

## Effect of stacking sequence on Free Vibration Analysis of Thick FRP Skew Specially Orthotropic Laminate with Circular Cutout

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### Abstract

The present investigation deals with the study of effect of stacking sequence of specially orthotropic nature in free vibration analysis of a thick four-layered Fiber Reinforced Plastic (FRP) skew laminated composite plate with a circular cut out. Three dimensional finite element model which uses the elasticity theory for the determination of stiffness matrices is modeled in ANSYS software to evaluate first five natural frequencies of the laminate. The effect of stacking sequence on the natural frequencies of the plate is discussed.

### 1. Introduction

Composite materials are replacing metals in many structural applications such as aerospace, transportation, novel and pressure vessels due to their high specific strength and specific modulus. The dynamic responses of these structures differ from that made of isotropic materials due to orthotropic nature of individual layers of the laminate. Free vibration analysis is a part of dynamic analysis where the natural frequencies of the structure can be estimated. The following paragraph provides a brief review of various research contributions on free vibration analysis of FRP composite structures.

Free vibration frequencies of symmetric and anti-symmetric laminated plate are studied using higher order shear deformation theories by Kant [1], Panda [2] and Viswanathan [3]. A higher order theory has been developed by Nayak [4] for the vibration characteristics of the laminated plate. Chao [5] presented the free vibration natural frequencies of rectangular plates. A general analytical model applicable to the dynamic behavior of a thin-walled channel section composite is developed by Jaehong [6]. Based on a higher order shear deformation theory Ajay [7] is presented for the free vibration analysis of sandwich skew laminates. The layer-wise shear

deformation theory has been extended by Shimpi [8-9] for the free vibration of two-layered cross-ply laminated plates and beams. Vibration analysis of angle-ply laminated beams subjected to different sets of boundary conditions is investigated by Metuin [10], Baharuymaz [11] and Li [12]. The effects of various parameters on the natural frequencies are presented by Chun [13] for the buckling and vibration behaviours of hybrid composite plates. Omer [14-15] is developed for static and dynamic analysis of thick symmetric cross-ply laminated composite plates based on the first-order shear deformation theory.

The present investigation intends to apply three-dimensional finite element techniques, based on theory of elasticity, for the free vibration analysis of clamped thick skew laminates with circular cutout. The lowest five natural frequencies are studied by varying the stacking sequence.

### 2. Problem modeling

#### 2.1 Geometric modeling

The geometry of the problem is shown in Figure 1. A rhombic plate with sides 'l' and 'b' equal to 2m is taken. The height of the plate is divided in to four layers of equal thickness (h/4), Where 'h' is the total thickness of the laminate, which is taken from the length-to-thickness ratio (s=10). The skew angle ( $\alpha$ ) is taken as 30°. The circular hole is placed at the geometric centre of the plate. The diameter of the hole is selected from the ratio d/l = 0.3.

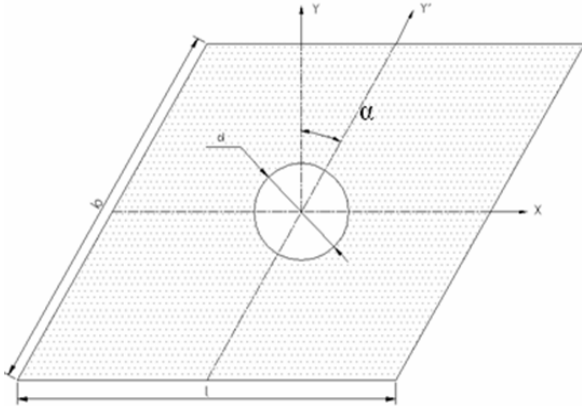


Figure 1. Skew plate with cutout

## 2.2. Finite Element Modeling

The element used for the present analysis is SOLID 95 of ANSYS, which is developed, based on three-dimensional elasticity theory and is defined by 20 nodes having three degrees of freedom at each node, translation in the node  $x$ ,  $y$  and  $z$  directions. The model with finite element mesh is shown in Figure 2.

