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Effect of Volumetric Heat Generation / Absorption on Convective Heat and Mass Transfer in Porous Medium Between Two Vertical Porous Plates

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"Abstract"

Aim of the paper is to investigate the effect of volumetric heat generation, which is a function of local species concentration on steady free convective mass transfer flow of a viscous fluid through a porous medium bounded by two stationary infinite vertical porous plates. The analytical solution to the problem is obtained and presented for different values of physical parameters through graphs. The skin friction coefficient, the heat and the mass transfer coefficients are derived, discussed numerically and presented through tables for different values of parameters.

1. Introduction

A large number of physical phenomena involve free/forced convection [Jaluria, (1)], which are enhanced and driven by internal heat generation. In such flows the buoyancy force is incremented due to heat generation resulting in modification of heat/mass transfer characteristics. Convection in the presence of internal heat generation/absorption has numerous applications in the fields of geophysical science, fire and safety engineering, nuclear science, chemical engineering etc. The volumetric rate of heat generation *Q* in the boundary layer flows generally has been presented in the literatures. Singh et al. [2] investigated the effect of volumetric heat generation/absorption on mixed convection stagnation point flow on an iso- thermal vertical plate in porous media taking the following form

$$Q = Q(T - T_\infty), \tag{1}$$

$$Q = Q(x, y, z). \tag{2}$$

The equation (1) represents volumetric heat generation/absorption [Vajravelu and Nayfeh (3) and Vajravelu and Hadjinicolaou (4)], which depends on local fluid temperature and is considered for exothermic/endothermic chemical reactions. The temperature distribution is not known a priori. The equation (2) represents the volumetric heat generation [Crepeau and Clarksean (5), Chamkha and Khaled (6)], which has priori known space distribution not controlled by flow pattern and local temperature distribution directly.
To find a solution the governing equations are made dimensionless and the volumetric rate of heat generation/ absorption discussed above would take the form $S\theta$, where *S* is heat source/sink parameter.
The local concentration of specie should also be a major factor affecting the volumetric rate of heat generation/ absorption. Singh et al. [2] proposed a new kind of volumetric heat generation/absorption

$$Q = Q(C - C_\infty) \tag{3}$$

In the dimensionless form it would take the form $S\phi$ where ϕ is the shape function i.e. the local concentration distribution.
Samad and Mohebujjaman [7] investigated MHD heat and mass transfer free convection flow along a vertical stretching sheet in presence of magnetic field with heat generation. Jha and Ajibade [8] discussed free convective flow of a heat generating/absorbing fluid between vertical porous plates with periodic

heat input. Abdallah [9] derived analytic solution of heat and mass transfer over a permeable stretching plate affected by chemical reaction, internal heating, Dufour-Soret effect and Hall effect. Ahmed et al. [10] discussed MHD free convective Poiseuille flow and mass transfer through a porous medium bounded by two infinite vertical porous plates.

Makinde [11] discussed heat and mass transfer by MHD mixed convection stagnation point flow toward a vertical plate embedded in highly porous medium with radiation and internal heat generation.

In the present paper the effect of volumetric heat generation, which is a function of local species concentration on steady free convective mass transfer flow of a viscous fluid through a porous medium bounded by two stationary infinite vertical porous plates is investigated.

2. Formulation of the Problem

The governing equations of continuity, momentum energy and specie are given by

$$\frac{dv}{dy}=0 \qquad \Rightarrow \quad v=-v_0 \quad \text{(Constant)}, \tag{4}$$

$$-v_0\frac{du}{dy}=\nu\frac{d^2u}{dy^2}+g\beta(T-T_\infty)+ g\beta_c(C-C_\infty)-\frac{\nu u}{\tilde{K}} \tag{5}$$

$$-v_0\frac{dT}{dy}=\frac{\kappa}{\rho C_p}\frac{d^2T}{dy^2}+Q, \tag{6}$$

$$-v_0\frac{dc}{dy}=D\frac{d^2C}{dy^2}, \tag{7}$$

where ρ is the density, ν is the kinematic viscosity, g is acceleration due to gravity, v_0 is suction/injection velocity, β is the coefficient of volume expansion for heat transfer, β_c is the coefficient of volume expansion for mass transfer, D is the chemical molecular diffusivity, $\tilde{K}$ is the permeability of the porous medium, C_p is the specific heat at constant pressure, T is the temperature, T_∞ is the fluid temperature far away from the plate, C is species concentration, C_∞ is the species concentration far away from the plate and u, v are velocity components along x, y-directions, respectively.

