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Effect of P-Delta Action on Multi-Storey Buildings

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***Abstract*— As urbanization increases worldwide, the construction of tall buildings in seismic regions is becoming increasingly common. In heavily populated cities, the available land for buildings is becoming scarcer and scarcer, and the cost of land is becoming higher and higher. A tall building is defined as one in which the structural system is adopted such that to make it sufficiently economical and also to resist lateral forces due to wind or earthquakes within the prescribed criteria for strength, drift and comfort of the occupants. Structure subjected to lateral loads often experience secondary forces due to the movement of the point of application of vertical loads. This secondary effect, commonly known as P-Delta effect plays an important role in the analysis of the structure. The P-Delta analysis is recommended by several design codes such as ACI-318, LRFD, etc. in lieu of the moment magnification method for calculation of more realistic forces and moments. Seismic analysis of a multi-storey RC building is analyzed by using STAAD structural analysis software. The building models with different storey have been analyzed to investigate the maximum response in buildings in terms of displacements, storey drifts, column moment, beam moment, column shear and beam shear.**

Keywords*— *P-Delta, (P-Δ) structure deformation, (P-δ) member deformation, (P-Δ-δ) with Structure and Member deformation, storey drift.

I. INTRODUCTION

In conventional first order structural analysis, the equilibrium is expressed in terms of the geometry\of the undeformed structure. In case of linearly elastic structure, relation between displacement and external force is proportional. In addition, stress-strain relationship of material is linear. Thus, by definition, this method excludes non-linearity, but it generally represents conditions at service loads very well. The first order elastic analysis is based on following assumptions: - (1) Material behaves linearly and hence all yielding effect can be ignored. (2) The member behave linearly, and the member instability effect such as those caused by axial compression (these are called P-δ effects), which reduces the member's flexural stiffness, can be ignored. (3) The frame also behaves linearly, and the frame instability effects, such as those caused by the moments due to horizontal frame deflection and gravity loads acting on the displaced structure (these are called P-Δ effects), can be ignored. Though the first–order elastic analysis provides an 'exact solution' that satisfies the requirements of compatibility and equilibrium of the undeformed structure, it does not provide any information about the influence of plasticity and stability on the behavior of the structure. Hence, these influences are normally provided indirectly in member capacity checks. A first-order elastic analysis is sufficient for normal framed structures, which are braced against sway, however, first-order elastic analysis will not yield sufficiently accurate results for some suspension systems, arches, tall buildings, and structure subjected to early localized yielding or cracking. All the structure exhibit significant non-linear response just prior to reaching their limit of resistance because of yielding and buckling effects associated with axial compressive forces. Hence, this non-linear behavior is accounted for by the code formulae (that makes allowance for non-linearity in some empirical or semi-empirical manner) or by supplementary theoretical or experimental studies. Second order effects on the frame are accounted by a combination of P-Δ effect, which corresponds to overall frame, and P-δ effect, which corresponds to individual members within the frame. Since both of these contribute to the deformation of the frame it is important to consider their combined effect.

II. SECOND ORDER ELASTIC ANALYSIS

A. Non-Linearity

Nonlinearity caused by large deformations is referred to as Geometric non-linearity. Linear stress-strain equations are assumed to hold in this category. Problems involving geometric nonlinearity arises both from nonlinear strain-displacements relations in theory of elasticity and from finite changes in geometry. In other words, this category encompasses large strains and large displacements. There are four sources of nonlinear behavior in case of structural analysis. The corresponding non linear effects are identified by the term geometric, material, force boundary conditions and displacement boundary conditions. The P-Delta analysis accounts for the effect of a large axial load upon the transverse bending behavior of frame elements. Axial compression reduces the flexural stiffness of frame elements, and axial tension stiffens them. This is a type of geometric nonlinearity known as the P-Delta effect. It does not include large strain or large rotation effects. Non linearity can also arise when the stress-strain relationship of the material is non-linear in the elastic or in the plastic range, this is called material nonlinearity. The methods used for second order analysis are

1) The effect Length Factor Method.
2) Approximate Buckling Analysis.
3) Approximate P-Delta Analysis.
   i) Direct P-Delta Method.
   ii) Negative Bracing Member Method.
   iii) Iterative P-Delta Method.

