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Effect of Shape Change and Angular Orientation of Slot Ends on Vibration Characteristics of Annular Discs

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Abstract— **In this paper we consider the effect of shape change and angular orientations of slot ends on vibration characteristics such as natural frequencies and mode shape of annular disc. Circular discs with different shapes of holes are widely used in the field of rotating components and structures. Especially, if holes are located eccentrically on the surface of disc it causes to change in natural frequency significantly. Also, this holes usually causes change in some of the properties of component such as load carrying capacity, decrease in weight of component and stresses developed in the component etc. Therefore in this study, An FEM investigation is done on this type of discs by using ANSYS simulation software and results are validated by experimental method such as impact hammer and FFT analyzer. The detailed procedure for FEM and Experimental work is given. Also the results of parameter variation and modes are investigated.**

Keywords—FEM; Vibration analysis; Circular disc; FFT analysis; Natural frequency; Mode shapes;

I. INTRODUCTION

The use of different types and shapes of holes in the various locations of body of annular discs (which intentionally induces residual stresses into the disc) will be analysed, quantified, and discussed as a means of making annular disc last longer and at the same time help realize solutions to problems with noise, natural frequency and fatigue life of the spinning discs. Knowing the value of the natural frequencies of the static and rotating discs as well as the value of the residual stresses and their locations by using FEA software will help to solve the need to quantify the stress effects and help the disc manufacturers incorporate these modifications into their future designs for better performed rotating machine components and products.

Circular plates with holes are widely used in mechanical parts. Circular plates with multiple circular holes have applications in many engineering structures. They are used either to decrease the mass of the structure by keeping vibration properties and material same or to upturn the range of inspection or to satisfy engineering applications. Circular geometries are also used because of the ease of their manufacture.

II. LITERATURE REVIEW

Ming-Hung Hsu [1] discussed an efficient numerical approximation technique that called the differential quadrature method. The free vibration problems of annular plates were numerically formulated using the modified differential quadrature method and differential quadrature method. The modified differential quadrature method does not use the δ method. Numerical results indicated that the accuracy of the calculated results is improved significantly by employing the modified differential quadrature method.

H. Rokni Damavandi Taher et.al [2] given the first nine frequency parameters of circular and annular plates with variable thickness and combined boundary conditions were computed for different thickness to radius ratios. Several combined boundary conditions were considered for inner and outer edges. Results of this paper were compared with works of other authors and the results of finite element method analysis which were in agreement. Data for thick circular and annular plates with linear and parabolic thickness variation and different combinations were presented for the first time.

J. N. Sharma et.al [3] considered the application of finite element method for the analysis of thermo elastic characteristics of a thin circular disk which was further subjected to a thermal load and an inertia force arising due to rotation of the disk. On the basis of the two dimensional thermo elastic theories, the axisymmetric problem was formulated in terms of second order ordinary differential equation which was solved by FEM. Further, it was assumed that the disk is vibrating. The effect of Kibel number on different components of stress, strain and displacement has also been discussed. The numerical results reveal that these quantities were significantly influenced by temperature distribution and angular speed of the disk.

S. Khare and N. D. Mittal [4] described numerical analysis of free vibration of thin circular and annular plate using finite element method. The first five natural frequencies were presented for uniform annular plates of various inner-to-outer radius ratios, with nine possible combinations of free, clamped and simply supported boundary conditions at the inner and outer edges of plates. Results show that natural frequency parameter increases as the inner-to-outer radius ratio increases except in case of free boundary condition, for which it decreases with the inner-to-outer radius ratio.

F. Kuratani, S. Yano [5] proposed a method in finite element analysis for estimating natural frequencies of a disk tensioned by rolling, without the use of eigenvalue analysis. Tensioning was used for improving the dynamic stability of circular saws. The optimal condition of rolling can be predicted from natural frequency characteristics. In the proposed method, the natural frequencies after rolling were easily estimated from the mode shapes of the disk before rolling and the stress distribution after rolling.

Charles B. Ponton [6] developed and evaluated finite element models of an industrial saw blade while undergoing the effects from rotating and cutting. In addition, the effects on the saw blades performance due to various numbers and lengths of expansion slots and saw blade tensioning were explored. The work developed by author was a first step toward characterizing the effects of specific mechanisms which can be used to design better, longer lasting saw blades.

Weisensel G. N. et.al [7] has analyzed the results of an extensive literature search and review of available sources of numerical natural frequency information for stationary circular and annular elastic plates.

