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# Effect of Thickness on Interlaminar Stresses In Simply Supported FRP Angle-Ply Laminate with A Circular Cut Out

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## Abstract

*The present research work deals with the prediction of interlaminar stresses in simply supported laminated FRP composite plate with a circular cut-out under transverse load using 3-D finite element analysis. The finite element analysis software ANSYS has been successfully executed and the finite element model is validated. The interlaminar stresses are evaluated by varying length-to-thickness ratio (s) and constraints. The effect of 's' and the boundary conditions on the interlaminar stresses is discussed. The present analysis will be useful in quantifying the effect of above said factors that helps in the safe and efficient design of the structural elements made of laminated FRP composites.*

**Keywords:** FRP, Interlaminar stresses, Angle-ply laminate, cut-out

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## Nomenclature

$E_1$ = Young's modulus of the lamina in the fibre direction
$E_2 = E_3$= Young's modulus of the lamina in the transverse direction of the fibre
$G_{12}= G_{13}$= Shear modulus in the longitudinal plane of the fibre
$G_{23}$= Shear modulus in the transverse plane of the fibre
$\nu_{12} = \nu_{13}$ = Poisson's ratio in the longitudinal plane of the fibre
$\nu_{23}$ = Poisson's ratio in the transverse plane of the fibre
I-1 = First interface i.e. interface between $45^0$ and $-45^0$ laminae
I-2 = Second interface i.e. interface between $-45^0$ and $-45^0$ laminae
I-3 = Third interface i.e. interface between $-45^0$ and $45^0$ laminae
l = Length of the Square plate
d = Diameter of the cut-out
t = Thickness of laminate
s = l/t = thickness-to-length ratio
d/l = diameter-to length ratio
SS-1 = plate simply supported along all the four edges
SS-2 = plate simply supported along edges parallel to Y-axis
SS-3 = plate simply supported along edges parallel to X-axis

## 1. Introduction

The increasing use of fiber reinforced laminates in space vehicles, aircrafts, automobiles, ships and chemical vessels have necessitated the rational of structures for their mechanical response. In addition, the anisotropy and non homogeneity and larger ratio of longitudinal to transverse moduli of these new materials demand improvement in the existing analytical tools.

FRP composites deliver more strength per unit of weight than most metals. In fact, FRP composites are generally 1/5th the weight of steel. The composite can also be shaped into one complex part, often times replacing assemblies of several parts and fasteners. The combination of these two benefits makes FRP composites powerful material system- structures can be partially or completely pre-fabricated at the manufacturer's facility, delivered on-site and installed in hours. The addition of the reinforcement to the polymer matrix increases the creep resistance of the properly designed FRP part.

As a result, the analysis of laminated composite structures has attracted many research workers and has been considerably improved to achieve realistic results. Depending upon the nature of application, these structural elements are acted upon by mechanical thermal loads of varied nature. Usually, the anisotropy in laminated composite structures causes complicated responses under different loading conditions by creating complex couplings between extensions, bending and shear deformation modes, it must be described by three dimensional elasticity theories.

In practical applications, composite plates with cut-outs are required for various purposes, such as joining of riveted and bolted joints. Till now there are number of approaches have been proposed to solve the three –dimensional elasticity equations of rectangular plates. The interlaminar stresses in symmetric laminates under uniform axial extension were first evaluated by Pipes and Pagano [1] by applying a finite difference technique to solve the Navier equations of elasticity for off-axis plies. Srinivas and Rao [2] and Srinivas et al. [3] presented a set of complete analytical analyses on bending, bucling and free vibration of plates with both isotropic and orthotropic materials. Pagano et al. [4] has given exact solutions for the deflections and stresses of a cross – ply laminated rectangular composites using elasticity theory. Following the approach used by Pipes and Pagano [ 1] the interlaminar stress distribution in a four layer composite laminate in bending was studied by salamon [ 5]. He predicted that the magnitudes of the interlaminar normal and shear stresses, although in general relatively small, rise sharply near the free-edges. Kong and cheung [6] proposed a displacement-based, three- dimensional finite element scheme for analyzing thick laminated plates by treating the plate as a 3-dimensional inhomogeneous anistropic elastic body.

