

# Effect of using Materials of Different Characteristics on the Performance of a Supply Chain

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**Abstract**— In an ever changing constrained and competitive environment, any form of flexibility or a way for industrial organizations to reconfigure their resources is a turning point for better adaptation to changes. This paper studies supply chain operational decisions for an industry that can produce single-item from one of alternative materials that are used separately in manufacturing lots. Each alternative material has its own quality, price, and requires different manufacturing time. The problem is modeled in mathematical Integer Linear Programming (ILP) to determine the optimal quantity mix from alternative materials to maximize profit for certain operating conditions. Robust optimization is also used to obtain feasible solutions against quality uncertainty of alternative materials. In addition, mathematical models are developed, based on cost elements, to determine the values at which the decision has to be switched from using one material to its alternative. Results showed that considering mix of alternative materials provides flexibility against system constraints and material quality. The model cost elements determine the threshold values where the decision of using any of the alternative materials is changed. Results of the robust model gave the same trend at lower profits. Several analytical and numerical solutions are obtained relating supply chain parameters to optimal decisions.

**Keywords**—Order quantity allocation; supply chain; integer programming; variable production rate.

## I. INTRODUCTION

A supply chain network is a set of organizations, responsible for fulfilling the downstream requests. Many of the researchers who focused on the supply side of the network and studied multi-supplier systems were concerned with the suppliers' evaluation criteria and the selection process. Quality of purchased parts, reliability of on time delivery, and price are the main factors that support supplier selection [1]. A list of several evaluation criteria and methods are included in the review by Moliné et al. [2]. Other researchers were concerned with the selection process itself, in an optimization context. Rosenblatt et al. [3] called it the *acquisition policy*, and the aim was to determine from whom should the firm buy the product, in what quantities, and how often? Yang et al. [4] considered it as a sourcing problem. In general, it is known as “*supplier selection and order allocation problem*” [5-8].

A similar field of research can be found in lot sizing optimization problems considering multiple suppliers. Basneta and Leung [9] studied an inventory lot sizing

problem with multiple suppliers, and their definition of the problem was to decide on what products to order, in what quantities, with which suppliers, and in which periods. Their problem mainly differs from the supplier selection and order allocation problem in that the products are sourced from a set of approved suppliers. Mostly, the evaluation process is not integrated in the lot sizing optimization models, since it is considered as a strategic management decision [10].

Chuang [11] addressed the problem of order allocation, while being able to handle multiple conflicting objectives and soft constraints. A solution procedure for supplier selection problem was developed, and goal programming along with stochastic programming were implemented. Yang et al [4] developed an algorithm to maximize the expected profit of an order allocation problem under single-period stochastic demand and suppliers' random yields. They justified the necessity for suppliers with low reliability to reduce their ordering costs. Rezaei and Davoodi [12] used a genetic algorithm to obtain order allocation decisions in a multiple supplier, multiple product system, with limited storage capacity, over a multi-period horizon, and the received items are of imperfect quality. Duan and Liao [13] integrated a hybrid meta-heuristic optimization algorithm with an inventory simulation model to obtain optimal replenishment policies (i.e., ordering patterns) for a two-tier supply chain. Detailed analysis was provided regarding the effect of different demand patterns, suppliers' capacity constraint, supply chain adopted control strategy, and different ranking allocation rules on the optimal solutions.

Long before the term supply chain management was extensively used; many researchers studied the effect of incorporating quality of the acquired materials, quality of products, and reliability of manufacturing processes in their models. Porteus [14] introduced a mathematical model that captured the relation between quality and lot size. Agnihothri and Kenett [15] explicitly stated that defects and rework are common occurrences in a manufacturing process; they modeled the number of defects as a random variable. Huang [16] considered imperfect quality items in a just-in-time (JIT) manufacturing environment. Khan et al. [17] extended Huang's [16] vendor-buyer model to include human errors (e.g. inspection and learning errors). J. T. Hsu and L. F. Hsu [18] investigated the effect of both imperfect production processes and Type I, II inspection errors on the