C. J. Luo et.al [8] given an analytical nonlinear solution for the asymmetric vibration of rotating disc by using a recently developed plate theory instead of von Karman model. The nonlinear solution can reduce to the linear one when nonlinear effects vanish. The symmetrical response was also recovered when the nodal diameter vanishes.

Yongchen Pei et.al [9] investigated the multi body dynamics of a rotating flexible annular thin disk subjected to double slider loading systems. System dynamic model is solved by Galerkin's method, and then natural frequency, dynamic stability and shape are determined with a quadratic eigenvalue problem. Effects of the distributing positions and interaction mechanism of the double slider loading systems on natural frequency, dynamic stability and mode shape were discussed and investigated.

Stanisław Kukla et.al [10] concerns axisymmetric free vibration of annular and circular plates of stepped thickness with elastic ring supports. Exact solution to the problem was obtained by dividing the considered plate into annular plates of uniform thickness and by using Green's function method. Analytical solution to vibration problem was used to perform numerical frequency analysis of an exemplary stepped annular plate.

Eihab M. Abdel-Rahman et.al [11] presented a numerical procedure to solve the axisymmetric vibration problem of statically loaded annular plates. They used the von Karman nonlinear plate model to account for large deformations and study the effect of static deflections on the natural frequencies and mode shapes for six combinations of boundary conditions. The shooting method was used to solve the resulting eigenvalue problem. Their results show that static deformations have a significant effect on the natural frequencies and small effect on the mode shapes of the plate.

Mhaiskar et.al [12] given a numerical technique used to model a disc of friction clutch and also to compute the natural frequencies and mode shapes. Natural frequencies calculation has been made for the various parameters by changing and was investigated on the vibration characteristics as well. Also the numerical approach is applied for the verification.

Thakare et.al [13] given an experimental method to determine the modal characteristics of a plate with multiple holes and slots was verified by finite element analysis with Ansys. Also the relationship between parameter variation and vibration modes was investigated. These results can be used as guidance for modal analysis and damage detection of circular plate with hole.

Raut et.al [14] Implemented finite element method tool for modal analysis of annular disc. Natural frequency and mode shapes of annular disc were found out by FFT analyser method.

Waikar et.al [15] has done optimization of existing tile cutter for better performance in vibration. It was done by analysing vibration characteristics like natural frequency and mode shapes of tile cutter with free boundary condition through optimization of no of diametral slots, diameter of slot end and number of cooling holes.

III. SIMULATION USING ANSYS

For present study, to determine numerically the natural frequencies of the disc and their corresponding mode shapes, Finite Element Analysis is performed in ANSYS. A model of the disc is drawn in CATIA and imported in ANSYS followed by parameters given in table 1 are applied. The clamping was obtained by fixed support on the face of inner edge of the disc as in the experimental test rig. The disc is meshed using sizing on the edges. The modal analysis is performed in ANSYS. This will finds both the mode shapes and their natural frequencies.

A. *Simulation with standard CRCA steel (Cold Rolled Close Annealed)*

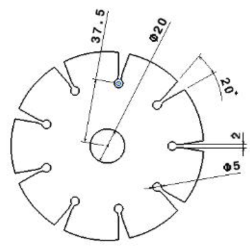
To determine numerically the natural frequencies of the disc, a simulation using the material properties of standard steel is performed in ANSYS. Table 1 shows the material properties of standard steel.

TABLE I. GEOMETRIC PROPERTIES, MATERIAL PROPERTIES

Geometric and Material properties	
Material	Mild steel (CRCA)
Outer Diameter	110 mm
Inner Diameter	20 mm
Thickness	1.2 mm
Modulus of Elasticity	200 Gpa
Density	7850 kg/m^3
Poisson's ratio	0.3

Following fig. 1 shows the sample dimensions of disc with 9 slots of 20° angular orientations of slots and slot end hole is located at Radius of 37.5mm. Slot end hole diameter is 5mm and distance between slots is 2mm. Thickness for all disc throughout this work are kept constant.

FIG. 1 SAMPLE DIMENSIONS OF DISC WITH 9 SLOTS OF 20° ANGULAR ORIENTATION.

