

Effect of Warping Torsion on the Behavior of Open and Closed Section Core Walls in High Rise Buildings

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ABSTRACT

In building structures, the elevator cores assembled with connected shear walls often provide full or partial resistance to lateral loading. The twisting of the core wall under lateral loading induces bimoment and causes the warping of the plane sections of core walls. Since the cores are restrained at the base and at floor levels, warping induces longitudinal warping strains and stresses throughout the height of the walls. St Venant's theory is inadequate for the analysis of concrete cores subjected to twisting. It is necessary to study the torsional behavior of such core walls subjected to torque caused by lateral loading. Open core wall sections are subjected to higher intensity of stresses and curvature compared to the closed sections. The partial closure of core walls by spandrels and floors increases the torsional stiffness thus restrains the warping the core walls to certain extent. The provision of spandrels reduces the curvature, displacement and warping stresses in the closed sections. Since computer application FEM softwares have limitation on considering the effect of bimoment, it is essential to accompany the warping stresses along with the direct stresses obtained from the computer FEM analysis. The computer aided FEM analysis plate stresses are accompanied with the warping stress. Consequently, stresses in the core wall increases. Therefore, in the buildings that are predominately supported by core walls for lateral resistance, the warping effects should be considered.

Keywords – Bimoment, Core walls, FEM analysis, Sectorial Properties, Warping stress

I. INTRODUCTION

Reinforced concrete cores usually comprise an assembly of connected shear walls forming a box section with openings that may be partially closed by beams and floor slabs. These cores are designed to resist the torsional loading in addition to lateral loading as already mentioned. If a building is subjected to twist, the torsional stiffness of the core can be significant part of the torsional resistance of the building. Due to twisting of core walls, the plane sections of core warp. Since the base slab section is prevented from warping by foundation, the twisting induces longitudinal warping strains and stresses

throughout the height of the core walls. Therefore, St. Venant theory is inadequate for the analysis of concrete cores. In the present practice, the buildings are analyzed using sophisticated software technologies. The computer aided analysis may be matrix methods or finite element analysis. Since most of the computer application FEM software have the limitation on the effect of bimoment in core wall buildings; a separate analysis check is essential to consider the net effect due to the warping moment and St Venant moment. This paper includes the work of analysis core walls by classical theory and computer aided FEM analysis of core wall supported building subjected to lateral loads. The aim of the subject to study the torsional behavior of the closed core wall and open core walls and accompanying the computer aided output with closed form solutions to obtain ultimate stress in the core walls.

II. LITERATURE REVIEW

There is limited experimental study is done on the analysis and building core walls with closed and open sections. However, in most of the studies, classical theory of thin wall sections has been used for analysis. Experimental tests on thin-walled U profiles loaded by shears and torsional forces relieved that analytical formulation can be considered as a suitable tool for structural analysis of high rise of high-rise buildings stiffened by thin wall open section bracings [1]. Closing of last storey in core wall shown significant improvement in controlling the drifts and story ration of about last quarter of building height. Also reduction in the beam share at last quarter height of the building [2]. Another study on warping analysis of cores [5] implies that under substantial torsional moment, the warping and header beam forces are significant and shall not be neglected in design. Other studies are involved in the finite element response of thin wall sections.

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III. WARPING TORSION THEORY

A thin walled section with an open cross section that is subjected to torsional moment presents a perceptible distortion in each cross-sectional plane. The longitudinal fibers of the wall section deformed along this length so that no cross section remain planar but undergoes warping. The distortion of cross section occurs due to torsional moment applied to an open section is taken up only by the shear flow in each of its constituent thin wall section (St. Venant) but also by their bending within the plane as shown in fig1.1. This creates bending rotations of sections of the elements within their plane and the synthesis of these bending rotation of the elements leads to warping of the cross section.

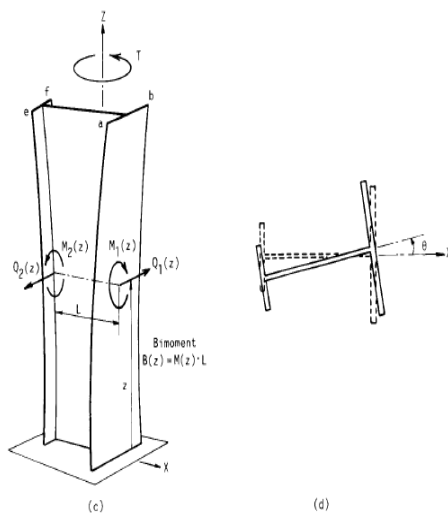


Figure 1.1 I section core subjected to Torque

A simple example of restrained warping of a thin core is I section cantilever fixed at its base. When a twisting moment T about the Z axis is applied to the top of the section in fig1.1. It twists about its shear center axis with the flanges bending in their planes about X axis and twisting about the longitudinal axis which ultimately leads to warping of the core wall. The twisting moment $T(Z)$ is resisted internally by a couple $Tw(Z)$ resulting from the shears in flanges and associated with their in-plane bending and a couple $Tv(Z)$ resulting from shear stresses circulating within the section and associated with the twisting of the flanges. Therefore