A.Srinivas et al [7] evaluated the displacements, in-plane, out-of-plane and interlaminar stresses in a four layered symmetric balanced angle-ply laminates subjected to longitudinal and in plane transverse loads. Using a double fourier series approach Kabir[8] presented the results of the variations of transeverse displacements and moments for various parametric effects for antisymmetric angle-ply ($45^0$/-$45^0$) and for symmetric angle-ply ($45^0$/-$45^0$) laminate plate[9] with simply supported boundary conditions at all edges. Chen and Kam [10] presented a two level optimization method for elastic constants identification of symmetric angle-ply laminates. To study the interlaminar stresses in cylindrical shells under static and dynamic transverse loads and to determine the dynamic magnification factors (DMF i.e. the ratio of the maximum dynamic response to the corresponding static response) Bhaskar and varadhan [11] used the combination of Navier's approach and a Laplace transform technique to solve the dynamic equations of equilibrium. Ravi kiran [12] presented the prediction of interlaminar stresses in simply supported laminated FRP cross-ply laminate with a circular cut-out under transverse load using 3-D finite element analysis. In the present analysis interlaminar stresses in simply supported laminated FRP angle-ply laminate with a circular cut-out under transverse load are analyzed using 3-D finite element analysis.

## 2. Problem Modelling

Three-dimensional finite element analysis of a four layered symmetric balanced angle–ply laminate has been taken up in the present work. The finite element model created in ANSYS software is validated and extended to evaluate the interlaminar stresses by varying the length-to-thickness ratio (s).

### 2.1 Geometric Model

A square plate of length 100 units is considered for the present analysis. Four layers of equal thickness with the fiber angle $45^0$/-$45^0$/-$45^0$/$45^0$ are arranged to observe the balance as well as symmetry across the thickness of the laminate. The thickness of the plate is selected from length-to-thickness ratio (s) which is varied as 10, 20, 30, 40 and 50. A circular cut out is considered at the centre of the plate with diameter-to-length ratio (d/l) = 0.2.

### 2.2 Finite Element Model

The finite element mesh is generated using Solid 45 [13] element as shown in figs. 1 and 2. Solid 45 is a second order brick element. It can tolerate irregular shapes without much loss of accuracy. Solid 45 elements have compatible displacement shapes and are well suited to model curved boundaries. This element is defined by 8 nodes having three degrees of freedom per node: translations in the nodal x, y and z direction. The element may have any spatial orientation and is suitable to model isotropic as well as orthotropic materials.

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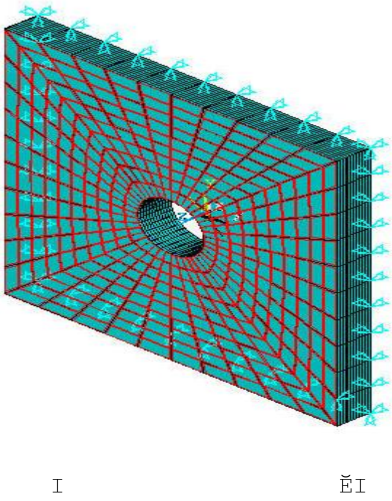

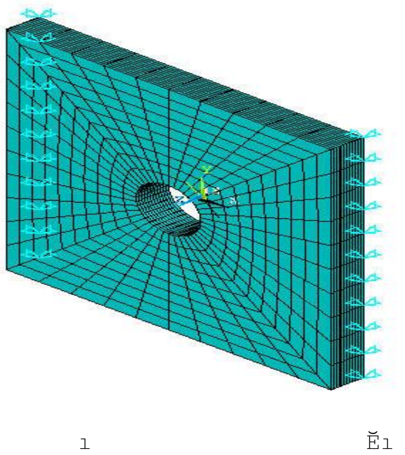

## 2.3 Material Properties

The following material properties are considered for the present analysis.

i) Young's Modulus $E_1$=127.5GPa, $E_2$=9.0GPa,
$E_3$= 4.8GPa.
ii)Poisson's Ratio $\nu_{12} = \nu_{13} = 0.28$, $\nu_{23} = 0.41$
iii) Rigidity Modulus $G_{12}$= $G_{13}$=4.8GPa,
$G_{23}$ = 2.55GPa.

## 2.4 Boundary Conditions and Loading

Simply supported boundary conditions are applied along all the four edges of the plate in SS-1, along edges parallel to Y-axis in SS-2, and along edges parallel to X-axis in SS-3. A uniform load of 1 MPa is applied on the top surface of the FE model.