B. P-Delta Effect

Engineers have been aware of the P-Delta for many years. However, it is only relatively recently that the computational power aided to provide analytical approximations to this effect, which has become widely available. It is an engineer's judgment as to how accurately the second order effect needs to be accounted for in determining design forces and moments.

Use of geometric stiffness matrix is general approach to include large deformation effects in the static as well as dynamic analysis of all types of structural system, which is commonly referred to as P-Delta analysis or second order analysis. In second-order elastic analysis, the material is assumed to have a linear elastic relationship. However, the equilibrium is calculated on the deformed geometry of the structure. A rigorous second-order analysis includes both the member curvature (P-δ) and side-sway (P-Δ) stability effects. (It may be of interest to note that the structural system becomes stiffer when its members are subjected to tension and it becomes softer when its members are subjected to compression.) The detrimental effects associated with second-order deformations due to compressive forces are considered to be important in structures subjected to predominant gravity loads. In first-order analysis, the unknown deformations can be obtained in a simple and direct manner, whereas second-order analysis requires an iterative procedure to obtain the solutions. This is because the deformed geometry of the structure is not known during the formation of the equilibrium and kinematic relationship. Thus, the analysis proceeds in a step-by-step incremental manner, using the deformed geometry of the structure obtained form a preceding cycle of calculation. For most practical case, accurate second–order design forces can be obtained by applying the loads in one or two increments, and only a few iterations are required to converge to an accurate solution.

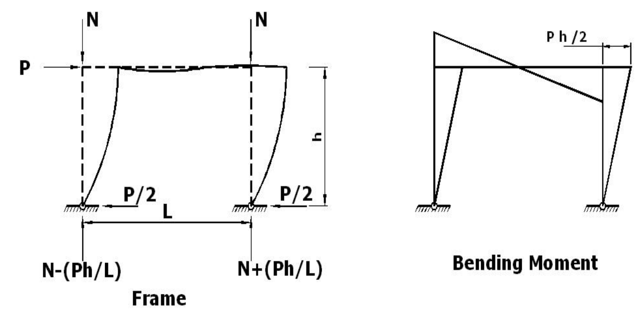


Fig. (a) Fig. (b)

First order Analysis

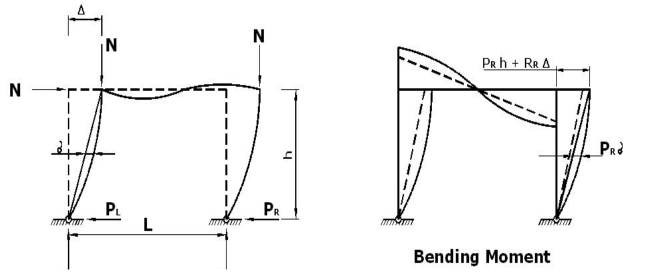


Fig. (c) Fig. (d)

Second order Analysis

Fig. 1.1 Simple Portal Frame

Considering the portal frame as shown in fig.1.1 (a), first-order analysis does not consider the second order moments, the bending moment distribution given by first order elastic analysis is linear as shown in fig. 1.1 (b) Elastic analysis assumes linear-elastic material behavior and may be done based on the equilibrium in the undeformed geometry of the structure (First-order elastic analysis).

In the deformed geometry of the structure (Second-order analysis) structure become even more slender and less resistant to deformation the need to consider the P-Delta effect increases. To reflect this, codes of practices and engineers refers to the use second order analysis in order that P-Delta and stress stiffening effects are accounted. P-Delta effect should be considered in all types of loading.

C. Iterative P-Delta Method

The iterative P-delta method is based on the simple idea of correcting first-order displacements, by adding the P-delta shears to the applied story shears. Since P-delta effects are cumulative in nature, this correction and subsequent reanalysis should be performed iteratively until convergence is achieved. At each cycle of iteration a modified set of story shears are defined as:

$$\sum \mathbf{V_i} = \sum \mathbf{V_1} + \left(\sum \mathbf{P}\right)_{\Delta_{i-1}} / \mathbf{h} \qquad (1)$$

Where $\sum V_i$ is the modified story shear at the end of i[th] cycle of iteration, $\sum V_1$ is the first-order story shear, $\sum P$ is the sum of all gravity forces acting on and above the floor level under consideration, Δ_{i-1} is the story drift as obtained from first-order analysis in the previous cycle of iteration, and h is the story height for the floor level under consideration. Iteration may be terminated when

$$\sum \mathbf{V_i} \cong \sum \mathbf{V_{i-1}} \quad \textbf{or} \quad \Delta_i \cong \Delta_{i-1} \qquad (2)$$

Generally for elastic structures of reasonable stiffness, convergence will be achieved within one or two cycles of iteration. One should note that since the lateral forces are being modified to approximate the P-delta effect, the column shears obtained will be slightly in error. This is true for all approximate methods which use sway forces to approximate the P-delta effect.