The boundary conditions are

$$y=0: u=0: T=T_0: C=C_0;$$

$$y=h: u=0: T=T_1: C=C_1. \tag{8}$$

3. Method of Solution

Introducing the following non-dimensional quantities

$$\eta=\frac{y}{y},\ f=\frac{u}{v_0},\ \theta=\frac{T-T_\infty}{T_0-T_\infty},\ \phi=\frac{C-C_\infty}{C_0-C_\infty}$$

$$Gr=\frac{hg\beta(T_0-T_\infty)}{v_0^2},\ Gc=\frac{hg\beta_c(C_0-C_\infty)}{v_0^2},\ K=\frac{\tilde{K}}{h^2},$$

$$Q=S\frac{(T_0-T_\infty)v_0}{\Pr\mathrm{Re}\,h}\phi$$

into the equations (5) to (7), we get

$$f''+\mathrm{Re}\,f'-\frac{f}{K}=-\mathrm{Re}(\theta\,Gr+\phi\,Gc), \tag{9}$$

$$\theta''+\Pr\mathrm{Re}\,\theta'+S\phi=0, \tag{10}$$

$$\phi''+Sc\,\mathrm{Re}\,\phi'=0, \tag{11}$$

where *Pr* is the Prandtl number, *Sc* is Schmidt number, *Re* is the cross flow Reynolds number, *Gr* is the Grashof number for heat transfer, *Gc* is the Grashof number for mass transfer, *Ec* is the Eckert number, *K* is the permeability parameter and *Q* is volumetric heat generation/absorption.

The corresponding boundary conditions are reduced in non-dimensional form as given below

$$\begin{aligned}&\eta=0:\ f=0,\ \theta=1,\ \phi=1;\\ &\eta=1:\ f=0,\ \theta=m,\ \phi=n.\end{aligned} \tag{12}$$

Equations (9) to (11) are ordinary second order differential equations and solved under the boundary conditions (12). Through straightforward calculations, the expressions of $f(\eta),\theta(\eta)$ and $\phi(\eta)$ are obtained.

$$f(\eta)=f_1(\eta)-f_2(\eta)-f_3(\eta), \tag{13}$$

$$\theta(\eta)=A_4+A_3\exp\{(-\Pr\mathrm{Re})\eta\}-B_1\eta+ B_2-B_3\exp\{(-\mathrm{Re}\,Sc)\eta\} \tag{14}$$

$$\phi(\eta)=A_2+A_1\exp\{(-\mathrm{Re}\,Sc)\eta\}, \tag{15}$$

4. Skin-friction coefficient

$$(C_f)_0 = \left(\frac{df}{d\eta}\right)_{\eta=0} \text{ and } (C_f)_1 = \left(\frac{df}{d\eta}\right)_{\eta=1} \tag{16}$$

5. Nusselt number

$$(Nu)_0 = \left(-\frac{d\theta}{d\eta}\right)_{\eta=0} \text{ and } (Nu)_1 = \left(-\frac{d\theta}{d\eta}\right)_{\eta=1} \tag{17}$$

6. Sherwood number

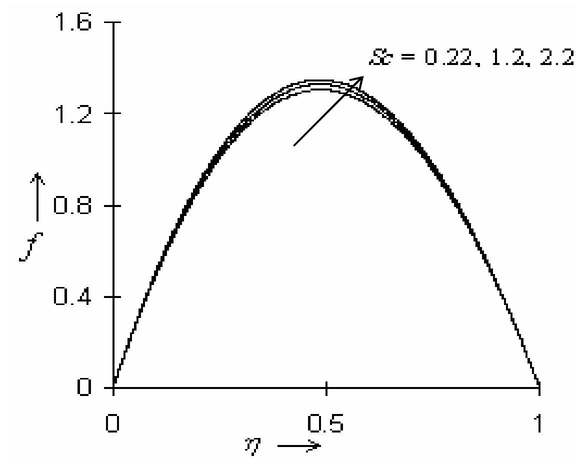
$$(Sh)_0 = \left(-\frac{d\phi}{d\eta}\right)_{\eta=0} \text{ and } (Sh)_1 = \left(-\frac{d\phi}{d\eta}\right)_{\eta=1} \tag{18}$$