$$T(z) = T_v(z) + T_w(z) \dots (3.1)$$

The derivative of the above equation is

$$\frac{dy_1}{dz}(z) = x_1 \frac{d\theta}{dz}(z) \dots (3.2)$$

$$\frac{d^2 y_1}{dz^2}(z) = x_1 \frac{d^2 \theta}{dz^2}(z) \dots (3.3)$$

$$\frac{d^3 y_1}{dz^3}(z) = x_1 \frac{d^3 \theta}{dz^3}(z) \dots (3.4)$$

In multistoried buildings of height H , torsional moments applied at the floor slab level Z along the height of the building. Therefore, solution of uniformly distributed torque is ideal for such applications. Then solutions of above equations for uniformly distributed torque results in

Curvature at height 'Z' given by

$$\theta(z) = \frac{mH^4}{EI_w} \left\{ \frac{1}{(\alpha H)^4} \left[\frac{(\alpha H \sinh \alpha H + 1)}{\cosh \alpha H} (\cosh \alpha z - 1) - \alpha H \sinh \alpha z + (\alpha H)^2 \left[\frac{z}{H} - \frac{1}{2} \left(\frac{z}{H} \right)^2 \right] \right] \right\} \dots (3.5)$$

Bimoment at height 'Z' given by

$$B(z) = -mH^2 \left\{ \frac{1}{(\alpha H)^2} \left[\frac{(\alpha H \sinh \alpha H + 1)}{\cosh \alpha H} (\cosh \alpha z) - \alpha H \sinh \alpha z - 1 \right] \right\} \dots (3.6)$$

Where

$$\alpha H = H \sqrt{\frac{GJ}{EI_w}} \dots \dots (3.7)$$

m= St.Venant Torque

H= height of the building

Warping stress at any section is given by

$$\sigma(s, z) = \frac{B(z) \cdot \omega(s)}{I_w} \dots \dots (3.8)$$

3.1 Sectorial properties of thin wall sections

The rotational counterpart of planar wall-frame behavior, restrained warping involves a set of sectorial parameters. These sectorial parameters such as sectorial coordinate, shear center, principal sectorial properties warping moment of inertia have been calculated in preceding sections.

a. Sectorial coordinate ω'

The sectorial coordinate at a point on the profile of a warping core is the parameter which expresses the axial response, i.e., displacement, strain, and stress at that point relative to the response at other points around the section. The value of the sectorial coordinate point at any point P, figure 3.1 on the profile is given by

$$\omega'(s) = \int_0^s h \cdot ds \dots \dots (3.8)$$

Where h is the perpendicular distance from the pole 'O' to the tangent to the profile at 'P' and 's' is the distance of 'P' along the profile from P₀. The sectorial coordinate ω' is equal to twice the area swept out by a radius vector O'P in moving from P₀ to P. It increases positively for a radius vector sweeping anticlockwise and negatively sweeping clockwise.

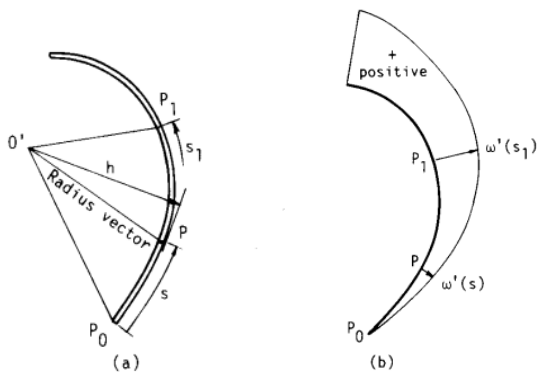


Figure 3.1 (a) Profile of section; (b) sectorial coordinate ω' diagram

a. Shear center

The shear center of a core is a point in the plane of its section through which a load transverse to the

$$\sigma(s, z) = \frac{B(z) \cdot \omega(s)}{I_w} \dots \dots (3.8)$$

core must pass to avoid causing torque. It can be calculated taking a ratio of sectorial moment of inertia to the second moment of inertia of the section.

$$\alpha_{x/y} = \frac{I_{\omega_{x/y}}}{I_{x/y}} \dots \dots (3.9)$$

Principal sectorial coordinate ω diagram

The ω diagram is related to the shear center 'O' as its pole and a point of zero warping deflections as an origin. The values of ω can be found by sweeping around the profile and taking twice the values of the swept areas or by the equation

$$\omega = \omega' - \alpha_x y \dots \dots (3.10)$$












c. Sectorial Moment of inertia

This parameter expresses the warping torsional resistance of the core's sectional shape. The sectorial moment of inertia derived from the principal coordinate distribution using.

$$I_w = \int \omega^2 \cdot dA \dots \dots (3.11)$$

This parameter can also be calculated with the help of product integral table with and sectorial coordinates.