optimal production quantities. Maddah and Jaber [19] derived an analytical expression for the optimal order quantity that maximizes the retailer's expected profit per unit time. In a non-capacitated problem, they analyzed the effect of unreliable suppliers and screening speed on the economic order quantity. Analysis showed that the order quantity is larger than the classical EOQ model when the variability of suppliers' yield rate is low. Sana et al. [20] considered a three layer supply chain model with multiple players in each layer. Multiple-products were produced from a combination of several raw materials. Imperfect quality was present in both the supplied materials and the produced products, and both the set-up and the screening costs were considered. The optimal solutions were replenishment lot sizes. Both the backward induction process as an optimization approach (i.e., determination of a sequence of optimal actions) and the collaborating system approach were used.

When process quality was considered; it was attributed to the used production rate, time, or random reasons. However, variable production rate and process quality can be related to the supplied material. An analogy can be found in crop planning problems, where the yield rate of crop production is a function of the crop being produced and soil characteristics; the utilization of the land for appropriate crops is the key issue for optimizing the problem [21-23]. Many industries face the decision of which material to select; from a potential list of materials that differ in quality, price, and the required production parameters.

There are many industrial applications for alternative materials to be used for the same production. For instance, cellophane and polypropylene can both be used alternatively for cigarette packet wrapping. The cellophane needs slower feed during the packaging process than the polypropylene; due to different properties it each yields different scrap percentages in processing. In textile industry, different fabrics may be handled differently and yield different waste rates in production. The present research concerns an order quantity allocation problem in a supply chain that produces single item product from two potential alternative materials of different manufacturing characteristics (manufacturing time and defective percentage of products) and prices. The supply chain network consists of two echelons: suppliers and manufacturers. It is a capacitated problem where only the perfect quality products are allowed to reach the network's customers. Optimal decisions are needed to maximize the profit of confirmed orders. The model provides decisions regarding the material(s) that should be selected, order quantities, and manufactured/delivered quantities. An integer linear program and robust optimization were used for solving the problem. A sensitivity analysis was conducted to study the behavior of each supply chain parameter on the results.

## II. PROPOSED MODEL

### A. Model assumptions

- Production rates are chosen according to the used material.

- Flows of products from predecessor nodes directed to successor nodes are divisible, i.e., any supplier can deliver to any manufacturer and any manufacturer can deliver to any retailer.
- Instantaneous 100% screening is carried out for produced items.
- Single item product is considered.
- The product has a fixed selling price; regardless of the material being used.
- The unmet demand is assumed to be lost.
- Transportation cost is charged on the incoming material regardless of their quality.
- Only the screened perfect quality products are transported to retailers.
- Transportation capacity is unlimited.
- Set-up cost is neglected.

### B. The integer programming model

The following are the used notation, proposed model objective function, and constraints.

#### Notation

$S$	: number of suppliers (index $s$ )
$M$	: number of manufacturers (index $m$ )
$R$	: number of retailers (index $r$ )
$i$	: type of alternative materials (a, b): <i>material "b"</i> takes long processing time (low production rate) and <i>material "a"</i> requires short processing time
$D_r$	: demand of retailer $r$
$FR$	: fill rate (i.e., limit of minimum service level)
$SP_m$	: unit selling price of perfect quality product supplied by manufacturer $m$
$Um_m$	: manufacturing capacity, in hours, of manufacturer $m$
$Tm_{im}$	: manufacturing time required by unit $i$ at manufacturer $m$
$CR_{is}$	: raw material cost per unit $i$ supplied by supplier $s$
$Cm_m$	: manufacturing cost rate at manufacturer $m$
$Cs$	: unit shortage cost
$Cd_{sm}$	: unit transportation cost between supplier $s$ and manufacturer $m$
$CD_{mr}$	: unit transportation cost between manufacturer $m$ and retailer $r$
$\gamma M_{im}$	: percent of imperfect items at manufacturer $m$ using material $i$
$U\gamma M_{bm}$	: upper measure for material "b" percent defective
$L\gamma M_{bm}$	: lower measure for material "b" percent defective
$UTm_{am}$	: upper measure for material "a" manufacturing time
$LTm_{am}$	: lower measure for material "a" manufacturing time
$qsm_{ism}$	: number of units $i$ transported between supplier $s$ and manufacturer $m$
$qm_{im}$	: number of manufactured units using material $i$ at manufacturer $m$
$qmr_{mr}$	: number of units transported between manufacturer $m$ and retailer $r$