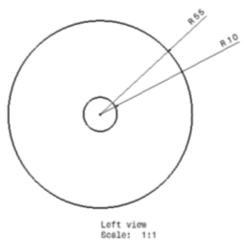


B. Effect of shape change on vibration characteristics of discs by using ANSYS

For analysis of effect of shape change on vibration characteristics of circular disc, four discs are chosen of specifications given in following fig. 2, 3, 4 & 5. For specimen 1 there is no any hole on the body of disc. Only annular disc with 20 mm inner diameter and 110 mm outer diameter is considered for analysis. For specimen 2, nine circular holes are created at diameter of 80 mm and radius of each hole is 5 mm. then in specimen 3, areas of 3 circular holes in one elliptical hole are combined. Therefore, total three elliptical holes are generated. These three elliptical holes are placed 120° apart from each other for symmetry. Lastly, for specimen 4 total areas of 9 circular holes are combined into 4 elliptical holes and these are placed at 90° apart.

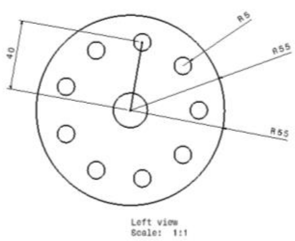
The first three natural frequencies and mode shape of these four specimens are analyzed and plotted in table 4 & 5.

FIG 2. SPECIMEN 1



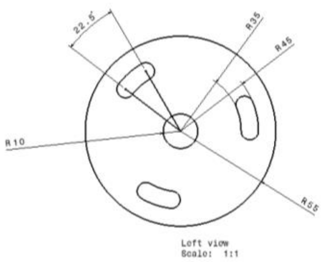
Disc without holes

FIG. 3 SPECIMEN 2



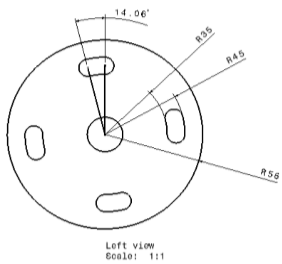
9 circular holes, radius = 5 mm, Location at R40

FIG 4. SPECIMEN 3



No. of elliptical holes= 3, Angle= 22.5°, Location= R40

FIG 5. SPECIMEN 4



No. of elliptical holes= 4, Angle= 14.06°, Location= R40

IV. EXPERIMENTAL MODAL TEST

A. Introduction to annular plates

An annular plate consists of a circular outer boundary and a concentric circular inner boundary. Throughout this work the radii a and b will define the outer and inner boundaries, respectively.

B. Natural frequency of disc

A single degree of freedom system with a mass (*m*), a damper (*c*) and a spring (*k*) is described by equation (1). The natural frequency *F* of this system is given by equation (2) (given $k >> \frac{c^2}{4m}$). A system with multiple degree of freedom system will have as many natural frequencies as it have degrees of freedom.

$$m\ddot{x} + c\dot{x} + kx = 0 \qquad .... \quad (1)$$

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad .... \quad (2)$$

C. Mode shape on disc

Discs diametrical mode shapes are described along the edge by cosines with maxima at the point of excitation, amplitude A and nodal diameter periods around perimeter of the disc. Nodal diameters are a line intersecting the disc centre where the mode shapes deflection amplitude is always zero. Each mode shape oscillates only transversely, meaning that the nodal diameters do not move relative to an excitation.

The transverse deflection ξ is a sum of the vibration of all mode shapes. Then, each mode shape oscillates with its particular frequency (f_{ND}). The transverse deflection of the perimeter of the disc is a function of time t and position (θ).

$$\xi(\theta,t) = \sum_{ND=0}^{\infty}(A_{ND}\cos(ND\cdot\theta)\cdot\sin(2\pi\cdot f_{ND}\cdot t)) \qquad ... (3)$$

In this work the name of the mode shapes will be the number of nodal diameter, e.g. a mode shape with two nodal diameters will be named ND2. The mode shapes studied in this article is presented in Figure 6, 7 & 8.

To get accurate believable results of the inner edge fixed and outer edge free vibration modal analysis, models of an annulus were analysed in non-rotating states. The same material properties of CRCA steel are used. No symmetry about any axis could be used to obtain vibration mode shape results. The reason no symmetry can be used is because the nodes and elements in a free-vibration modal analysis are not all symmetric modes. The modes of vibration include both symmetric and asymmetric mode shapes.

Due to the symmetric shape of the given discs, many modes are repeated and are actually the same mode. However, when solving the eigenvalue matrix equation, each repeated root (eigenvalue) will be unique mathematically. For example, Fig. 6 (a) & 6 (b) shows the repeated mode shapes of mode one and mode two. In the physical world these models are considered the same mode, but they are a unique solution to the eigenvalue problem. Other examples of the 20 modes reported in the analysis of the annulus that can be considered the same mode are modes four and five, six and seven, eight and nine, 10 and 11, 13 and 14, 15 and 16, 17 and 18, and 19 and 20.