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## 2.5 Validation of FE model

The present finite element model is validated by computing the out of plane stresses at the free surface i.e. at the bottom surface of the plate (z=0). The computed stresses are found to be close to zero (Table 1).

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| Bottom center | $\sigma_z$ | $\tau_{yz}$ | $\tau_{zx}$ |
|---|---|---|---|
| SS-1 | 0.00010931 | 0.0030434 | 0.0023665 |
| SS-2 | 0.0010606 | 0.0069009 | 0.00079798 |
| SS-3 | 0.00076937 | 0.0010608 | 0.0022889 |

## 3. Discussion of Results

Variation of the interlaminar stresses with respect to length-to-thickness ratio (s) is shown in Figs. 3-11. These figures shows the variation of interlaminar normal and shear stresses $\sigma_z$, $\tau_{yz}$ and $\tau_{zx}$ at the bottom ,middle and top interfaces of the laminate under consideration.

From Figs. 3-5 it is observed at all the interfaces, the interlaminar normal stress $\sigma_z$ is rapidly incresing with 's'. It is observed that the interlaminar normal stress $\sigma_z$ is maximum and same for SS-2 and SS-3 at all interfaces. $\sigma_z$ is minimum for SS-1 at all interfaces for all values of 's'

Variation of the interlaminar shear stress $\tau_{yz}$ with respect to 's ' is shown in Figs. 6-8. It is observed that the interlaminar shear stress $\tau_{yz}$ is rapidly increasing with 's' at all the interfaces. The shear stress $\tau_{yz}$ in SS-3 is maximum for all 's' at all interfaces. And $\tau_{yz}$ is minimum in SS-1 at all interfaces for the values of 's'.

Variation of the interlaminar shear stress $\tau_{zx}$ with respect to 's' is shown in Figs. 9-11. It is observed that the interlaminar shear sress $\tau_{zx}$ is rapidly increasing with 's' at all the interfaces. The shear stress $\tau_{zx}$ in SS-2 is maximum for all 's' at all interfaces. $\tau_{zx}$ is minimum in SS-1 at all interfaces for the values of 's'.