### Model formulation

The proposed order quantity allocation problem is mathematically formulated and solved using integer linear programming to maximize the total profit. The total profit is the total income generated from selling finished products to retailers (1) after deducting the incurred cost elements (2). The total cost includes raw material cost, manufacturing cost, transportation cost, and shortage cost, respectively.

$$\text{Income} = \sum_{m=1}^M \sum_{r=1}^R qmr_{mr} SP_m \quad (1)$$

Total cost=

$$\begin{aligned} & \sum_{i=1}^2 \sum_{s=1}^S \sum_{m=1}^M qsm_{ism} CR_{is} + \sum_{i=1}^2 \sum_{m=1}^M qm_{im} Tm_{im} Cm_m + \\ & \left( \sum_{i=1}^2 \sum_{s=1}^S \sum_{m=1}^M qsm_{ism} Cd_{ism} + \sum_{m=1}^M \sum_{r=1}^R qmr_{mr} CD_{mr} \right) + \\ & Cs \sum_{r=1}^R \left( D_r - \sum_{m=1}^M qmr_{mr} \right) \end{aligned} \quad (2)$$

The profit is subjected to the following constraints:

$$\sum_{i=1}^2 qm_{im} Tm_{im} \leq Um_m, \forall m \quad (3)$$

Constraint (3) restricts the manufacturer's production to its available capacity.

$$qm_{im} = \sum_{s=1}^S qsm_{ism}, \forall i, m \quad (4)$$

Equation (4) balances the production to the supplied material.

$$\left| \sum_{i=1}^2 (1 - \gamma M_{im}) qm_{im} \right| = \sum_{r=1}^R qmr_{mr}, \forall m \quad (5)$$

Equation (5) balances the delivered amount (*integer value*) to production, no access inventory is allowed.

$$\sum_{m=1}^M qmr_{mr} \leq Dr_r, \forall r \quad (6)$$

Constraint (6) limits the total delivered amount to the retailer's requirement.

$$\sum_{m=1}^M qmr_{mr} \geq FR \times Dr_r, \forall r \quad (7)$$

Constraint (7) guarantees a minimum fill rate fulfillment.

$$qsm_{sm}, qm_{im}, qmr_{mr} \geq 0, \text{ and integer } \forall s, m, r. \quad (8)$$

### III. RESULTS AND ANALYSIS

The developed IP mathematical model is a general model used to obtain optimum ordered, manufactured, and delivered quantities. It accommodates complex network structures. For simplicity and ease of interpretation, single

manufacturer and single retailer are assumed for further analysis. The manufacturer can source from two suppliers each provides alternative material. Material "a" provides higher production rate but has higher order costs than material "b". Material "b" yields different percent defectives during manufacturing processes, while material "a" yields zero percent defective. Products manufactured from alternative materials are of the same value.

#### A. Analytical Model for Material Usage

A flow chart is developed (Fig. 1) that highlights the role of system parameters on the behavior of the optimal solution. It depicts ranges for problem parameters at which the optimum decision of using any of the alternative materials is changed.

In Fig. 1, Equation (11) is derived from comparing the variable cost of introducing the use of material "a" (9) to the variable cost of maintaining the use of material "b" (10).

$$v_a = Cd_{asm} + CR_a + (Cm_m Tm_{am}) + CD_{amr} \quad (9)$$

$$v_b = \left( \frac{Cd_{bsm} + CR_b + (Cm_m Tm_{bm})}{1 - \gamma M_{bm}} \right) + CD_{bmr} \quad (10)$$

Therefore, to use material "b",  $v_b$  must be less than  $v_a$ .

$$U\gamma M_{bm} = \max \left( 0, 1 - \frac{Cd_{bsm} + CR_b + (Cm_m Tm_{bm})}{Cd_{asm} + CD_{amr} + CR_a + (Cm_m Tm_{am}) - CD_{bmr}} \right) \quad (11)$$