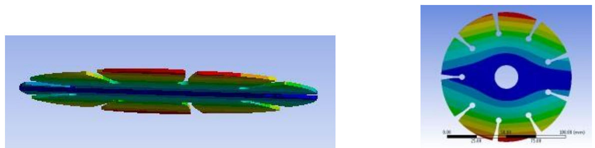


FIG. 6 (A) SYMMETRIC MODE SHAPES OF ANNULUS (MODE 1)

FIG. 6 (B) SYMMETRIC MODE SHAPES OF ANNULUS (MODE 2)

A result after convergence for specimen 1 gives (a) Mode 1 and (b) Mode 2 with inner edge boundary condition.

Mode three is a unique mode shape and due to its axisymmetric shape can easily be modelled (computationally) as an axisymmetric model. In fact, the bowl-shape mode modal analysis of the 3-D non-rotating disc model using linear reduced hexahedral elements agreed quite well with the axisymmetric first mode shape.

FIG. 7 Mode Three (3-D Model) - Bowl Shape Mode.

FIG. 8 TWO PEAK MODE

FIG. 9 THREE PEAK MODE

Fig 8 and 9 shows two and three peak mode respectively.

D. Description of experimental rig

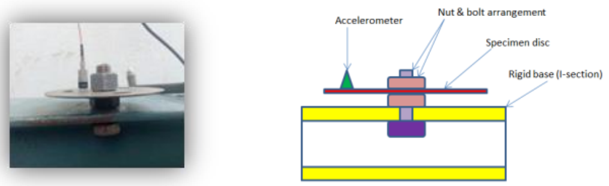
In vibration test, annular disc is fixed on fixture as shown in fig consist of test specimen (Annular disc), fixture mounted on rigid base, accelerometer, impact hammer, eight channel vibration analyser and laptop with DEWESOFT software for data analysis. Four marking lines are shown on which impact is to be made by using impact hammer for modal test. The experimental test rig build in the vibration lab at SKN Sinhgad College of Engineering, Korti, Pandharpur Dist. Solapur (MH) under the supervision of Dr. R. T. Vyavahare.

As the plates are fixed on inner boundary it can be used for following type of tests as below:

E. Impact hammer test

Clamping was obtained by using two nuts and one bolt is fastened above and below the disc for impact hammer test. Bolt is used to restrict the movement at inner edge of annular plate in x, y and z direction. The fixture was rigidly fixed on heavy weighted I section channel painted by blue colour as shown in fig. 10

FIG. 10 FIXTURE FOR IMPACT HAMMER TEST



F. Procedure of experimentation

The mode shapes of the disc are identified (connected to the diks natural frequencies) using the impact hammer method. Measuring and analysing (FFT analysis) the response of the disc to an impact, will reveal the natural frequencies of the disc. However, the natural frequencies corresponding mode shape will not be clear. To identify the corresponding mode shapes, the disc is impacted on its edge by impacting a hammer from a constant height on various positions relative to the accelerometer. This will excite all mode shapes to vibrate at their natural frequencies. Since the mode shapes are defined by the impact position, the mode shapes amplitude measured by the accelerometer will vary with the impact position.

The impact positions are set so that particular mode shape amplitude is zero at the position of the accelerometer. This is done by setting the impact position so that the accelerometer is on a nodal diameter. In theory, one natural frequency will not appear in the analysis (FFT) of the measurements acquired from such an impact position. That natural frequency can then be identified as the natural frequency of the mode shape the impact position was set for. However, because of inaccuracies in the impacts, the natural frequencies will not disappear completely from the analysis, but they will be clearly lower than they are for other impact positions.

For the mode shapes studied in this work (ND1, ND2 and ND3), the impact position are calculated using an equation derived from equation (3), and are shown in Table 2 and in Fig. 11. In addition to these positions, the disc is impacted at 180° from the accelerometer for reference. Measurements from this impact will make a good reference since all mode shapes will have amplitude maxima (or minima) at the position of the accelerometer. All impacts are repeated three times. The measurements (4 impacts, 3 repetitions) are made by the accelerometer with a sample rate of 20000 Hz.

Equation derived from equation (3) is,

$$\theta = \frac{180^\circ - Arc\cos(0)}{ND} = \frac{180^\circ - 90^\circ}{ND} \quad \ldots \quad (4)$$

FIG 11. IMPACT POSITIONS IN IMPACT HAMMER EXPERIMENT. ACCELEROMETER (BLACK), 180° (ORANGE), 90° (RED), 135° (GREEN) AND 150° (BLUE).

