified into Tuned Sloshing Dampers (TSD's), Tuned Liquid Column Dampers (TLCD's) and controllable TLD's. TSD's are rectangular or circular tanks partially filled with liquid (usually water), and is typically located either at the terrace level or immediately below it. When the tank is excited through structural motion, the fluid in the tank begins to slosh, imparting inertial forces into the structure, out of phase with the structural motion, thereby reducing the movement. Tuned Liquid Column Dampers (TLCD's) dissipates structural vibration due to the motion of the liquid in the tube as a result of gravity action and by the loss of hydraulic pressure due to the orifice installed inside the container. Controllable TLD's are used to increase the effectiveness of the damper when the forces that act on the structure are spread over a band of frequencies. This is done by active or semi active control devices such as controlling the angle of baffles provided in the tank or by using propellers powered by a motor.

In this study, TSD's were only used and since no other variant of TLD's are henceforth mentioned, the term 'Tuned Liquid Dampers' (TLD's) are used for TSD's from this point. Optimum performance of TLD is obtained by tuning its fundamental sloshing frequency to the structure's natural frequency which causes a large amount of sloshing & wave breaking. A TLD can be classified as shallow water type and deep water type depending on the height of water in the tank. If the ratio of the height of water against the length of tank in the direction of excitation is less than 0.15, it is a shallow water type else a deep water type TLD. Shallow water type TLD's have large damping effect for small levels of externally excited vibrations, but it is difficult to analyse the same for large amplitude vibrations since sloshing of water in a tank exhibits nonlinear behaviour. For deep water type TLD's, the sloshing can be described by a linear behaviour for large amplitude external forces.

Bauer [3] was the first to suggest a new damping device consisting of a liquid container filled with two immiscible liquids. He showed that the motion of the interface was able to dampen the structure effectively. Modi & Welt [12] were among the first to suggest the use of a TLD in buildings to reduce overall response during strong wind or earthquakes. Fujii, et al. [4] have found that wind-induced vibrations of two actual tall towers, at Nagasaki Airport Tower and Yokohama Marine Tower, were reduced to about half upon installation of Tuned Liquid Damper. Sun, et al. [15] could

successfully develop an analytical model for TLD, based on shallow water wave theory, which proved to be very effective if the wave is non-breaking. They extended this model to account for effect of breaking waves by introducing two empirical coefficients identified experimentally. Modi & Seto [13] studied the effects of wave dispersion, floating particles at the free surface, and wave breaking. Banarji et al. [2] defined appropriate design parameters of the TLD that is effective in controlling the earthquake response of a structure. These parameters include the ratio of the linear sloshing and structure's natural frequencies, the ratio of the masses of water and structure, and the water depth to the TLD tank-length ratio.

Tait et al. [16] discussed the numerical flow model of TLD behaviour including the free surface motion, the resulting base shear forces along with the energy dissipated using slat screens. Kaneko & Ishikawa [8] conducted analytical study on TLD with submerged nets. They found that the optimal damping factor, as in the case for Tuned Mass Dampers (TMD's), can be produced by nets, and the TLDs with submerged nets are more effective in reducing structural vibration than TLDs without it. Fujino and Sun [5] studied Multiple Tuned Liquid Dampers (MTLD's), and showed that MTLD's are more efficient than TLD's in the small amplitude range, while they perform more or less at the same level in large amplitude range. Kim et al. [9] found that TLD and MTLD effectively dampened frequency responses but MTLD gave better frequency ratio, off-tuning ratio and damping ratio than the TLD. Tait et al. [18] studied the ability of TLD to operate in two directions and the results indicate that by choosing the appropriate aspect ratio for the TLD it can be used to reduce structural responses in both directions simultaneously with no penalty on its performance. Sorkhabi, Kristie and Mercan [1] confirmed that introducing multiple TLD's to the MDOF structures and tuning them with respect to modal properties of the structure results in improved control of vibrations in comparison where only a single TLD is employed. Love and Tait [11] considered nonlinear energy dissipation associated with the damping screens and the nonlinear coupling amongst the sloshing modes for MTLD tanks.