Equation (13) is derived from comparing the profit of introducing the use of material "a" to the profit from maintaining the use of material "b" (12).

$$\begin{aligned} \frac{1}{Tm_{am}} (SP_m + Cs - v_a) & > \frac{1}{Tm_{bm}} (1 - \gamma M_{bm}) (SP_m + Cs - v_b) \\ \frac{1}{Tm_{am}} \left[ SP_m + Cs - Cd_{asm} - \right] & > \\ \left[ CD_{amr} - CR_a \right] & \end{aligned}$$

$$\frac{1}{Tm_{bm}} \left[ (1 - \gamma M_{bm}) (SP_m + Cs - CD_{bmr}) - \right] \quad (12)$$

Therefore, Constraint (12) must hold for introducing material "a".

$$L\gamma M_m =$$

$$\max \left( 0, 1 - \frac{\frac{Tm_{bm}}{Tm_{am}} \left( SP_m + Cs - \right) - Cd_{asm} + CR_b + Cd_{bsm} + CR_b}{(SP_m + Cs - CD_{bmr})} \right) \quad (13)$$

The analytical conditions in Fig. 1 provide ranges of the percent defective and corresponding decisions while the rest of the parameters remain unchanged. It can be modified to

obtain other parameter ranges, such as the manufacturing time (14,15) while the percent defective and the rest of the parameters are fixed at certain values.

$$LTm_{am} = \frac{1}{Cm_m} \left[ \left( \frac{Cd_{bsm} + CR_b + (Cm_{mt} Tm_{bm})}{(1 - \gamma M_{bm})} \right) - \left( Cd_{asm} + CD_{amr} + CR_a - CD_{bmr} \right) \right] \quad (14)$$