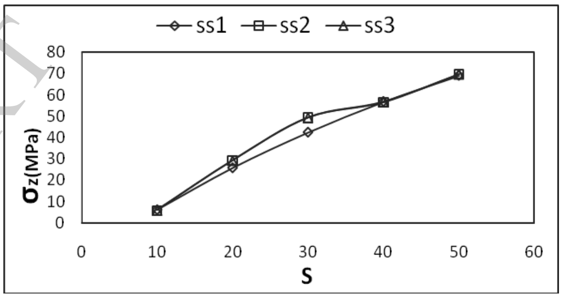

Fig 3: Variation of $\sigma_z$ with 's' in I-1

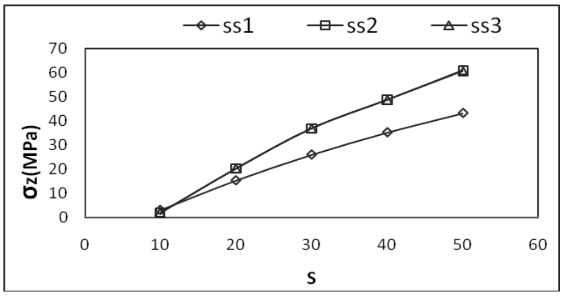

Fig 4: Variation of $\sigma_z$ with 's' in I-2

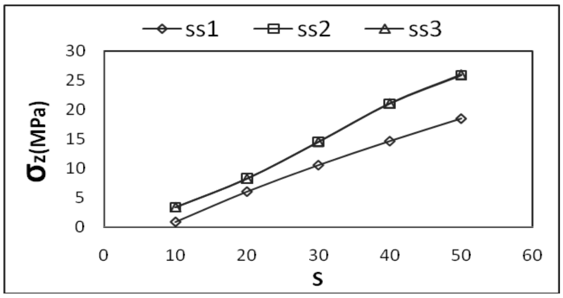

Fig 5: Variation of $\sigma_z$ with 's' in I-3

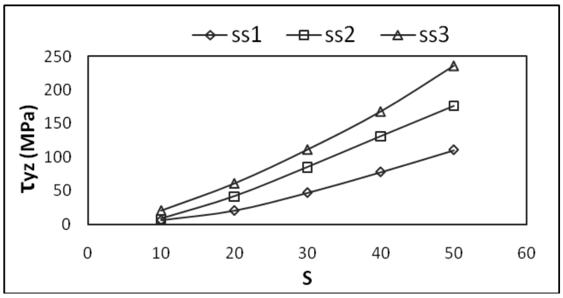

Fig 6: Variation of $\tau_{yz}$ with ‘s’ in I-1

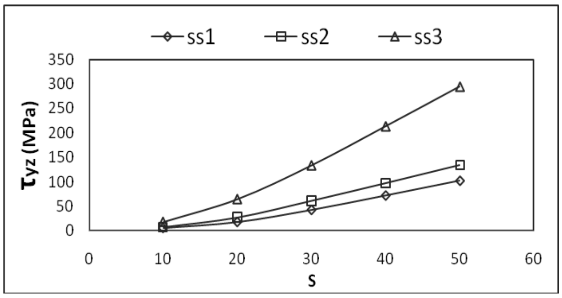

Fig 7: Variation of $\tau_{yz}$ with ‘s’ in I-2

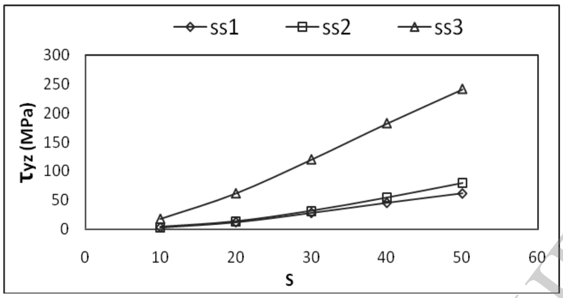

Fig 8: Variation of $\tau_{yz}$ with ‘s’ in I-3

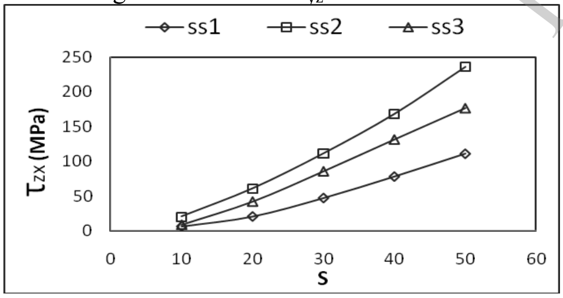

Fig 9: Variation of $\tau_{zx}$ with ‘s’ in I-1

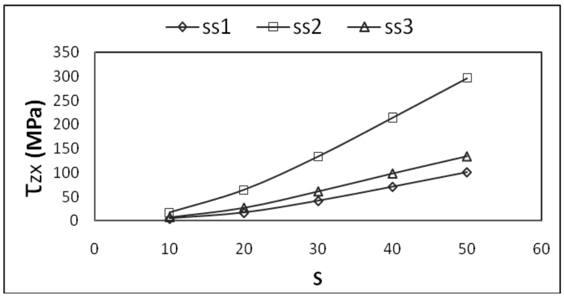

Fig 10: Variation of $\tau_{zx}$ with ‘s’ in I-2

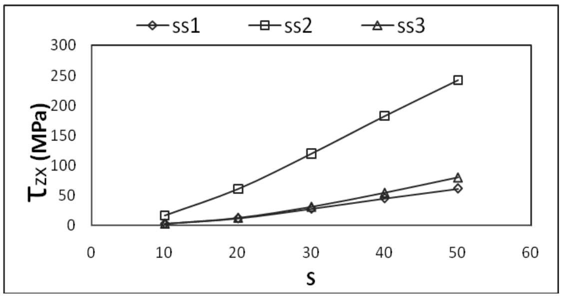

Fig 11: Variation of $\tau_{zx}$ with ‘s’ in I-3

## 4.Conclusions

Three dimensional finite element analysis is carried out for the pridiction of interlaminar stresses in a 4-layered angle-ply laminate subjected to out-of-plane transverse loads. The following conclusions are drawn.

- $\sigma_z$ is maximum at bottom interface.
- $\tau_{yz}$ is maximum at middle interface.
- $\tau_{zx}$ is minimumm at middle interface.
- Dominating stresses are interlaminar shear stresses $\tau_{yz}$ and $\tau_{zx}$.

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