Table 3.1 Product Integrals

PRODUCT INTEGRAL TABLE $\int_0^L F_1(x) F_2(x) dx$				
$F_1(x)$ \ $F_2(x)$				
	abl	$\frac{1}{2}abl$	$\frac{1}{2}abl$	$\frac{a}{2}(b_1 + b_2)$
	$\frac{bl}{2}(a_1 + a_2)$	$\frac{bl}{6}(a_1 + 2a_2)$	$\frac{bl}{6}(2a_1 + a_2)$	$\frac{L}{6}(2a_1b_1 + a_1b_2 + a_2b_1 + 2a_2b_2)$
	$\frac{1}{2}abl$	$\frac{1}{3}abl$	$\frac{1}{6}abl$	$\frac{1}{6}aL(b_1 + 2b_2)$
	$\frac{2}{3}abl$	$\frac{1}{3}abl$	$\frac{1}{3}abl$	$\frac{1}{3}aL(b_1 + b_2)$
	$\frac{1}{3}abl$	$\frac{1}{4}abl$	$\frac{1}{12}abl$	$\frac{a}{12}L(b_1 + 3b_2)$
	$\frac{2}{3}abl$	$\frac{5}{12}abl$	$\frac{1}{4}abl$	$\frac{a}{12}L(3b_1 + 5b_2)$
	$\frac{Lb}{6}(c + 4d + e)$	$\frac{Lb}{6}(2d + e)$	$\frac{Lb}{6}(c + 2d)$	$\frac{L}{6}[b_1(c + 2d) + b_2(2d + e)]$

d. Shear Torsion constant J

The twisting rigidity of open section given by GJ. When an open section is subjected to torque, the shear stresses are distributed linearly across the thickness of the wall. The effective lever arm of these stresses is equal to the two-third of the wall thickness. The torsion constant for this thin open section given by

$$J = \frac{1}{3} \sum_{n=1}^n b t^3 \dots\dots\dots (3.12)$$

Where,

‘b’ is the width and ‘t’ is thickness of wall.

When a closed section subjected to torque, the shear stresses circulate around the profile and uniform across the walls. The total shear torsion constant for closed sections given by

$$J = \frac{1}{3} \sum_{n=1}^n b t^3 + \frac{\Omega^2}{\oint ds / t} \dots\dots\dots (3.13)$$

Where

Ω = Twice the area enclosed by the profile of the section

$\oint \frac{ds}{t}$ = Line integral taken around the profile

IV. PROBLEM- CASE STUDY

Several computer programs consider only St.Venant torsion under the effect of twisting and neglect the warping moments. This paper involves studying the effect of warping moment in open and closed section core walls and combining the effect of warping stress with computer-aided output i.e., longitudinal stress to get the ultimate stress in core walls due to lateral twist. The figure 4.1 shows an example of 30 storey building. The plan is assumed to be typical for each floor. The structural system is building frame system as per UBC 97. Wind loads are applied along with gravity loads. Wind load in positive Y direction considered to be predominant. The floor diaphragm is assumed to be rigid and wind load acts at the center of mass of the building. The corewall is eccentrically located in the X direction of the building. Due to this eccentricity, building is subjected to twisting about its longitudinal axis. The bottom support idealized as fixed support to the core walls and columns. Computer modelling generated for with and without spandrel. For model without spandrel, the slab is connected to the core walls, thus slab thickness act like shear collector. Table 4.1 describe the basic data considered for building analysis.