In literature, studies relating to the effectiveness of TMD's are plentiful, however the use of TLD's to control vibration is an area still undergoing various modifications. Performance of TLD's against wind induced lateral loads was studied more specifically, while works relating to seismic performance were found to be scarce. Studies related to the effectiveness of TLD's for a proposed high rise building were absent. This work mainly intends to familiarise the software Ansys Workbench in detail and model a proposed 40 storeyed building, to be constructed in India. Eigen Value Analysis (EVA) was performed to find out the fundamental natural frequency of the structure. Modelling of TLD into the structure was then done through Fluid Structure Interaction (FSI) and frequencies of structure without TLD and with

TLD's were compared. Optimum mass ratio of TLD required for the proposed structure was found out by Response Spectrum Analysis by using the time history data of EL-Centro Earthquake. Details of TLD tanks required were then proposed.

II. MATHEMATICAL FORMULATIONS

While developing a mathematical model, some assumptions are made to simplify the analysis. They are, mass of columns and flexibility of slab are ignored and the beam column joint is assumed to be rigid. By these assumptions, the possibility of lateral deformation is due to only the rigid beam /slab. Such a model is called as shear building model. This shear building idealisation is necessary for mathematical formulation of vibration problems.

Consider a spring mass system under the influence of viscous damping and subjected to a harmonic force of $F \sin \omega t$ as shown in Fig 1. The governing equation of motion can be written as

$$m\ddot{x} + c\dot{x} + kx = F \sin \omega t \quad (1)$$

$$\ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = \frac{F}{m} \sin \omega t \quad (2)$$

where m , c and k represents the mass, damping coefficient and stiffness of the structure. The general solution $x(t)$ can be divided into complimentary solution $x_c(t)$ and particular solution $x_p(t)$. Assuming the system is underdamped, i.e. $\rho < 1$, the solution is obtained in the following form

$$x(t) = \sqrt{A^2 + B^2} e^{-\rho\omega_d t} \sin(\omega_d t - \phi_c) + \frac{\frac{F}{k}}{\sqrt{(1 - \beta^2)^2 + (2\rho\beta)^2}} \sin(\omega t - \phi_p) \quad (3)$$

In the above equation ω_n and ω_d refers to frequency of the system and damper while β and Φ represent frequency ratio and phase angle respectively.

A. Modelling of Wave Sloshing in TLD

Considering a 2-dimensional wave fluid as shown in Fig. 2 (X-O_Z plane), liquid depth be h , and $z=0$ be the still liquid surface. η represents the free surface elevation, which is a function of location x and time t . L and H express wave length and wave height respectively. The wave amplitude is assumed to be infinitesimally small, so that the wave motions can be regarded as linear.

Liquid motion is assumed to be inviscid, irrotational, and incompressible. The velocity potential Φ , therefore exists and is satisfied to Laplace equation, i.e.,

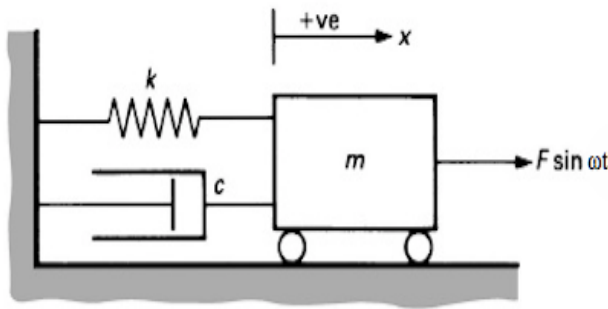


Fig 1: Damped Harmonic Oscillator

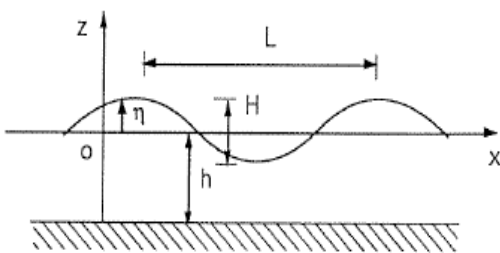


Fig 2: Schematic Diagram Showing Wave Motion in TLD

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial z^2} = 0 \quad (4)$$

At the bottom of the tank

$$w = \frac{\partial \phi}{\partial z} = 0 \quad (z = -h) \quad (5)$$

On the free surface $z = \eta(x,t)$, there are two kinds of boundary conditions; one is the dynamic boundary condition