III. PROBLEM CONSIDERED

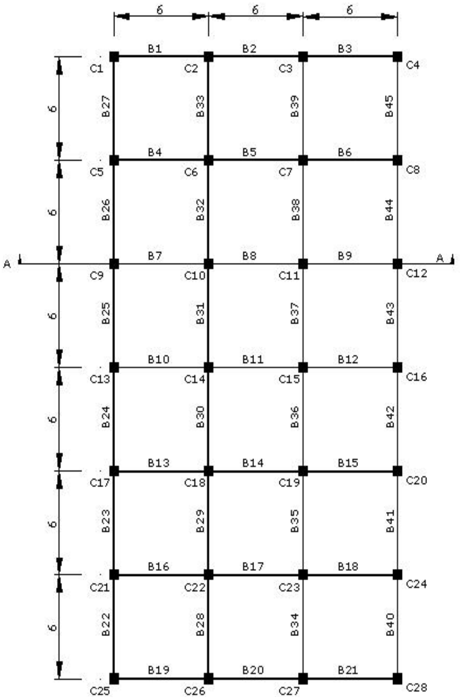
In the present study the method of P-Δ (structure deformation) and P-Δ-δ (structure with member deformation) effect in multi-storied structures are identified floor wise and the significance of building responses like displacement, drift, column moment, beam moment, column shear and beam shear are studied in detail. Seismic analysis is carried out as per IS-1893 (Part-I) 2002 guidelines. Equivalent static force method is adopted.

The stress resultants are displacement, bending moment and corresponding shear force. Linear elastic plane frame analysis is performed for the different models of the building using *STAAD* software. The frame members are modeled with rigid end zones.

A building has plan dimensions 18 m x 36 m with bay width 6 m both sides is selected. The building is located in Zone III Solapur district as per IS 1893 (Part-I) – 2002).

TABLE 1.1 DATA OF THE EXAMPLE

Live Load	4 kN/m^2 at typical floor 1.5 kN/m^2 at terrace.
Floor Finish	1.0 kN/m^2
Water Proofing	2.0 kN/m^2
Terrace Finish	1.0 kN/m^2
Location	Solapur City (Zone III for earthquake)
Earth quake load	As per IS:- 1893-2002
Type of soil	Type II medium as per IS:-1893-2002
Storey height	Floor height 3.2 m
Walls	230 mm thick brick masonry 12 mm plaster on both sides
Column	0.50 m x 0.50 m
Beams	0.30 m x 0.60 m
Slab	0.125 m



(All dimensions are in m)

Fig. 1.2 Plan.

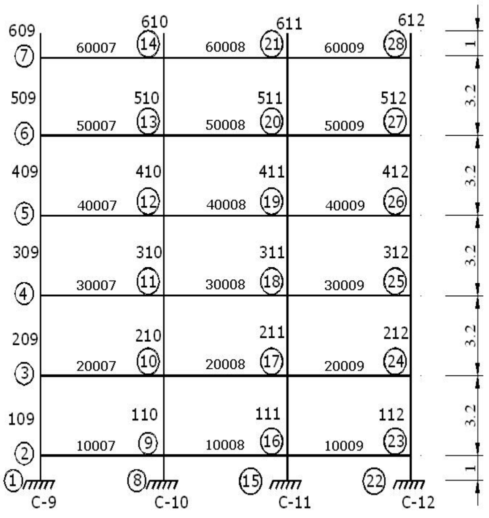


Fig. 1.3 Beam and Column Numbering

In the analysis of structures, neglecting the second order effects may overestimate the strength and stiffness of a member or frame. The elastic forces generated within a member or frame can be more accurately predicted with the use of an elastic second-order analysis. The second-order effects (amplified moments and deflections) are of increasing importance as lighter, and more flexible structures are constructed. The use of higher strength materials and less rigid non-structural elements is producing more flexible structures where second-order effects are of greater importance. The behavior of multistory frames subjected to either gravity loads only or under combined gravity and lateral loading cannot be accurately predicted by a first-order elastic analysis, when the compressive axial load level is significant the lateral displacements may be rather large and may not be within the acceptable range of lateral displacements for tall buildings. An elastic second-order analysis using computer software's can be used to provide an accurate.