4.1 Building properties

Table 4.1

Sl	Description	Data/Details
1	Length x Breadth	27mx25m
2	Height	90m
3	Building system	Building frame system
4	Materials	$f_{ck}=M32/40$,
5	Elastic modulus, E_c	26587 kN/m ²
6	Shear modulus, G_c	1100 kN/m ²
7	No of floors	30
8	Column size	600x600
9	Beam size	300x500
10	Core wall thickness	300mm
11	Thickness of slab	250mm
12	Spandrel size	300x750mm
13	Wind load	BS 6659
14	Basic Wind speed	30m/s

3D view of core wall and building

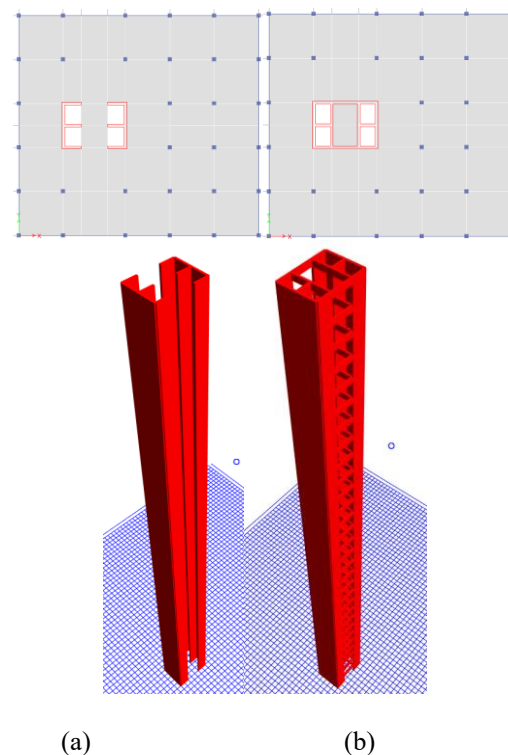
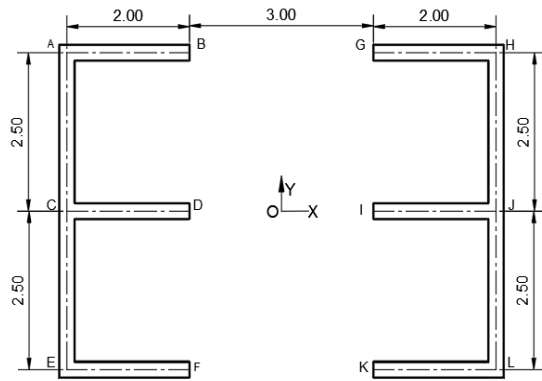


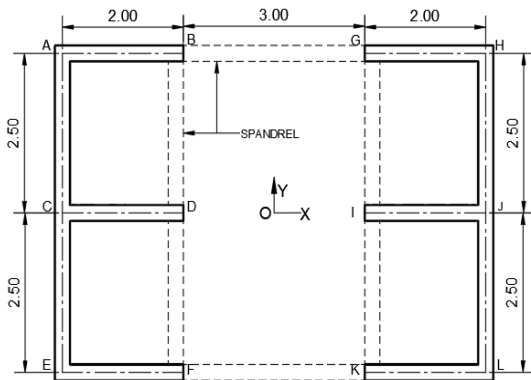
Figure 4.1. Computer Mathematical model;
(a) 3D-building; (b) 3D-Core wall

4.2 Geometry of the core wall

The figure 4.2 shows the geometry of open and closed core walls. The thickness of the core wall is 300mm. The height of the wall between floors 3m. The closed core wall section consists of a spandrel of size 300x750mm forming the closed geometry.



(a) Open section



(b) Closed section

Figure 4.2 Core wall geometry-Plan view

Table 4.2 Warping properties

Property	Data
Torsion constant, J	0.198(Open Section) 18.42(Closed section)
Warping M.I, m ⁶	89.36

The wind load acts at distance 4.72m from the shear center of the core wall. The uniformly distributed torsional moment along the height of corewall is 83kNm anticlockwise.

i) Bimoment for Open section

$$\alpha H = 90 \sqrt{\frac{1.10 \times 10^7 \times 0.10}{2.658 \times 10^7 \times 74.71}} = 2.12$$

$$B(z) = -39 \times 90^2 \left\{ \frac{1}{(2.12)^2} \left[\frac{(2.12 \sinh 2.12 + 1)}{\cosh 2.12} (1) \right] \right\}$$

$$B(z) = 91123 kNm^2$$

ii) Bimoment for a closed section

$$\alpha H = 90 \sqrt{\frac{1.10 \times 10^7 \times 14.50}{2.658 \times 10^7 \times 75.89}} = 25.30$$

$$B(z) = 11993 kNm^2$$

iii) Warping stress at point B in Figure 4.1a for open section core wall

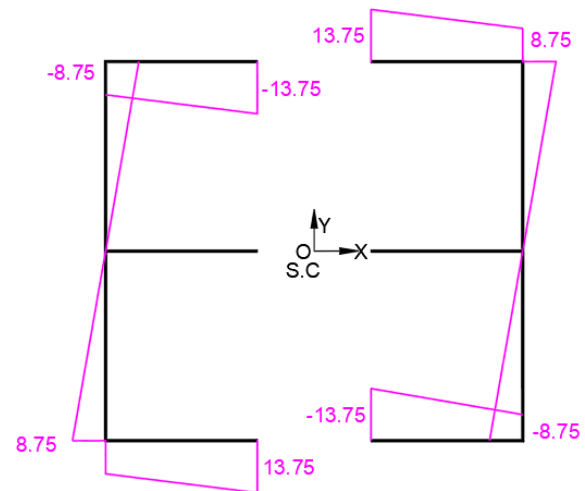
$$\sigma(B, z) = \frac{91123 \times 3.45}{74.71} = 4207 kN/m^2$$

Similarity at other points warping stress have been computed using equation 3.8.