$$UTm_{am} =$$

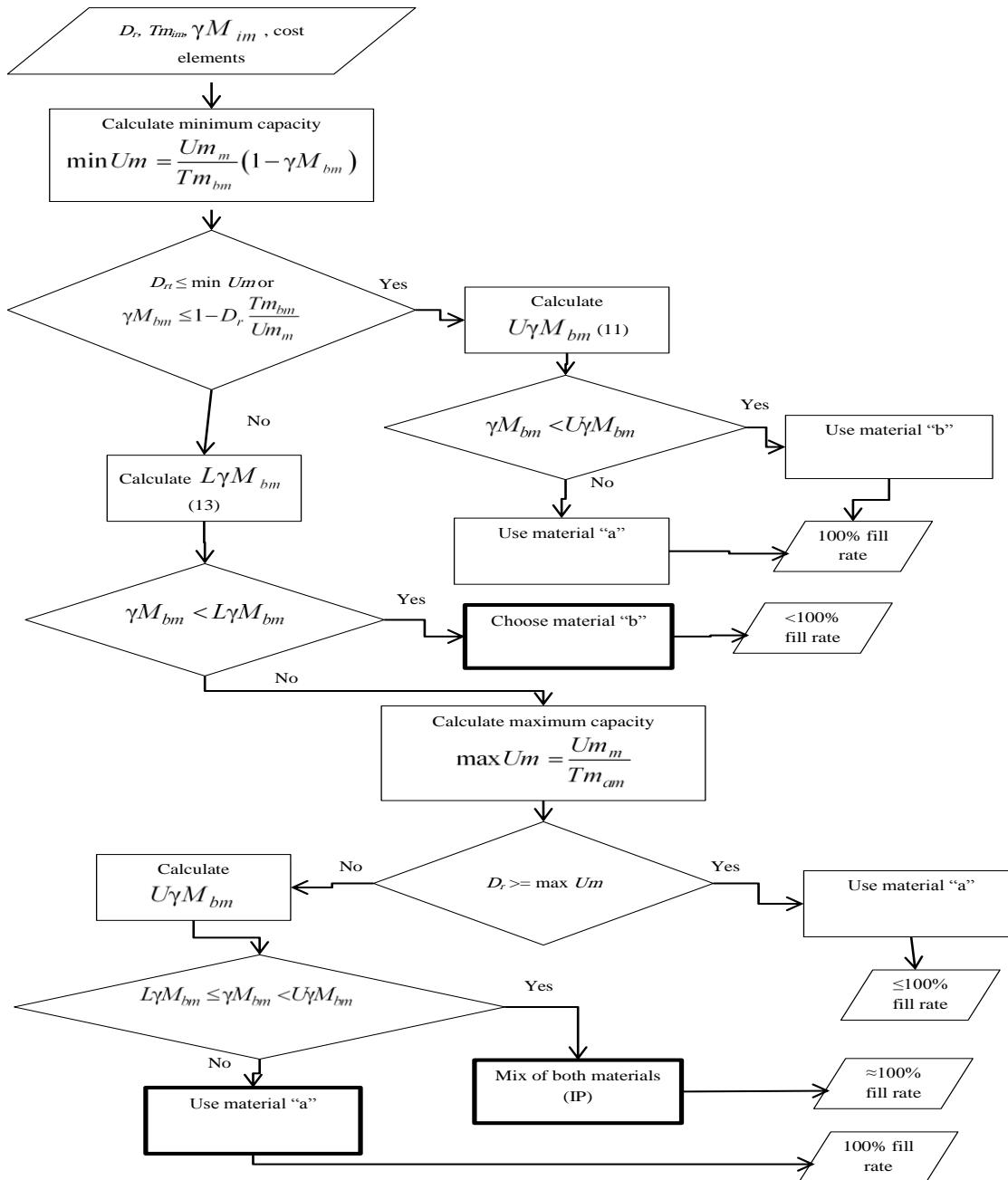


Fig. 1. Flow chart represents the effect of model parameters on material usage

#### B. Effect of different manufacturing parameters on the supply chain performance

Different factors and parameters affect the optimization decisions; such as adopted strategies, voice of customer,

availability of resources, cost elements, and quality considerations. In this section, numerical examples are performed to study the effect of different model parameters on the effectiveness of adopting the strategy of utilizing alternative materials. All the results were obtained using the

branch and bound algorithm accessed via FICO-Xpress Software v7.8. The default values for the used parameters are given in Table (1).

TABLE I. PARAMETER VALUES

Parameters	value	dimension
Demand	480	units/period
Manufacturing time for material "a" = (1/production rate)	0.5-0.8	hr/unit
Manufacturing time for material "b"	1	hr/unit
Percent defective of material "a"	0	%
Percent defective of material "b"	0,variable	%
Cost of material "a"	30	\$/unit
Cost of material "b"	10	\$/unit
Manufacturing cost	35	\$/hr
Manufacturing capacity	480	hrs/period
Shortage cost (lost sales cost)	10	\$/unit
Transportation cost	10	\$/unit
Selling price	100	\$/unit

*Effect of demand on material usage at limited capacity:* In case of limited demand, higher quantities of low quality material can be used (Fig. 2) for better profit (Fig. 3). For limited capacity and high demand, an increasing percent of high quality, high cost material should be used to achieve the required fill rate while decreasing the profit.