$$p = p_0 = 0 \quad (z = \eta) \quad (6)$$

and the other is the kinematic boundary condition

$$\frac{D\eta}{Dt} \equiv \frac{\partial \eta}{\partial t} + u \frac{\partial \eta}{\partial x} = w \quad (z = \eta) \quad (7)$$

Let η take the form of

$$\eta = \frac{H}{2} \sin(kx - \omega t) \quad (8)$$

Following the derivation given by Sun Limin [13], we get the dispersion equation as

$$\omega^2 = gk \tanh(kh) \quad (9)$$

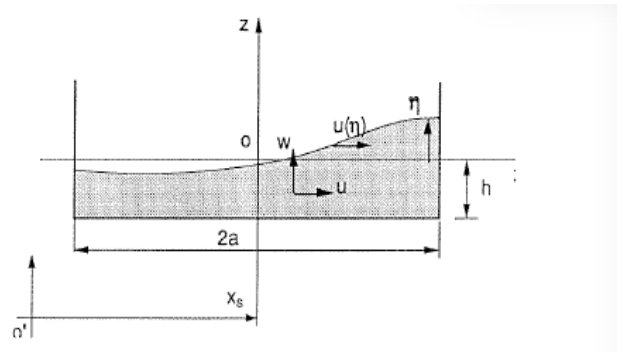
Therefore, the velocity potential, Φ can be expressed as

$$\Phi(x, z, t) = -\frac{gH}{2\omega} \frac{\cosh(k(z+h))}{\cosh(kh)} \cos(kx - \omega t) \quad (10)$$

The profile of Φ along the z direction is a function of $\cosh(k(z+h))$

B. Linear Natural Frequency of Liquid Sloshing in a Rectangular Tank

The rigid rectangular tank in Fig 3 of length $2a$ and mean liquid depth h is subjected to a horizontal motion x_s . The local Cartesian coordinate system (O-X-Z) has its origin at the center of the mean liquid surface as shown. For the liquid sloshing in this rectangular tank, the boundary conditions on the side walls are



$$u = \frac{\partial \Phi}{\partial x} = 0 \quad (x = \pm a) \quad (12)$$

Fig 3: Liquid Sloshing in Rectangular Tank under Horizontal Motion

Since the walls of tank are vertical, the liquid sloshing can be regarded as the superposition of a progressing wave and its reflection wave, which have opposite phase and travelling in opposite directions. Eq 8 can thus be modified as

$$\begin{aligned} \eta &= \frac{H}{2} \{ \sin(kx + \omega t) - \sin(kx - \omega t) \} \\ &= H \cos(kx) \sin(\omega t) \end{aligned} \quad (13)$$

Corresponding to this, velocity potential Φ is rewritten as

$$\Phi(x, z, t) = \frac{gH}{\omega} \frac{\cosh(k(z+h))}{\cosh(kh)} \cos(kx) \sin(\omega t) \quad (14)$$

To satisfy the boundary condition, Eq. 12, letting

$$\cos(kx) = 0 \quad (x = \pm a)$$

$$k = \frac{2n-1}{2a} \pi \quad (n = 1, 2, \dots) \quad (15)$$

where k is wave number. The natural frequency of liquid sloshing in a rectangular tank is derived from Eq. 9

$$f_n = \frac{\omega_n}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{2n-1}{2a} \pi g \tanh\left(\frac{2n-1}{2a} \pi h\right)} \quad (n = 1, 2, \dots) \quad (16)$$

where n denotes the various modes of liquid sloshing. The fundamental natural frequency ($n=1$) is

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{\pi g}{2a} \tanh\left(\frac{\pi h}{2a}\right)} \quad (17)$$