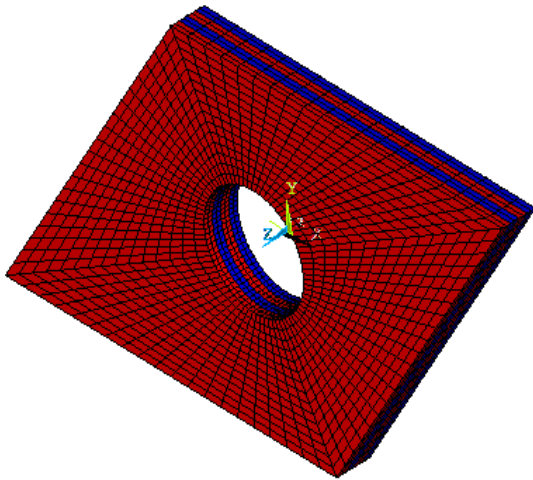


Figure 2. Finite Element mesh on the skew plate showing 0/90/0/90 layers

## 2.3. Material Properties

Each layer is an unidirectional carbon FRP possessing the following engineering constants.

Elastic modulus in the longitudinal direction of the fiber,  $E_L = 175$  GPa.

Elastic modulus in the transverse direction of the fiber,  $E_T = 7$  GPa.

Shear modulus in the longitudinal plane of the fiber,  $G_{LT} = 3.5$  GPa.

Shear modulus in the transverse plane of the fiber,  $G_{TT} = 1.4$  GPa.

Poisons ratios,  $\nu_{LT} = \nu_{TT} = 0.25$

Mass density,  $\rho = 1.6 \times 10^3$  kg/m<sup>3</sup>

## 2.4. Boundary Conditions

The sides of the skew laminate considered for the present analysis are clamped i.e. all the three degrees of freedom of the nodes along the four sides of the skew plate are constrained.

## 2.5 Validation of FE model

The finite element model is validated with the results available in the literature and found good agreement. (Table 1.)

Table 1. Validation of the finite element results\* ( $\varpi$  of Rectangular laminated composite plate)

$E_1/E_2$ (0/90) <sub>2</sub>	3	10	20	30	40
Present	6.4245	8.1681	9.4247	10.3672	10.8699
Kant [1]	6.4319	8.1010	9.4338	10.2463	10.7993
Panda [2]	6.3601	8.0335	9.3894	10.2143	10.7747

\* $E_1=40E_2$ ,  $G_{12}=G_{13}=0.6E_2$ ,  $G_{23}=0.5E_2$ ,  $\nu_{12}=0.25$

Where the normalized frequency,  
 $\varpi = \omega a^2 / h(\rho/E_2)^{1/2}$ ,

$\omega = \text{Natural Frequency}$

## 3. Analysis of results

First five natural frequencies are predicted by varying stacking sequences of the skew laminate. The variation of first five natural frequencies with respect to the mode number is shown in Figures 3-7. The natural frequency increases with respect to mode number. As the mode number increases, the number of curvatures in flexure increases resulting in the increase in stiffness of the structure which in turn increases the frequency. The arrangement of contour plots for the five modes of natural frequencies is shown in Figure 5.

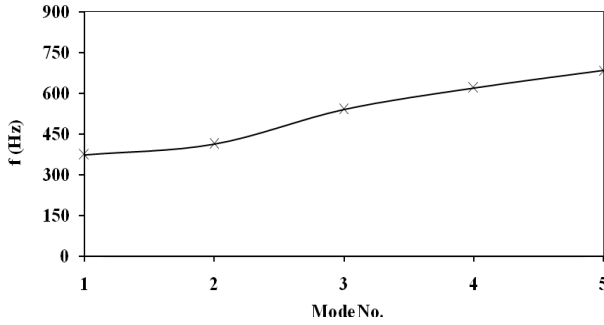


Figure 3. Variation of natural frequency with respect to mode (0/0/0/0)

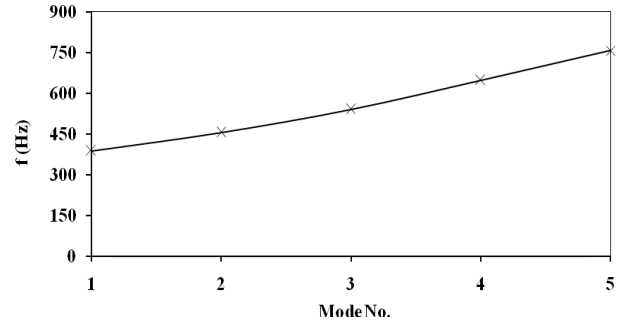


Figure 7. Variation of natural frequency with respect to mode (90/0/0/90)