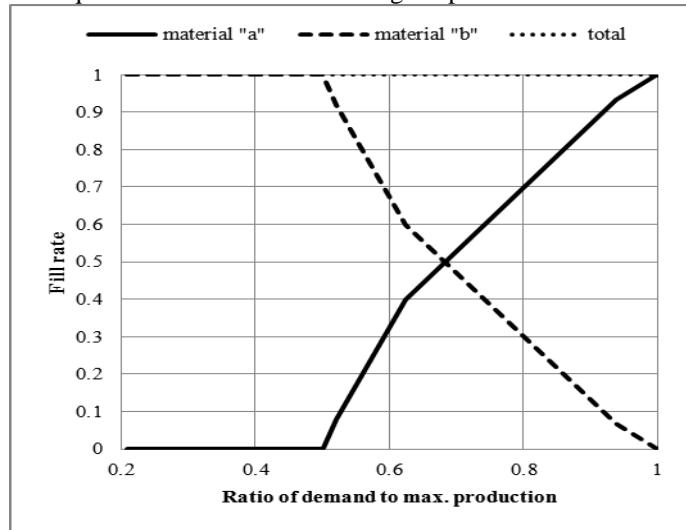


Fig. 2. The effect of material usage on the fill rate at different demand

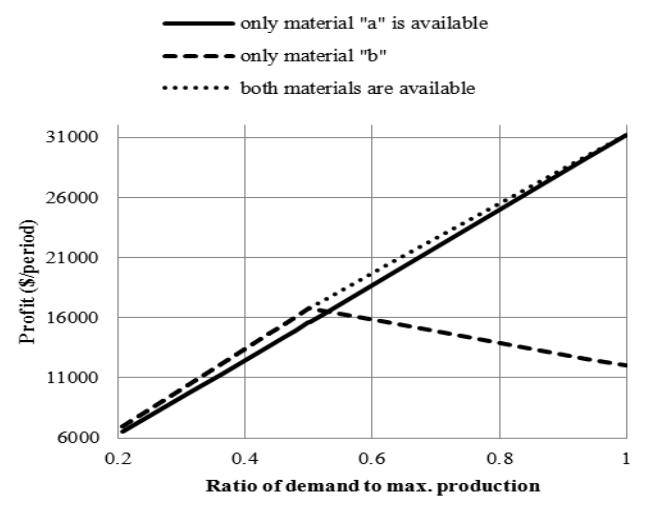


Fig. 3. The effect of material usage on the profit at different demand

*Effect of production rate of the high quality material at different percent defective of the low quality material:* In case that the production rate of both materials are close to each other with different output defect percentages, it can be more economic to go for the low cost material "b" (Figs. 4, 5) while achieving almost the same fill rate (Fig. 6). This is true especially in case the percent defective obtained from processing low quality material ( $\gamma M_{bm}$ ) is of small value. Fig. 7 shows that at low production rates of material "a" the profit entirely depends on the usage of material "b".

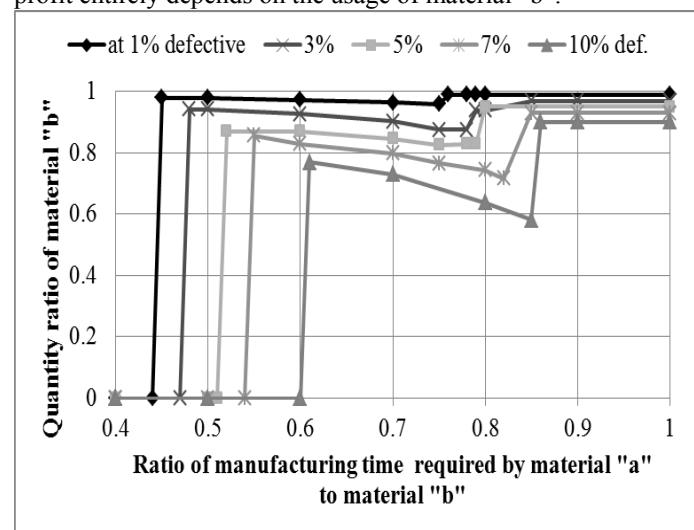


Fig. 4. The effect of production rate of the high quality material on optimum quantities

The inflection points of the optimum integer quantities, in Fig. 4, can be explained as follows: from the right, at slow production rates for material "a" ( $Tm_{am} > UTm_{am}$ ) the manufacturer will use the inexpensive material, since the expensive material "a" has lost its advantage. When  $Tm_{am} < UTm_{am}$ , the optimization increases the use of the expensive material in return of higher fill rates.