III. METHODOLOGY

A proposed 40 storeyed building in Calicut, Kerala, was considered for this project work. No of storeys for the building is 40 with each storey of 3 m height. Beam and column dimensions were 300 x 800 mm and 400 x 1200 mm respectively. The building was modelled with the help of the software Ansys Workbench. Centre line diagram was first drawn. Points were generated at the intersections using their coordinates and then lines were created by joining the points. Geometry for the first floor was drawn and then the upper floors were created by using the translational command. Beams and columns were created as 1D bar elements. The orientation of beams and columns were fixed as per plan. Slabs and shear walls were modelled as 2D plates of thickness 150 mm and 200 mm respectively. Once the full geometry of the building was created, finite element discretisation was done followed by meshing. Then support conditions were given. The support of the structure was made fixed by assigning all translational and rotational degrees of freedom to zero. Fixed supports were provided at the bottom of each column. Material of the structural elements was assigned as concrete with compressive strength 40 N/mm². Fig 4 shows an isometric view of the 40 storeyed building that was modelled in this work.

Once the structure was modelled, Eigen Value Analysis (EGA) was carried out. This was to determine the natural frequencies and mode shapes of the structure with damping neglected. Natural frequencies and mode shapes are functions of the structural properties and boundary conditions. Hear the building can be considered analogous to a cantilever beam (fixed at one end and free at the other) and hence it's mode shapes will also be equivalent. The next step was to integrate damper into the building. For this TLD tanks were to be designed for each mass ratio. Designing of TLD includes fixing the tank dimensions such as length and width and determining the number of similar tanks required. Calculating the depth of water was done such that the sloshing frequency (Eq. 17) becomes equal to the fundamental natural frequency of the structure. TLD's were designed for mass ratios varying from 0.2% - 1.6%.

The designed TLD's were then incorporated into the building using the Fluid Structure Interaction (FSI) inbuilt into Workbench software. Two-way FSI simulations between the mechanical component and fluid component (Fluent) were done using the system coupling tool. For ease of analysis, the mechanical component and Fluid component were set up and analysed independently before adding the complexity of the coupled analysis. Before analysing in the two components, the modelled geometry had to be divided into two parts. The first



Fig 4: 3-D model of the building studied

part, called the structural part involved the structural components alone and the second part contained the Fluid bodies. The geometry of the model was shared between the Transient and Fluent Systems and each part was separately analysed in their respective component. The time duration for coupled analysis was given as 30 seconds with each sub step measuring 0.1 seconds. This was because the time history values up to 30 seconds of El-Centro Earthquake were applied to the model for analysis. After analysing both the systems, their setups were coupled using system coupling and analyzed again to model sloshing effect in TLD tanks.

In order to find out the optimum mass ratio for the building under consideration, Response Spectrum Analysis (RSA) was carried out first on the building without damper and then on the same building with TLD's installed of mass ratios varying from 0.2 % to 1.6 %. RSA provides insight into dynamic behaviour of the building by measuring displacement, velocity or acceleration as a function of structural period for a given time history and level of damping. The provided time history was that of El-Centro Earthquake. The acceleration data was applied to all nodes at the base and RSA curves for the maximum displacement node at the top floor were taken. Inferring these response spectrum curves, the most optimum mass ratio was found out. TLD tanks corresponding to the optimum ratio were then proposed.

IV. RESULTS AND DISCUSSIONS

The normal mode analysis yields the mode shapes and the corresponding frequencies are shown in Table 1. The first mode shape is shown in Fig 5. The dominant frequency that produces the maximum effect on the building compared to other frequencies is obtained as 0.293 cycles. The tuned liquid damper is to be designed such that its tuning frequency is also 0.293 cycles. As per IS 1893:2002, the structural damping ratio ξ of any building can be taken as 5%. The modal mass for the first mode along the Z direction is obtained as 2.166×10^7 kg.