4.3 Calculation of sectorial properties of core wall section

The sectorial properties have been obtained as per the concept explained in section III. The figure 4.3, 4.4, 4.5 and 4.6 shows sectorial properties of open and closed sections.

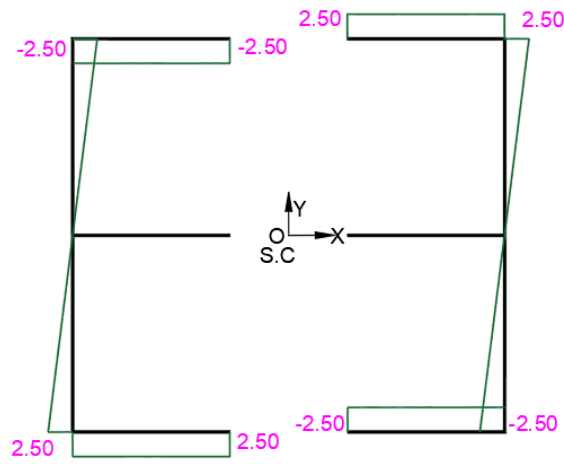
i.) Sectorial coordinate diagram for open section



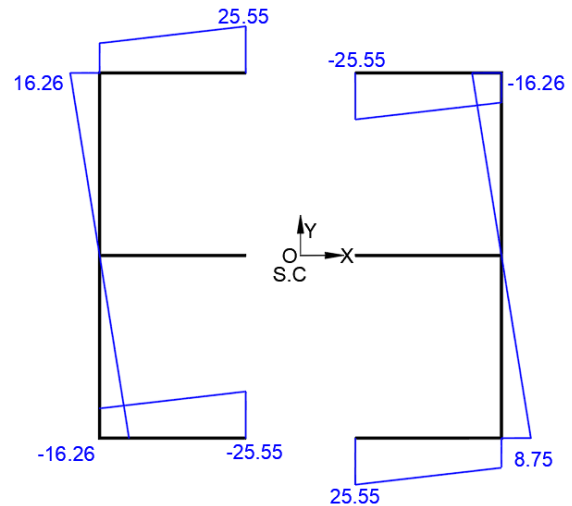
(b)

Figure 4.3 (a) ω' diagram; (b) y diagram

Y coordinate diagram



ii) Principal sectorial diagram

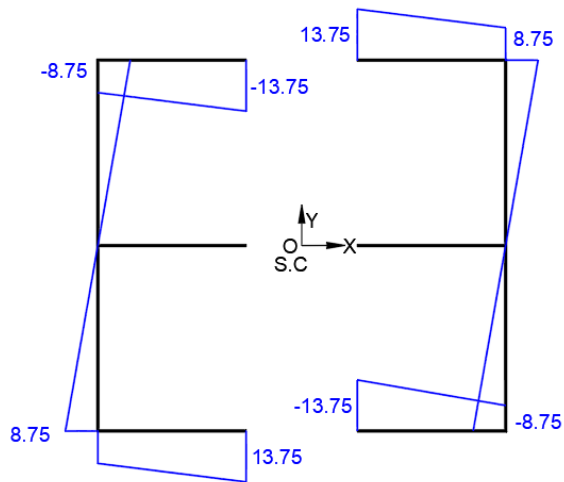


(a)

(b)

Figure 4.6 (a) ω diagram; (b) warping stress, MPa

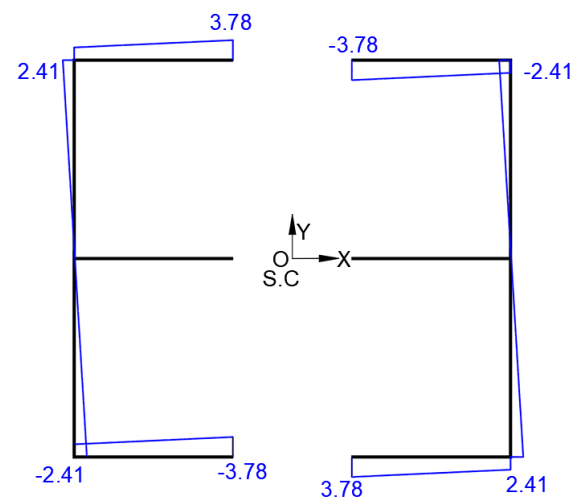
Warping stress diagram for closed section



(a)

(b)

Figure 4.4 (a) ω diagram; (b) warping stress, MPa



V. RESULTS AND DISCUSSIONS

Bimoment, curvature, stresses and displacement of open and closed sections have been studied in this section. Also, FEM analysis outputs of closed and open sections are included in the observation. Various graphs have been plotted to understand the significance difference between the closed and open sections.