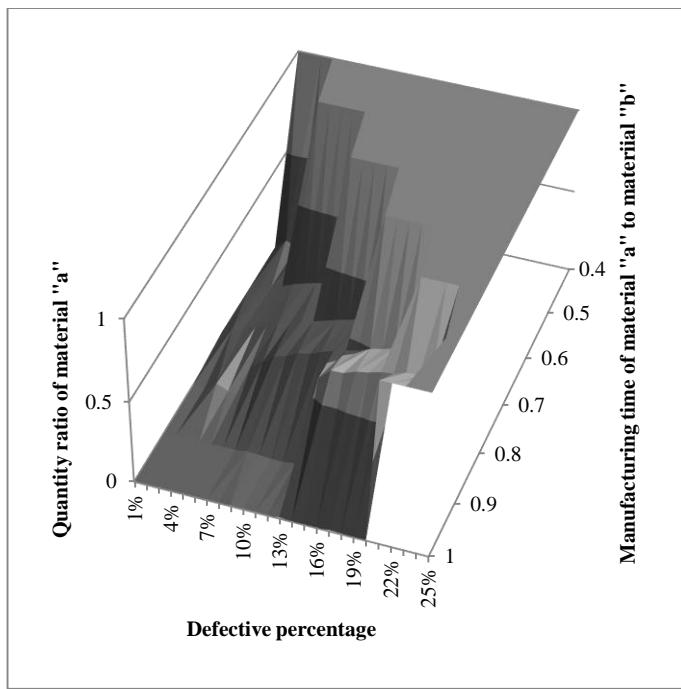


Fig. 5. The effect of production rate and material quality on optimum quantities

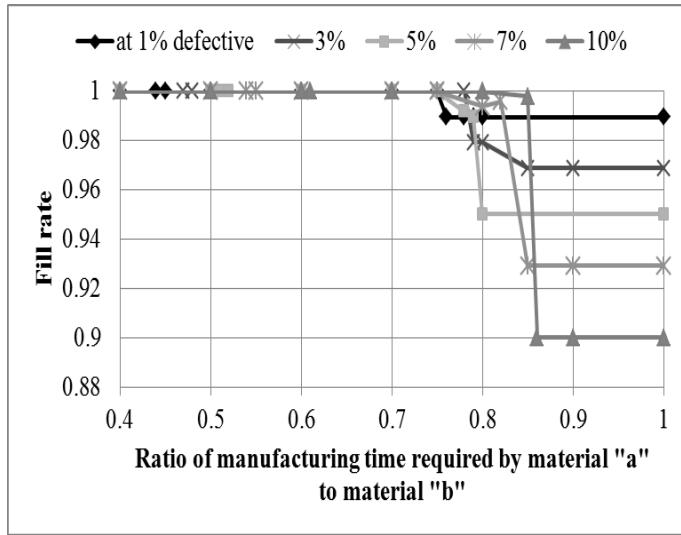


Fig. 6. The effect of production rate of the high quality material on the fill rate

Fig. 7.

At range  $LTm_m < Tm_m < UTm_m$  the added quantities from material "a" free extra capacity to be used by the inexpensive material "b". The maximum point, where material "b" has the highest quantity relative to its preceding and succeeding points, achieve 100% fill rate. That is to say, the manufacturer is able to achieve the requested fill rate without utilizing extra amounts from the expensive material. When  $Tm_m < LTm_m$ , it is more profitable to switch completely to material "a".