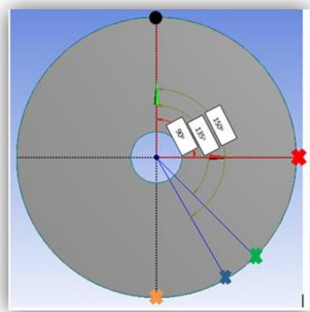


TABLE 2 IMPACT POSITIONS OF HAMMER ON DISC.

Mode shape	Impact position (θ)
ND 1	90°
ND 2	135°
ND 3	150°

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In post processing, all measurements are analyzed using Fast Fourier Transform (FFT). For each impact point, an average FFT result is calculated from the FFT results of the repetitions. The FFT results will show peaks at the natural frequencies of the disc.

The magnitudes (amplitude) of these peaks will vary with impact position. For each of the three impact positions in Table 2 there should be one natural frequency that is significantly lower than it is for the other impact positions. This will indicate that the frequency is the natural frequency of the mode shape the impact position was set for, see Table 2.

G. Effect of shape change on vibration characteristics of discs by using Experimental modal test

For analysis of effect of shape change on vibration characteristics of circular disc using experimental modal test, four discs are precisely manufactured of specifications given in following fig. 12, 13, 14 & 15 by laser cutting method. For specimen 1 there is no any hole on the body of disc. Only annular disc with 20mm inner diameter and 110 mm outer diameter is considered for analysis. For specimen 2, nine circular holes are created at diameter of 80 mm and radius of each hole is 5mm. then in specimen 3, areas of 3 circular holes in one elliptical hole are combined. Therefore, total three elliptical holes are generated. These three elliptical holes are placed 120° apart from each other for symmetry. Lastly, for specimen 4 total areas of 9 circular holes are combined into 4 elliptical holes and these are placed at 90° apart.

FIG 12 SPECIMEN 1

Disc without holes, inner dia.= 20mm, Outer dia.= 110mm

FIG 13 SPECIMEN 2

9 circular holes, radius = 5 mm, Location at R40

FIG 14 SPECIMEN 3

No. of elliptical holes= 3, Angle= 22.5°, Location= R40

FIG 15 SPECIMEN 4

No. of elliptical holes= 4, Angle= 14.06°, Location= R40

V. RESULTS AND DISCUSSION

A. Effect of no. of slots and angular orientation of slot ends on first three natural frequencies

For this analysis we have chosen two different no. of slots i.e. 9 & 10 and three different no. of angular orientation of slot ends i.e. 15°, 20° & 25°.

TABLE 3 NATURAL FREQUENCY FOR DIFFERENT NO. OF SLOTS AND ANGULAR ORIENTATION

No. of slots	Angular orientation of slot end (deg.)	Natural frequency (Hz)		
		Fn1	Fn2	Fn3
9	15	439	460	516
	20	432	452	507
	25	450	471	529
10	15	443	464	519
	20	447	467	523
	25	453	474	531

B. Effect of different sizes on first three natural frequencies:

FIG. 16 NATURAL FREQENCY VS ANGULAR ORIENTATION OF SLOTS FOR 9 NO. OF SLOTS

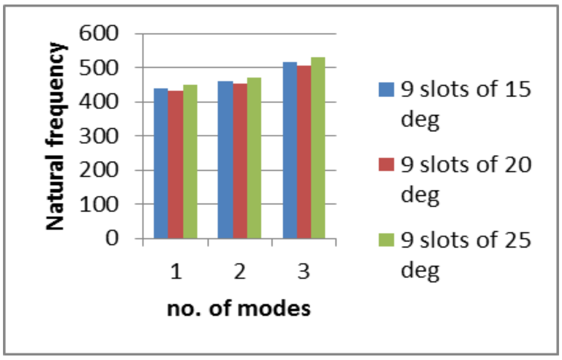
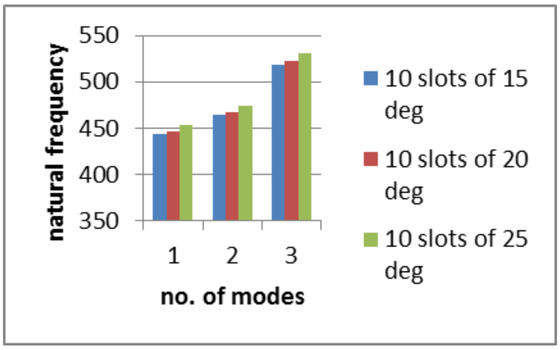