Table 1: Natural Frequencies of various modes

Mode	Frequency
1	0.29338
2	0.35676
3	0.38145
4	0.98627
5	1.1279
6	1.203
7	1.8946
8	2.0303
9	2.2121
10	2.7969

TLD's were then integrated into the building model by FSI simulations and normal mode analysis was carried out for building with dampers for varying mass ratios. The effects of liquid dampers in the natural frequencies of the building can be observed from Table 3. As can be observed from the table, the fundamental frequency of the building increase with increasing mass ratios. In other words, the building without damper could be excited with an exciting force of very low frequency i.e. 0.293 Hz. When dampers were installed, the same building will not be excited with the same force. When TLDs of mass ratio 1.6 % were installed, a forcing frequency of 0.322 Hz to is required to excite the building.

To find out the optimum mass ratio for the building under consideration, Response Spectrum Analysis (RSA) was performed. In this study, the time history data for El-Centro Earthquake, California, NS 1940 with maximum recorded ground acceleration of about 0.33g, where g is the acceleration due to gravity was used. The recorded acceleration was for the first crucial 30 seconds. RSA analysis was performed for the structure without damper. The maximum amplitude of 0.371 m was obtained at frequency of 0.293 Hz. Then RSA analysis was performed for structure with integrated TLD tanks of mass ratios varying between 0.2 to 1.6 %. The frequency bandwidth taken for analysis is 0.25 - 0.35 Hz. The plot between displacement (m) and frequency ratio (Hz) for all mass ratios is shown in Fig 6.

Table 2: Details of TLD dimensions

Mass Ratio %	Volume of water required (m ³)	Length of tank (m)	Height of tank (m)	Width of tank (m)	Number of Tanks
0.2	43.32	5	0.985	4.25	2
0.4	86.64	5	0.985	4.25	4
0.6	129.96	5	0.985	4.25	6
0.8	173.28	5	0.985	4.25	8
1	216.6	5	0.985	4.25	10
1.2	259.92	5	0.985	4.25	12
1.4	303.24	5	0.985	4.25	14

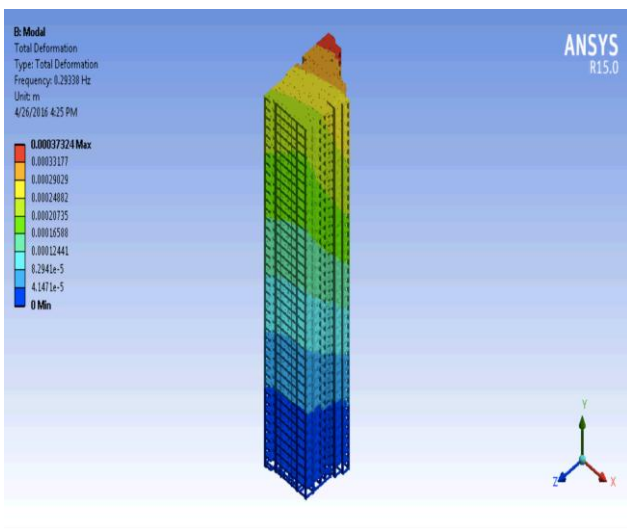


Fig 5: First Mode Shape

TLD's for varying mass ratios were then designed such that the sloshing frequency becomes equal to 0.293 Hz. Designing of TLD includes fixing the tank dimensions such as length and width, calculating the depth of water, and determining the number of similar tanks required. Based on the guidelines described by Banerji et. al. [2], TLD's were designed for mass ratios varying between 0.2 - 1.6 %. The ratio of water height to the length of the tank was fixed at 0.197 while the ratio of width to length was taken as 0.8. Fixing the length of tank as 5m and height of water as 0.985 m, we get sloshing frequency (Eq. 17) as 0.293 Hz. Table 2 shows the details of TLD tanks required for each mass ratio.