The building model has been analyzed for 5 to 27 storeys with 2 storey interval. The maximum response in building models occurring at certain height of floor levels had been studied. Lateral load for the selected frame has been carried out as per IS-1893 (Part-I) 2002. The analysis has been carried out for without P-Delta effect to locate the maximum responses, and then same has been analyzed for P-Δ (structure deformation) effect with number of iterations. Again analysis for P-Δ-δ (with structure and member deformation) effect with number of iterations for same model had been carried out. The maximum response values are compared to notify the P-Delta effect. As Iterative P-Delta method has been used numbers of iterations have been carried out for each building model with P-Δ & P-Δ-δ till convergence occurs. It is found that convergence of results occurred for third iteration hence comparison has been done for third iteration only.

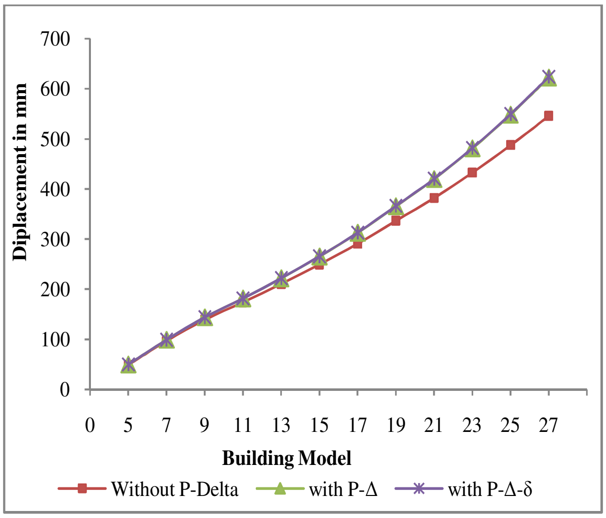
The significance of building responses like displacement, drift, column moment, beam moment, column shear and beam shear are studied in detail

IV RESULTS

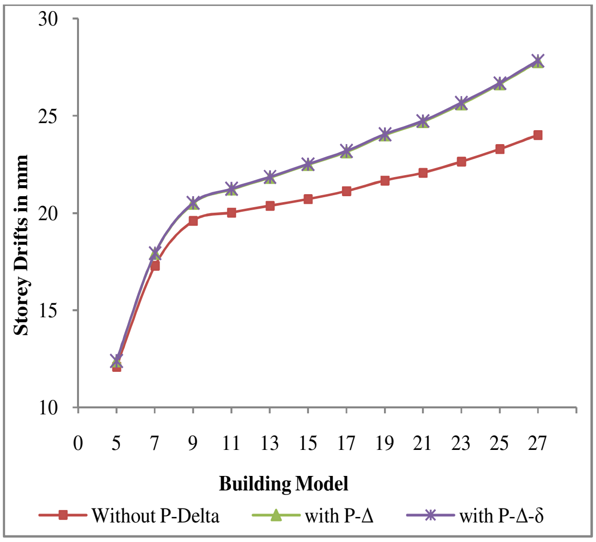
A. Displacement and Storey Drift.

TABLE 1.2 MAXIMUM DISPLACEMENT & STOREY DRIFTS

Bldg. Model with nos. of storey	**Maximum Displacement in mm**			**Maximum Storey Drifts in mm**		
	Without P-Delta	*With P-Δ*	*With P-Δ-δ*	*Without P-Delta*	*With P-Δ*	*With P-Δ-δ*
5	228.978	228.978	228.993	12.096	12.373	12.384
7	293.810	298.702	298.744	17.290	17.905	17.928
9	311.886	319.528	319.579	19.592	20.485	20.511
11	315.789	324.944	324.961	20.013	21.210	21.242
13	318.670	330.350	330.366	20.360	21.813	21.853
15	321.483	334.824	334.841	20.721	22.475	22.514
17	324.074	339.422	339.418	21.119	23.132	23.179
19	327.347	344.596	344.591	21.675	23.995	24.045
21	328.936	348.178	348.163	22.069	24.687	24.740
23	331.190	352.590	352.571	22.640	25.601	25.659
25	333.298	357.021	357.002	23.285	26.628	26.690
27	335.256	361.638	361.617	24.006	27.776	27.843