FIG. 17 NATURAL FREQENCY VS ANGULAR ORIENTATION OF SLOTS FOR 10 NO. OF SLOTS



C. Inferences from above results

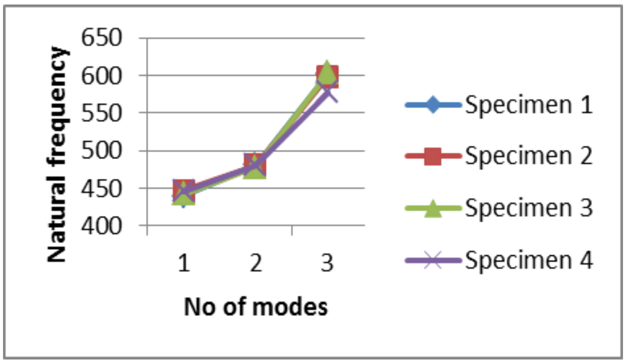
Above fig. 16 & fig. 17 shows that by keeping no of slots constant, if we increase angular orientation of slots, natural frequency is increases.

D. *Effect of shape change on vibration characteristics of discs by using ANSYS:*

TABLE 4 EFFECT OF SHAPE CHANGE ON NATURAL FREQUENCY OF CIRCULAR DISC

Spec no.	No of holes	Shape of hole	Angle (Deg.)	Location (mm)	Natural frequency (Hz)		
					Fn1	Fn2	Fn3
1	Nil	Nil	Nil	Nil	440	481	602
2	9	Circular	Nil	R40	447	481	599
3	3	Elliptical	22.5°	R40	442	478	603
4	4	Elliptical	14.06°	R40	446	481	578

Fig. 18 Effect of shape change on nat. Frequency



E. *Inferences from above results:*

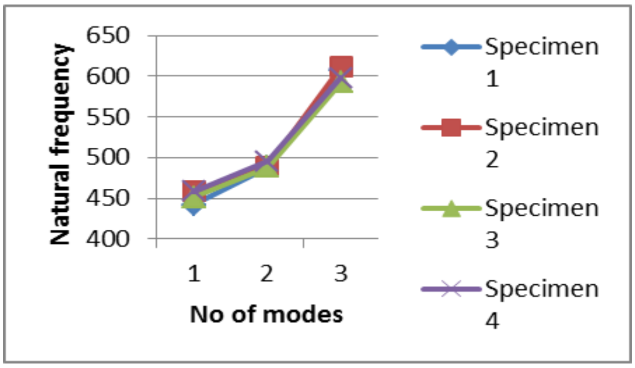
From table 4, it is observed that, if we add circular holes in geometry of circular disc, natural frequency of disc increases. If we change shape of holes from circular to elliptical by keeping same volume of material removal, then natural frequency of the disc is decreasing more for three holes and less for 4 holes because the concentration of areas are more for disc with three holes and less for disc with four holes.

F. *Effect of shape change on vibration characteristics of discs by using Experimental modal test:*

TABLE 5 EFFECT OF SHAPE CHANGE ON NATURAL FREQUENCY OF CIRCULAR DISC

Sp	No of holes	Shape of hole	Angle (Deg.)	Location	Natural frequency (Hz)		
					Fn1	Fn2	Fn3
1	Nil	Nil	Nil	Nil	442	488	607
2	9	Circular	Nil	R40	458	489	611
3	3	Elliptical	22.5°	R40	451	488	592
4	4	Elliptical	14.06°	R40	458	495	598

Fig. 19 Effect Of Shape Change On Natural Frequency



G. *Inferences from above results:*

From table 5, it is observed that, if we add circular holes in geometry of circular disc, natural frequency of disc increases. If we change shape of holes from circular to elliptical by keeping same volume of material removal, then natural frequency of the disc is decreasing more for three holes and less for 4 holes. Hence ANSYS and Experimental methods are validates each other by good agreement.

VI. CONCLUSION

1) Effect of no. of slots and angular orientation of slot ends on first three natural frequencies shows that by keeping no of slots constant, if we increase angular orientation of slots, natural frequency is increases.

2) Results from effect of shape change of holes on natural frequency of annular disc shows that, if we add circular holes in geometry of circular disc, natural frequency of disc increases. If we change shape of holes from circular to elliptical by keeping same volume of material removal, then natural frequency of the disc is decreasing for three and four no. of holes.

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