7. Results and Discussion

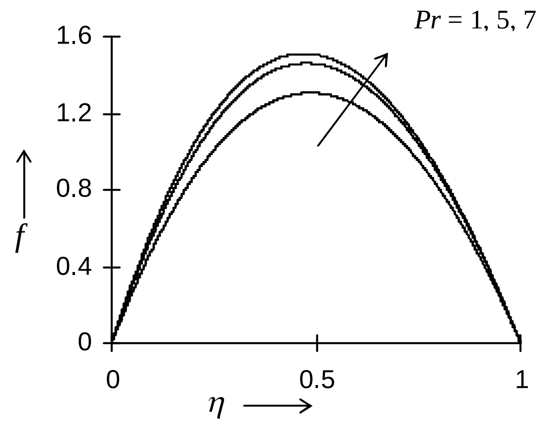
To get a physical view into the problem, the numerical values of velocity distribution, temperature distribution, species concentration distribution have been obtained and shown graphically.

The variation in the velocity field under the influence of Schmidt number *Sc* or Prandtl number *Pr* or Reynolds number *Re* or permeability parameter *K* are presented in figures 1, 2, 3 and 4 respectively. The fluid velocity increases with the increase of *Sc*, *Pr*, *Re* or *K*. Further the effects of the Grashof number for heat transfer *Gr* or Grashof number for mass transfer *Gc* are found from the figure 5 and figure 6. Which show that the fluid velocity enhances with the increment of *Gr* or *Gc*. It is observed from figure 7 that with the increase in parameter *S*, the fluid velocity increases.It is observed from figures 8, 9 and 10 that the fluid temperature increases with the increase in heat source parameter *S* or Schmidt number *Sc*, Prandtl number Pr. Figure 11 shows that species concentration increases with the increase in Schmidt number *Sc*.

It is observed from table 1 that increment in rate of heat generation/absorption parameter causes increase in skin friction coefficient and Nusselt number at first plate and decrease in the same at second plate. The skin friction coefficient initially increasing and decreasing later with the increment in Schmidt number. Again it is increases with the increase in Prandtl number whereas decreases with the increment of permeability parameter. The Sherwood number increases at first plate and decreases at the second plate with the increase in Schmidt number.



"Figure 1. Velocity distribution versus η when Gr = 5.0, Gc = 2.0, Pr = 1.0, Re = 1.0, m = 2.0, n = 2.0, K = 2.0, S = 0.0".



"Figure 2. Velocity distribution versus η when Gr = 5.0, Gc = 2.0, Sc = 0.22, Re = 1.0, m = 2.0, n = 2.0, K = 2.0, S = 0.0".