5.1 Bimoment

The figure 5.1 displays the bimoment plot between the Open section and closed section core walls. The bimoment at the base of the building for open and closed sections are -166083kJNm^2 and -24603kJNm^2 respectively. Open section experiences high intensity of bimoment under the twisting moment.

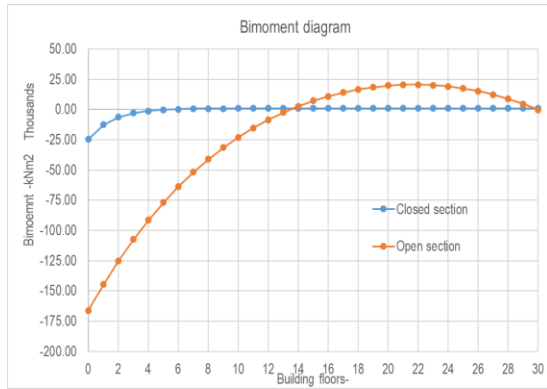


Fig 5.1 Bimoment plot diagram

5.2 Curvature

The figure 5.2 represents the curvature of the core walls. Open section exhibits high curvature of 0.07851 at top of the building and least curvature of 0.0002821 at first floor level. Closed section exhibits high curvature of 0.001533 at top of the building and least curvature 0.00004946 at first floor. The curvature plot between two core walls shows that closed sections are stiffer than open sections.

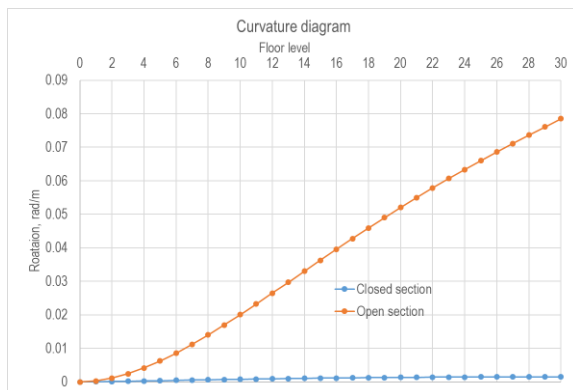


Figure 5.2 Bimoment curvature plot

5.3 Displacement

Below figure 5.3 shows that the open section is more highly displaced than the closed core wall sections. The highest displacement value is 275mm for an open section and 5mm for a closed section at the top of the building.

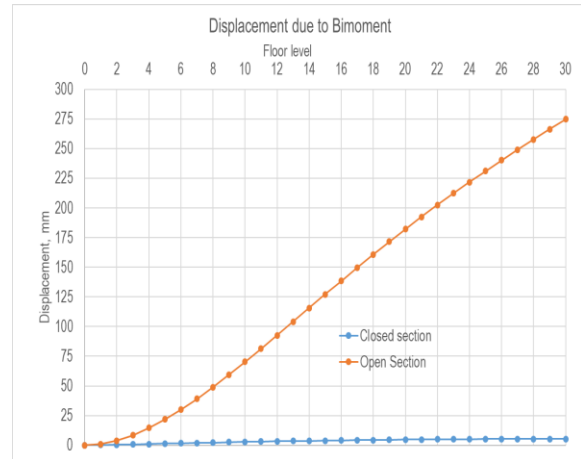


Figure 5.3 Bimoment- displacement plot

FEM analysis displacement output from a computer program is shown in figure 5.4. The plot shows that there is hardly a difference in the value of displacement for open and closed sections. The maximum displacement at top of building for open and closed sections are 49mm and 48mm respectively. This implies that the effect of bimoment is absent.