Graph- 1.1 Maximum Displacements in Building Models

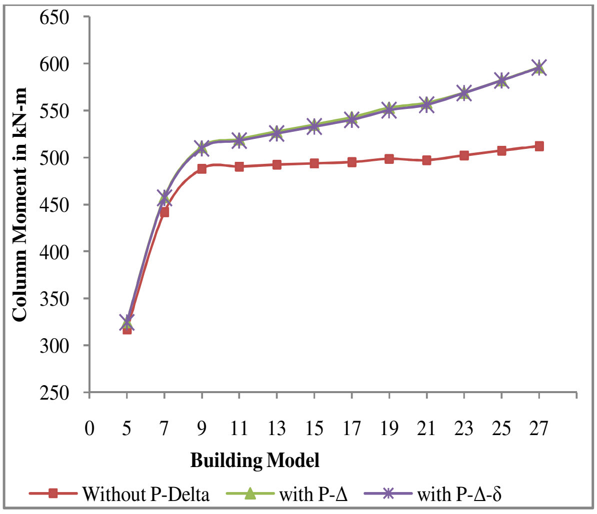


Graph- 1.2 Maximum Storey Drifts in Building Models

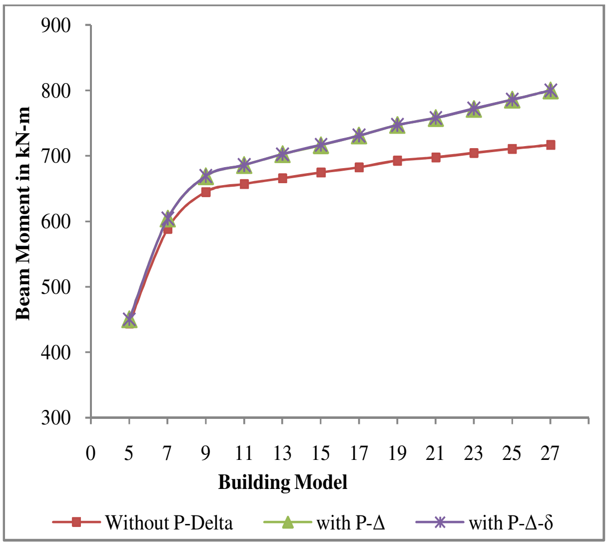
B. Column and Beam Moments

TABLE 1.3 MAXIMUM COLUMN & BEAM MOMENTS

Bldg. Model with nos. of storey	Maximum Column Moments in kN-m			Maximum Beam Moments in kN-m		
	Without P-Delta	*With P-Δ*	*With P-Δ-δ*	*Without P-Delta*	*With P-Δ*	*With P-Δ-δ*
5	317.12	324.99	324.52	443.89	450.37	450.47
7	442.06	458.04	457.16	588.57	603.86	604.02
9	488.07	511.28	510.01	644.99	668.88	669.09
11	490.53	519.59	518.01	657.06	685.69	685.80
13	492.39	527.46	525.57	666.02	702.55	702.67
15	493.88	535.13	532.93	674.66	716.52	716.66
17	495.14	542.76	540.25	682.61	730.78	730.86
19	498.75	553.22	550.37	692.69	746.98	747.06
21	497.14	558.09	556.09	697.49	758.11	758.17
23	502.37	568.97	568.70	704.37	771.85	771.90
25	507.44	582.41	582.13	710.81	785.77	785.84
27	512.10	595.96	595.67	716.93	800.19	800.26



Graph- 1.3 Maximum Column Moments in Building Models.