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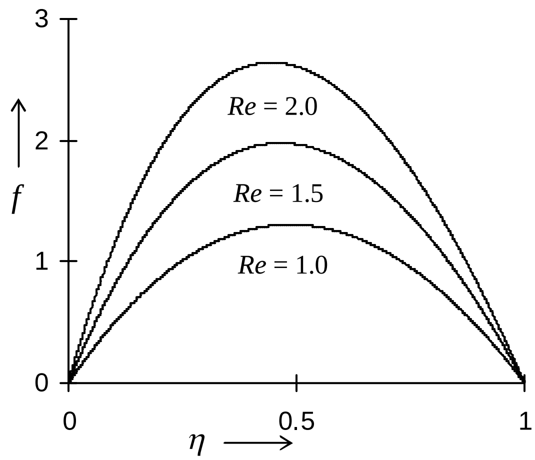
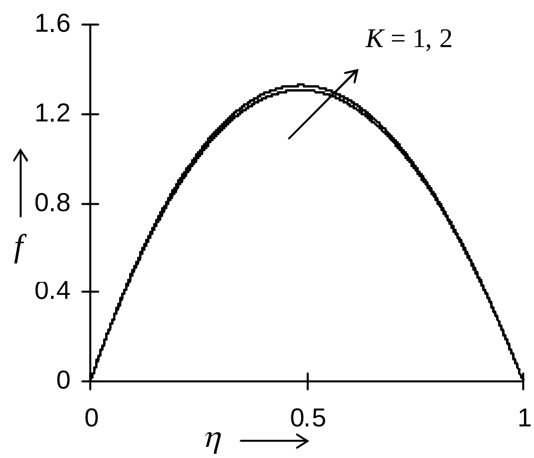
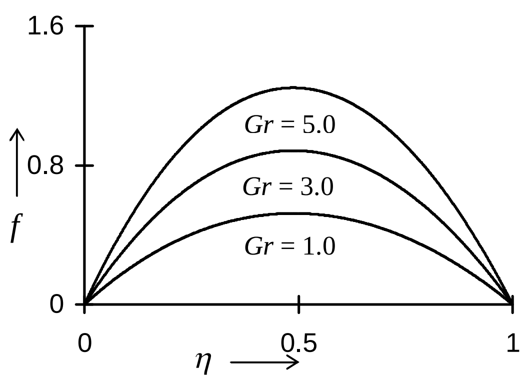


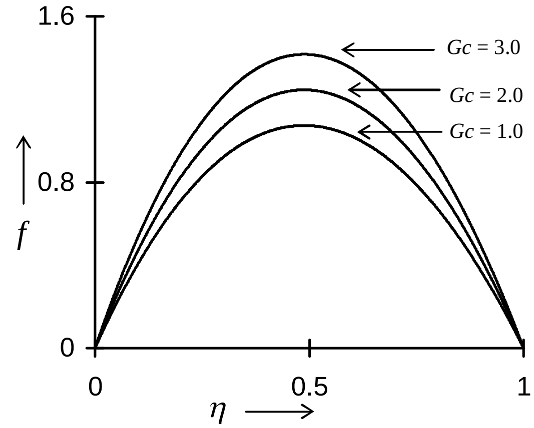
"Figure 3. Velocity distribution versus η when Gr = 5.0,Gc = 2.0,Sc = 0.22, Re = 1.0, m = 2.0, n = 2.0, K = 2.0, S = 0.0".



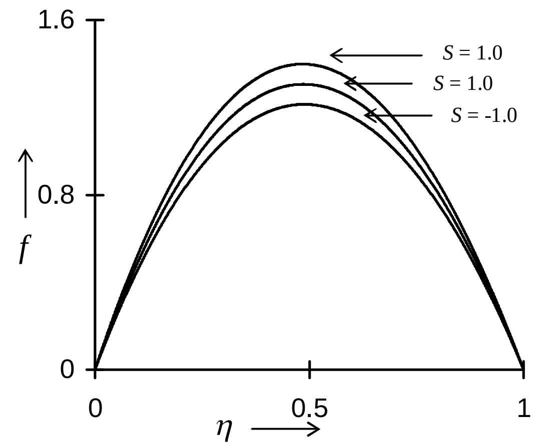
"Figure 4. Velocity distribution versus η when Gr = 5.0,Gc = 2.0, Sc = 0.22, Pr = 1.0, m = 2.0, n = 2.0, S = 0.0".



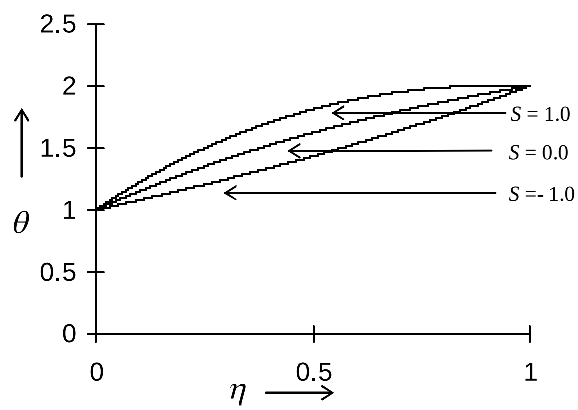
"Figure 5. Velocity distribution versus η when K = 1.0,Gc = 2.0, Sc = 0.22, Pr = 1.0, m = 2.0, n = 2.0, S = 0.0".