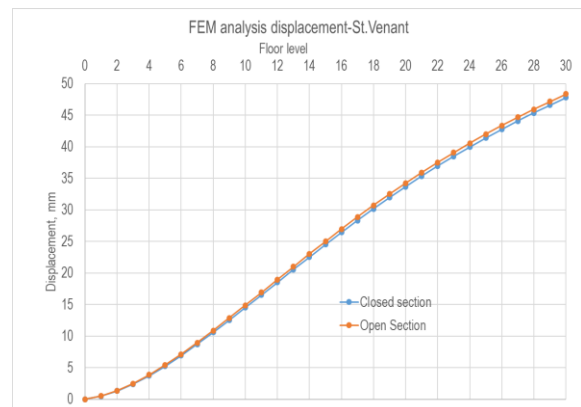


Figure 5.4 FEM analysis- displacement plot

The total displacement of core walls is obtained by summation of displacement due to bimoment and St.Venant displacement from FEM analysis outputs. Below plot in figure 5.5 shows the total displacement values. It shows that additional displacement due to bimoment in open section is substantially higher than closed sections and much beyond the limits specified in code standards. Whereas in closed sections, total displacement due to bimoment and St Venant torsion is 53mm. The accompanied displacement of bimoment increased the total displacement in both open and closed sections. Therefore, the effect of bimoment should not be avoided.

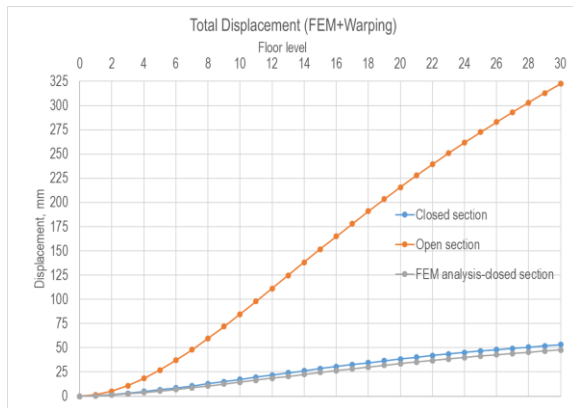


Figure 5.5 Total displacement plot

5.3 Warping stress plot

The figure 5.3 shows the envelope of stress diagram for open and closed section. The open sections are subjected to higher warping stress intensity. Discontinuous ends of open core walls susceptible for higher stresses. Closing sections with spandrel makes the shear flow uniform within the sections, thus effect bimoment is controlled to the greater extent.

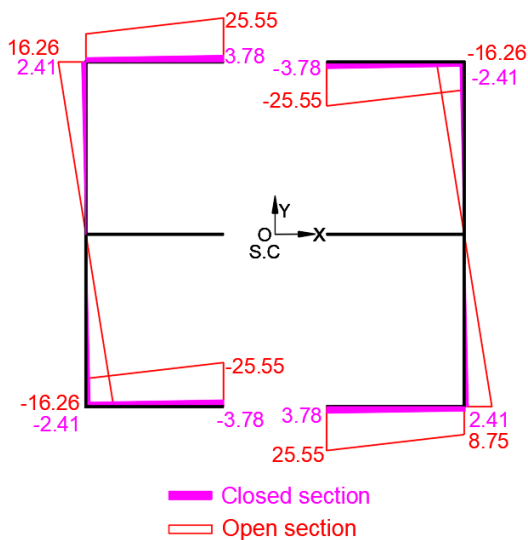


Fig 5.3 Envelope stress diagram for closed and open section

5.4 Longitudinal direct stress diagram from Computer program

Longitudinal stress for open and closed sections obtained from FEM analysis shown in below figure. For both types of sections, the longitudinal stress pattern is same. This also confirms the absence of

effect of bimoment in computer output. The equivalent 2D stress distribution diagram is shown in figure.