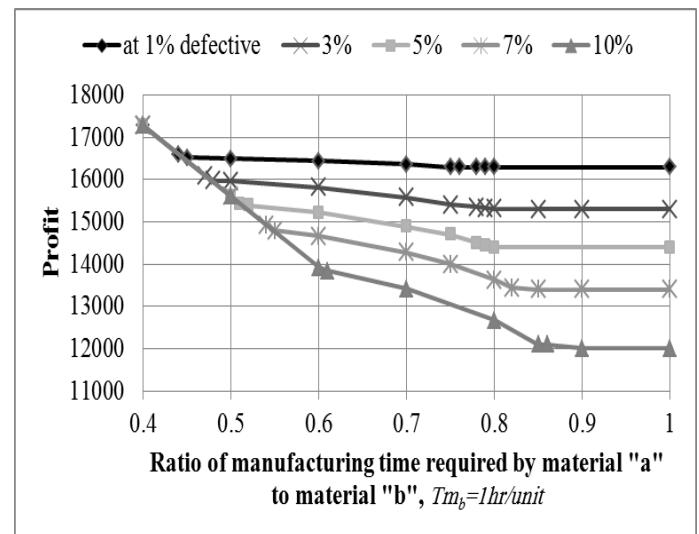


Fig. 8. Effect of production rate of the high quality material on the profit

### C. Optimal decision based on percent defective variation using robust optimization technique

The robust model achieves higher fill rate (Fig. 8) with reduction in profit (Fig. 9) which may be more practical for the decision maker. For example, in Figs (8,9) at  $\pm 2\%$  uncertainty the price of robustness is small—price of robustness is the difference in the objective value when using nominal/uncertain values [24]. On the other hand, at 5% defective, the robust solution achieves higher fill rates. Hence, the conclusion "there are ranges of variability where the robust model provides more practical decisions".

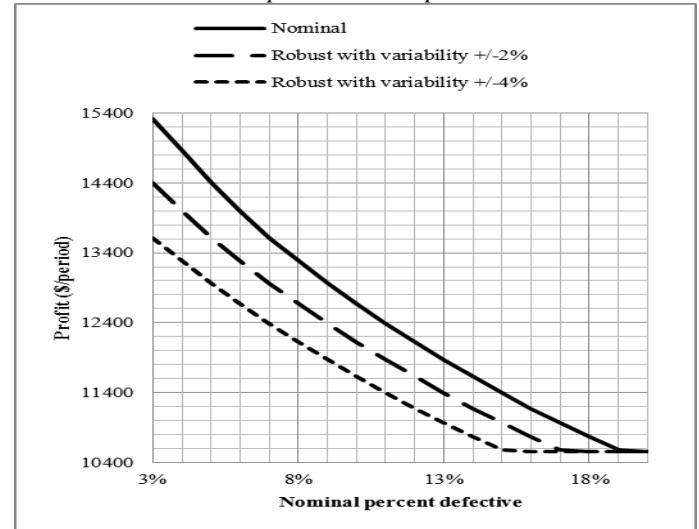


Fig. 9. The effect of robust optimization on the profit

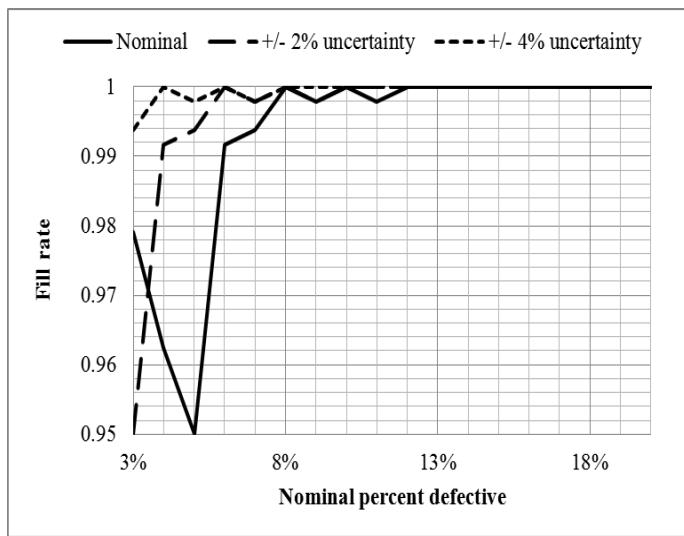


Fig. 10. The effect of robust optimization on the fill rate

#### IV. CONCLUSIONS

There are different mitigation strategies that can help in overcoming system constraints. In this work, the studied mitigation strategy is based on the following assumptions: 1) alternative materials can be separately manufactured and turned into same product with same quality; 2) these materials yield different scrap percentages when manufactured, 3) they are manufactured using different manufacturing times, and 4) they have different purchase prices. Analysis of a single-period model for two-echelon supply chain and two-material setting states that cost and quality are not the sole drivers for orders allocation, as capacity restriction increases. Supply chain performance represented in higher profit and/or higher fill rates and delivering the required quality is achievable through using alternative materials, if feasible. The capacity limitations oblige the decision maker to go for high quality material for higher profit. Robust optimization against quality variation gave same trends at lower profits; it can provide more practical decisions. This work can be extended to stochastic parameters, and joint price and lot sizing decisions can be considered.

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