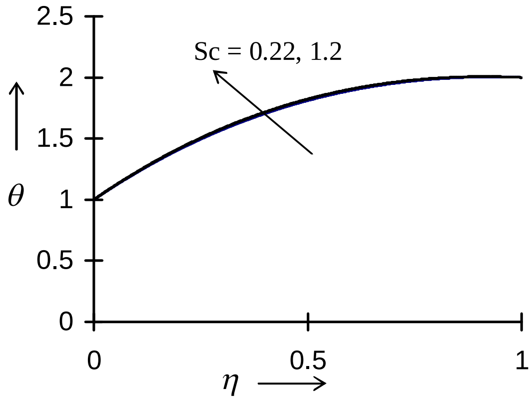
"Figure 6. Velocity distribution versus η when Gr = 5.0, K = 2.0, Sc = 0.22, Pr =1.0, m = 2.0, n = 2.0, S=0.0".



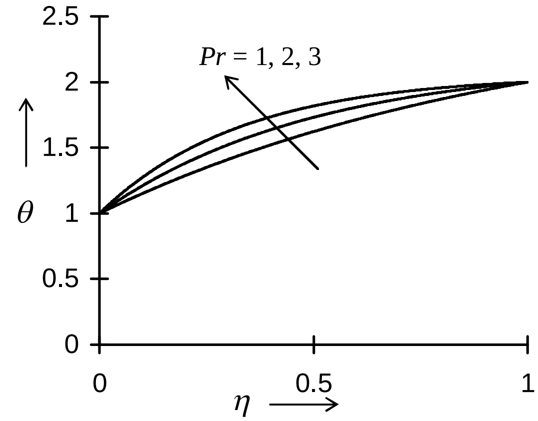
"Figure 7. Velocity distribution versus η when Gr = 5.0,Gc = 2.0, Pr = 1.0, Re = 1.0, m = 2.0, n = 2.0, K = 2.0, Sc = 0.22".



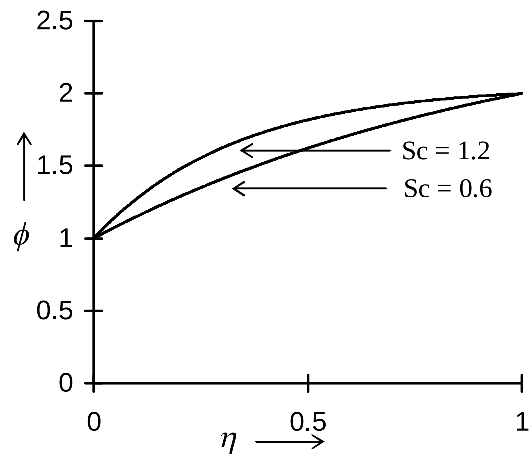
"Figure 8. Temperature distribution versus η when Pr = 1.0, Sc = 0.22, Gr = 5.0, Gc = 2.0,m = 2.0, n = 2.0".



"Figure 9. Temperature distribution versus η when Pr = 1.0, S = 1.0, Gr = 5.0, Gc = 2.0, m = 2.0, n = 2.0".



"Figure 10. Temperature distribution versus η when S = 1.0, Sc = 0.22, Gr = 5.0, Gc = 2.0, m = 2.0, n = 2.0".



"Figure 11. Species concentration versus η when Re = 1.0, n = 2.0".

Table 1. Numerical values of skin friction coefficient (Cf), Nusselt number (Nu) and Sherwood number (Sh) at both the plates for various values of physical parameters.