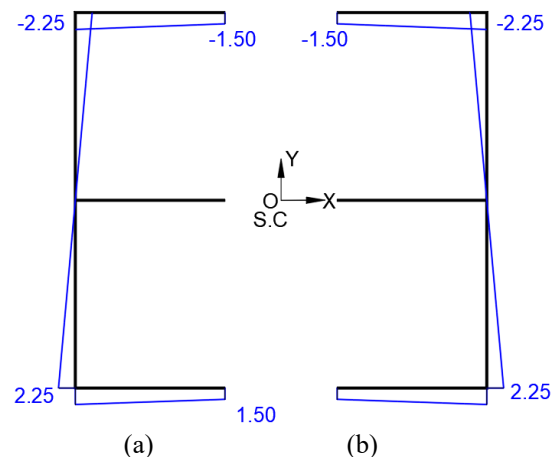
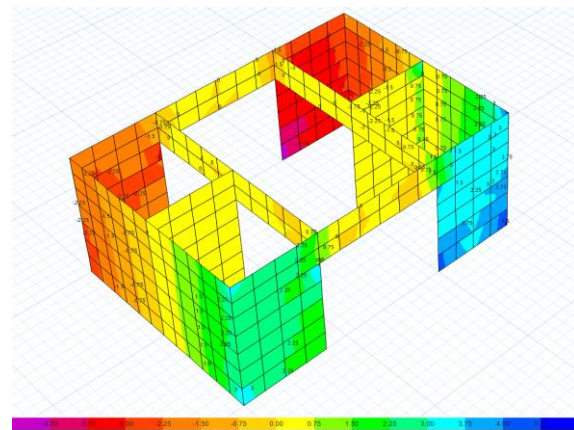
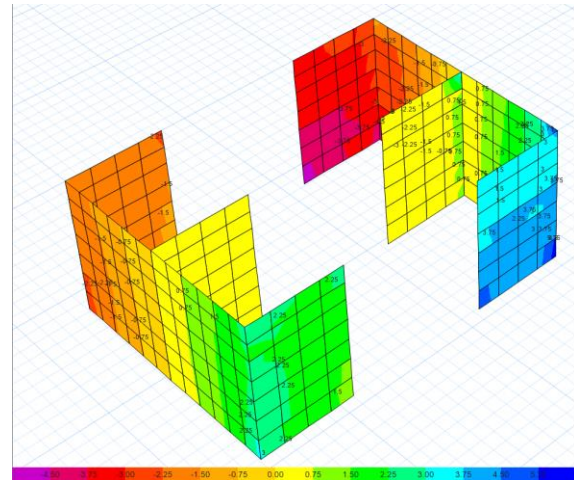


Fig 5.4 Longitudinal direct stress; (Approximate plot) (a) Plate stress, FEM output (b) Plate stress intensity in 2D view.

5.5 Superposition of FEM plate stress and warping stress

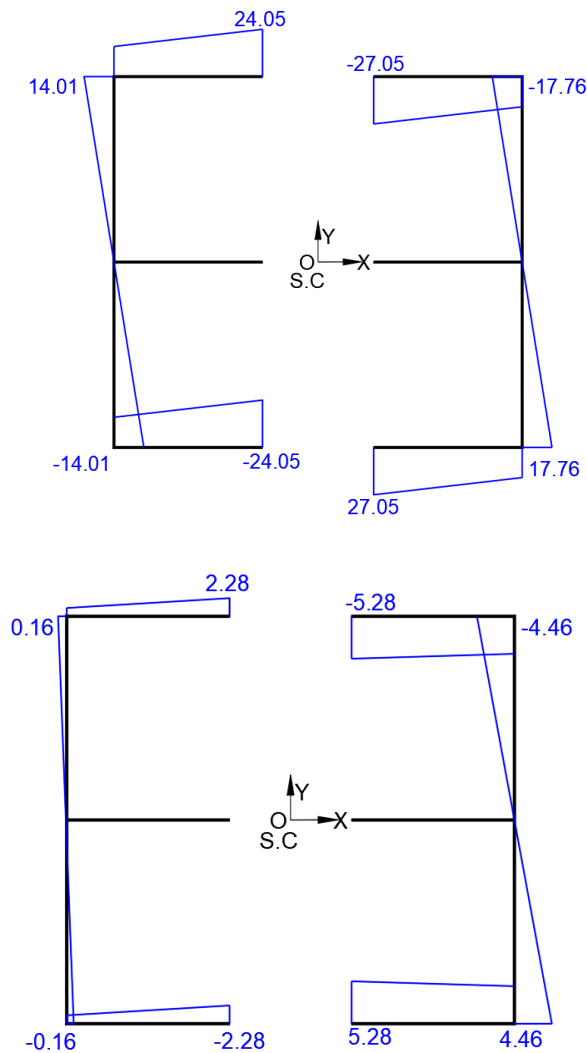


Fig 5.5 Ultimate stress diagram;

The longitudinal stress diagram obtained from computer program FEM analysis has been considered to understand the complete response of corewalls in terms of displacement and stresses. The accompanied effect of bimoment to the FEM analysis output results in enhanced curvature, displacement and stress. Thus, implies the importance of bimoment in corewall buildings. An additional analysis is essential to understand the complete response of corewall buildings under lateral twisting. Therefore, in buildings that are predominantly depending on open section core walls for torsional resistance, the warping effects should not be neglected.

superimposed with the warping stress. The total ultimate stress intensity here increases for both open and closed sections. However, closed sections exhibit lower intensity compared to open sections. The ultimate stress should be less than code specified limits.

VI. CONCLUSIONS

Open section core walls exhibit substantially higher deformation under twisting moment. Non-existence of spandrels at the opening locations within the core walls generates considerably higher bimoment, thus increased curvature and displacement. Provision of spandrels at the opening location controls the deformation of corewalls to greater extent. However, the effect of bimoment is to be

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