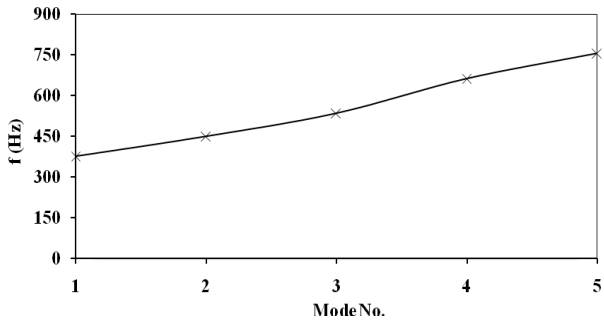


Figure 4. Variation of natural frequency with respect to mode (0/90/90/0)

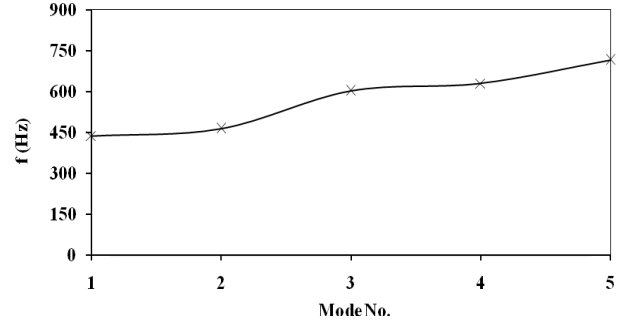


Figure 8. Variation of natural frequency with respect to mode (90/90/90/90)

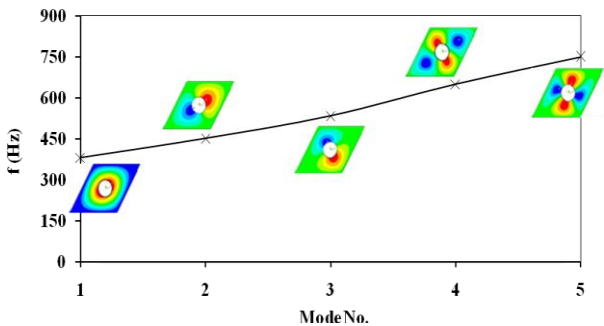


Figure 5. Variation of natural frequency with respect to mode (0/90/0/90)

### 3.1. Effect of stacking sequence

Variation of the first five natural frequencies with respect to stacking sequence is shown in Table 2. In all the modes, the least frequency is observed for stacking sequence of laminate with all 0° layers and the maximum frequency is observed for stacking sequence with all 90° layers in all the modes. In remaining cases, the order of increase of natural frequency is (0/90/90/0), (0/90/0/90), (90/0/90/0) and (90/0/0/90).

Table 2. Variation of natural frequency with respect to stacking sequence

Stacking sequence	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5
0/0/0/0	373.55	413.05	540.29	620.47	683.92
0/90/90/0	375.35	449.16	534.18	661.81	754.54
0/90/0/90	380.98	451.32	534.05	650.66	750.45
90/0/90/0	381.23	451.68	534.41	651.24	750.97
90/0/0/90	388.65	456.67	541.46	648.96	757.10
90/90/90/90	437.58	463.86	603.41	630.64	716.09

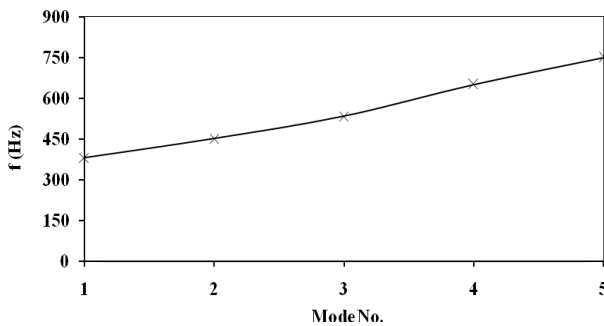


Figure 6. Variation of natural frequency with respect to mode (90/0/90/0)

### 4. Conclusions

The effect of stacking sequence in free vibration analysis of a thick four-layered FRP skew laminated

composite plate with a circular cut out at the centre is analyzed for natural frequencies. The following conclusions are drawn.

- The frequency increases with increase in mode number.
- The least frequency is observed for stacking sequence (0/0/0/0) in all the modes.
- The maximum frequency is observed for stacking sequence (90/90/90/90) in all the modes.

## 5. References

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