Table 3: Normal Mode Analysis of Building with TLD tanks for varying mass ratios

Building with TLD tanks for mass ratios (First 10 modes)								
0 %	0.2%	0.4%	0.6%	0.8%	1.0%	1.2%	1.4%	1.6%
0.293	0.296	0.299	0.303	0.307	0.311	0.313	0.317	0.322
0.357	0.359	0.362	0.368	0.372	0.374	0.378	0.383	0.389
0.381	0.383	0.385	0.385	0.388	0.392	0.394	0.408	0.414
0.986	0.995	1.004	1.078	1.105	1.132	1.155	1.183	1.213
1.128	1.137	1.146	1.197	1.218	1.243	1.261	1.297	1.328
1.203	1.207	1.212	1.243	1.254	1.272	1.287	1.317	1.342
1.895	1.912	1.930	1.956	1.983	2.000	2.023	2.046	2.063
2.030	2.047	2.060	2.080	2.097	2.113	2.126	2.159	2.168
2.212	2.217	2.223	2.240	2.249	2.269	2.284	2.321	2.325
2.797	2.821	2.847	2.886	2.917	2.958	2.996	3.055	3.104

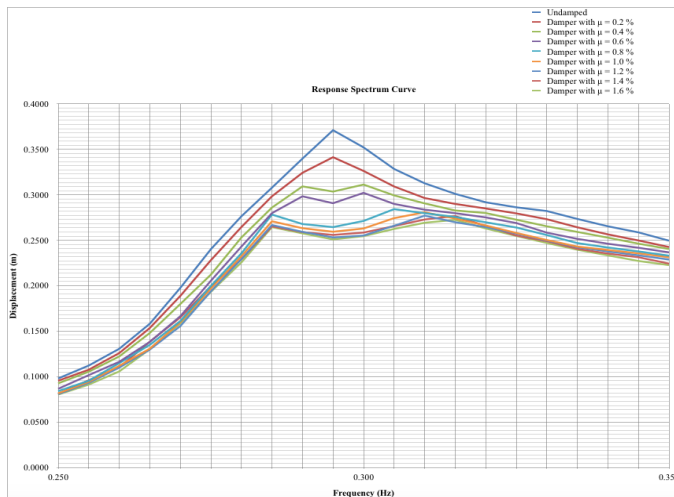


Fig 6: Response Spectrum Analysis curve for building with damper for varying mass ratios

Table 4: Percentage Reduction achieved for various mass ratios

Description	Maximum Displacement (m)	Percentage Reduction
Without damper	0.3710	-
Damper with mass ratio	0.2	8.0054
	0.4	18.1941
	0.6	21.6442
	0.8	28.7332
	1.0	30.1078
	1.2	31.8059
	1.4	31.0512
	1.6	32.3720

Table 4 shows the percentage reduction in the amplitude for varying mass ratios. It was found that the maximum peak amplitude of 0.371 m was reduced when the dampers were installed. When the dampers were installed, it was found that there was considerable reduction in the maximum peak amplitude for mass ratios 0.2, 0.4, 0.6 and 0.8 %. Then it was found that the reduction in amplitude for the remaining mass ratios were almost same, the differences very marginal. Also, from structural perspective, it is advisable not to provide any extra mass in the building greater than 1 %. Thus 0.8 % mass ratio is taken as the most optimum considering the structural, damping and economy factors. The reduction in maximum amplitude achieved was 28.73 %. So 8 tanks was proposed with 5 x 4.25 x 3 m dimensions. The water depth in each tank is to be maintained at 0.985 m.

V. CONCLUSIONS

Current trends in construction industry demands taller and lighter structures, which are more flexible and have low damping value. This increases failure possibilities and problems considering serviceability point of view. Several techniques are available today to minimize the vibration of the structure, out of which concept of using of TLD is studied in this work. TLD's were designed to have a sloshing frequency equal to the fundamental natural frequency of the structure. Designed TLD's were for mass ratios ranging from 0.2 % to 1.6 %. Following conclusions could be made from the results.

1. From this study, it has been found that the TLD can be successfully used to control vibration of the structure.
2. After carrying out the normal mode analysis of the structure with TLD tanks, it was found that the structural frequency increase with increasing mass ratios, making it less vulnerable to exciting forces.
3. An attempt was done to optimise the mass ratio of TLD's for the particular structure. It was found that 0.8 % would be the most optimum considering the structural, damping and economy factors.
4. The reduction in amplitude was found as 28.73 %.
5. 8 tanks are proposed with 5 x 4.25 x 3 m dimensions. The water depth in each tank is to be maintained at 0.985 m.

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