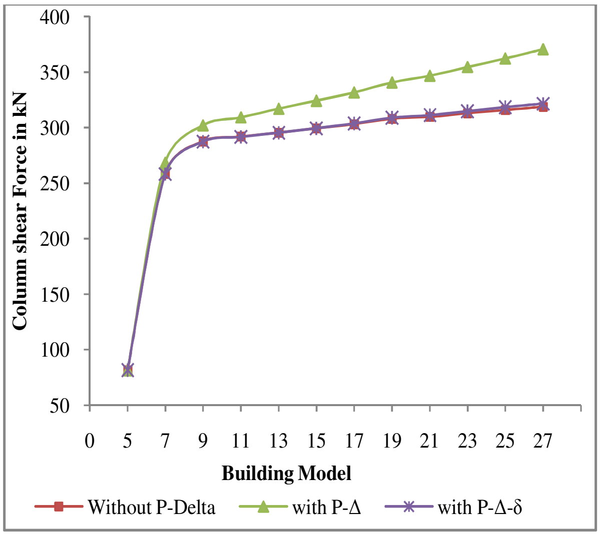


Graph- 1.4 Maximum Beam Moments in Building Models

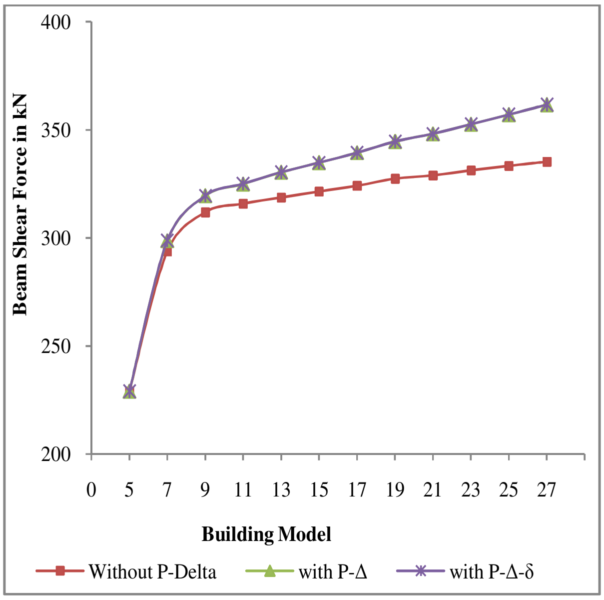
C. Maximum Shear Force in Column & Beam Sections

TABLE 1.4 MAXIMUM COLUMN SHEAR & BEAM SHEAR

Bldg. Model with nos. of storey	Maximum Column Shear Force in kN			Maximum Beam Shear Force in kN		
	Without P-Delta	*With P-Δ*	*With P-Δ-δ*	*Without P-Delta*	*With P-Δ*	*With P-Δ-δ*
5	81.72	81.72	81.746	228.98	228.98	228.99
7	258.83	268.46	258.51	293.81	298.70	298.74
9	287.76	302.31	287.39	311.89	319.53	319.58
11	291.99	309.41	291.80	315.79	324.94	324.96
13	295.66	317.20	295.70	318.67	330.35	330.37
15	299.50	324.31	299.73	321.48	334.82	334.84
17	303.24	331.92	303.78	324.07	339.42	339.42
19	308.19	340.87	309.11	327.35	344.60	344.59
21	309.94	346.82	311.29	328.94	348.18	348.16
23	313.09	354.58	314.90	331.19	352.59	352.57
25	316.15	362.52	318.51	333.30	357.02	357.00
27	318.95	370.88	321.91	335.26	361.64	361.62



Graph- 1.5 Maximum Column Shear in Building Models



Graph- 1.6 Maximum Beam Shear in Building Models

V DISCUSSION

The result of analysis of building models with increasing storey numbers shows that the effect of P-Delta will be considerable when lateral load exists on the structure. The effect of P-Delta is very negligible when only gravity loading exits on structure.
The graphs 1.1 to 1.6 shows that the P-Delta effects are not predominant up to seven storey buildings hence can be neglected. The P-Δ (structure deformation) and P-Δ-δ (with structure and member deformation) graphs shows variation for displacements which is linear with the height of building, where as the other responses shows change in nature of graph after 9th storey building model. Displacement, bending moment, shear force, are increased by considering P-Delta effect.

VI CONCLUSIONS

The following conclusions are made considering the effects of P-delta action

1) Generally, P-delta effects are negligible up to 7 storey buildings where only gravity loads are governing load combinations.
2) As it is iterative method, three iterations are required for convergence of the results.
3) As number of stories increases means height of building increases the P-Delta effect becomes more and more predominant from respective parameters like displacement, storey drifts, column & beam moments column & beam shear forces.
4) The analysis with P-Δ (structure deformation) and P-Δ-δ (with structure & member deformation) does not differ for displacement, storey drifts, and column & beam moments, beam shear except column shear.
5) In case of column shear only P-Δ (structure deformation) gives maximum column shear than P-Δ-δ (with structure & member deformation). Column shear reduces when member deformations are also considered in the analysis.

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