K	S	Pr	R_e	Sc	$(Sh)_0$	$(Sh)_1$	$(Cf)_0$	$(Cf)_1$	$(Nu)_0$	$(Nu)_1$
1	1	1	1	.22	1.114030	0.89403	4.19308	-6.206771	2.38399	-0.134329
1	1	1	1	.62	1.341830	0.721830	4.768734	-5.564032	2.404193	-0.147145
1	1	1	1	.92	1.529558	0.609558	4.725550	-5.485690	2.149317	-0.156291
1	1	1	1	1.2	1.717215	0.517215	3.647557	-5.740068	2.433275	-0.164404
1	1	5	1	.22	1.114030	0.894030	4.841201	-6.451462	6.198525	-0.319794
1	1	10	1	.22	1.114030	0.894030	5.115754	-6.470849	11.32797	-0.190340
1	1	15	1	.22	1.114030	0.894030	5.205654	-6.458614	16.38902	-0.129296
1	1	1	1.5	.62	1.536057	0.606057	9.209730	-7.155386	2.831617	-0.277488
1	1	1	2	.62	1.744966	0.504965	15.09481	-8.325213	3.290634	-0.310145
1	0	5	1	.62	1.341830	0.721830	5.122525	-5.649926	5.033918	0.033918
1	-1	5	1	.62	1.341830	0.721830	4.822035	-5.493314	3.840644	0.391983
1.5	1	5	1	.22	1.114030	0.894030	4.277225	-7.177588	6.198525	-0.319794
2	1	5	1	.22	1.114030	0.894030	3.714125	-7.769261	6.198525	-0.319794

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8. Appendix

$f_1(\eta) = A_1 \exp(-m_1\eta) + A_2 \exp(-m_2\eta)$,

$f_2(\eta) = \mathrm{Re}\, Gr\{C_1 + C_2 \exp\{(-\mathrm{Pr}\,\mathrm{Re})\eta\} + C_3\eta + C_4 + C_5 - C_6 \exp\{(-\mathrm{Re}\, Sc)\eta\}$,

$f_3(\eta) = \mathrm{Re}\, Gc\{C_7 + C_8 \exp\{(-\mathrm{Re}\, Sc)\eta)\}$,

$A_1 = D_1 - A_2$,

$A_2 = \{D_1 - D_2 \exp(m_2)\}/\{1 - \exp(m_1 - m_2)\}$,

$A_3 = (B_4 - B_5)/\{1 - \exp(-\mathrm{Pr}\,\mathrm{Re})\}$,

$A_4 = B_4 - A_3$,

$A_5 = (1 - n)/\{1 - \exp(\mathrm{Re}\, Sc)\}$,

$A_6 = 1 - A_5$,

$B_1 = SA_6 / \mathrm{Pr}\,\mathrm{Re}$,

$B_2 = (SA_6)/(\mathrm{Pr}\,\mathrm{Re})^2$,

$B_3 = (SA_5)/\{(\mathrm{Re}\, Sc)^2 - (\mathrm{Re}^2\, \mathrm{Pr}\, Sc)$,

$B_4 = 1 - B_2 + B_3$,

$B_5 = m + B_1 - B_2 + B_3 \exp(-\mathrm{Re}\, Sc)$,

$C_1 = -KA_4$,

$C_2 = A_3 / \{(\mathrm{Pr}\,\mathrm{Re})^2 - \mathrm{Pr}\,\mathrm{Re}^2 - 1/K\}$,

$C_3 = KB_1$,

$C_4 = K^2 B_1\, \mathrm{Re}$,

$C_5 = -KB_2$,

$C_6 = B_3 / \{(\mathrm{Re}\, Sc)^2 - \mathrm{Re}^2\, Sc - 1/K\}$,

$C_7 = -KA_2$,

$C_8 = A_1 / \{(\mathrm{Re}\, Sc)^2 - Sc\, \mathrm{Re}^2 - 1/K\}$,

$C_9 = C_1 + C_2 + C_4 + C_5 - C_6$,

$C_{10} = C_1 + C_3 + C_4 + C_5$,

$D_1 = \mathrm{Re}\, Gr\{C_9 + \mathrm{Re}\, Gc(C_7 + C_8)\}$,

$D_2 = \mathrm{Re}\, Gr\{C_{10} + C_2 \exp(-\mathrm{Pr}\,\mathrm{Re}) - C_6 \exp(-\mathrm{Re}\, Sc)\} + \mathrm{Re}\, Gc\{C_7 + C_8 \exp(-\mathrm{Re}\, Sc)\}$

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