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Effect of Multiple Surface Cracks under Moving Load in a Semi-Infinite Elastic Medium

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Abstract - This study presents an analytical investigation of stress distribution in a semi-infinite elastic medium containing multiple surface cracks subjected to a moving point load. The model considers anti-plane shear (Mode-III) deformation, where two or more surface cracks of finite length are located at specified intervals on the free surface. The moving load travels with constant velocity, generating shear waves that interact with the cracks and induce complex interference patterns. A Fourier transform technique is employed to derive the out-of-plane displacement and shear stress fields. Parametric analyses are conducted to examine the influence of crack spacing, number of cracks, and load velocity on the stress amplification at the surface. The results reveal that crack interference plays a significant role in modifying the surface stress distribution, with critical spacing conditions leading to constructive or destructive stress amplification. A non-dimensional crack interference factor is introduced to quantify this effect. Validation with benchmark solutions for single crack cases confirms the accuracy of the present model. The findings offer insights relevant to the design and monitoring of subsurface fracture networks, high-speed railbeds, and pavement structures.

Keywords
Multiple Cracks, Crack Interference, Moving Load, Semi-Infinite Elastic Medium, Mode-III Shear, Fourier Transform, Stress Amplification

1 INTRODUCTION

The dynamic response of elastic media containing surface discontinuities under moving loads is a subject of great interest in applied mechanics and geotechnical engineering. Cracks, whether caused by fatigue, environmental effects, or mechanical overloading, often appear in clusters rather than as isolated entities. In such scenarios, the interaction between multiple cracks and incident shear waves can significantly alter the stress distribution within the material, influencing structural integrity, failure patterns, and long-term durability.

In critical infrastructures such as highways, high-speed rail networks, and airfields, repeated dynamic loads traverse the surface, interacting with pre-existing surface cracks. These interactions may result in localized stress amplification, crack coalescence, or fatigue propagation, especially when the cracks are closely spaced. Consequently, understanding the effect of multiple surface cracks on stress distribution under dynamic conditions is essential for safe design, monitoring, and rehabilitation planning.

Earlier analytical and numerical studies have considered single surface or subsurface cracks under moving loads [1–4], or wave interaction with buried discontinuities in elastic or piezoelectric media [5–7]. However, most of these works assume a single crack, often of fixed geometry and isolated from other defects. In reality, multiple cracks may be distributed periodically or randomly, leading to constructive or destructive interference of stress waves depending on spacing, load frequency, and wavefront geometry.

Mode-III (anti-plane shear) deformation provides a mathematically tractable yet physically meaningful framework to model such scenarios. The governing equation reduces to a scalar wave form, enabling the use of Fourier transform methods to solve complex crack–wave interaction problems analytically. Despite the tractability, to the best of the authors' knowledge, no analytical work has modeled the combined effect of multiple surface cracks under a moving load in a semi-infinite elastic medium using such an approach.

The present study addresses this gap by developing an analytical solution for the stress field in a semi-infinite, isotropic elastic half-space containing two or more surface cracks. A moving point load, modeled as a Dirac delta function, travels at constant velocity along the surface, generating shear waves that interact with the cracks. The Fourier transform technique is used to derive

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the displacement and stress fields. Special emphasis is placed on analyzing the effects of crack number, spacing, and load velocity. A new non-dimensional parameter - the crack interference factor - is introduced to quantify the amplification or shielding effect caused by crack interactions. The results offer novel insights into wave–crack interference phenomena, with direct applications in pavement design, fracture diagnostics, and seismic hazard zones.

2 LITERATURE REVIEW

The interaction of moving loads with internal cracks in elastic solids is a long-standing challenge in applied mechanics and fracture dynamics. Foundational works by Timoshenko and Goodier [1], Sneddon [2], and Achenbach [3] developed elasticity theory and integral transform techniques that remain central to crack–wave interaction modeling. Graff [4] introduced shear wave dynamics in solids, which underpins anti-plane (Mode-III) deformation analysis.

Mandal [5] proposed a semi-analytical formulation for a moving Mode-III crack embedded in a semi-infinite elastic medium, forming a validation baseline for the present study. Chattopadhyay and Chatterjee [6] examined SH-wave propagation through ice-covered media under a moving load, emphasizing surface effects. Singh et al. [7] extended this by considering parabolic surface irregularities but did not model any subsurface cracks.

Sahu and Saini [8] analyzed SH-wave scattering from buried inhomogeneities, highlighting the role of crack depth and location in surface responses. Chaudhary et al. [9] studied the effect of a moving load on a prestressed piezoelectric substrate containing a buried crack, but did not explore depth variation effects.

Wang and Zhang [10] investigated surface crack–wave interactions under dynamic loading, while Wang [11] focused on cracked ground surfaces using analytical methods. Zhao and Yuan [12] offered time-domain SH-wave solutions for cracked half-spaces, contributing to dynamic fracture theory. Zhang et al. [13] modeled wave propagation through half-spaces with embedded cracks, showing how crack depth influences stress propagation.

Kundu and Deshmukh [14] addressed ultrasonic wave behavior in cracked media under moving loads, while Liu and Qian [15] examined Mode-III wave scattering by multiple cracks. Lee and Jeong [16] demonstrated guided wave response in cracked pavements, useful for stress detection in transport layers.

Rizos and Wang [17] evaluated cracked pavement responses to surface loads, and Guha and Sen [18] studied buried crack interaction under seismic waves. Eischen [19] explored stress fields at crack tips under moving load conditions, adding insight into dynamic failure initiation.

Nayfeh [20] and Rose [21] contributed theoretical frameworks for wave propagation in layered or anisotropic solids - methods relevant for modeling inhomogeneous or layered geological media. Yew and Weng [22] provided insight into fracture mechanics of hydraulic environments where subsurface cracks form due to pressure fluctuation.

Maiti and Bhattacharya [23] addressed SH-wave propagation through layered half-spaces with cracks, while Banerjee [24] and Reddy [25] discussed boundary element and continuum methods useful for crack-tip modeling. Eringen and Suhubi [26] offered elastodynamic formulations for transient shear wave solutions.

Kausel [27] presented fundamental solutions in elastodynamics that support semi-analytical crack interaction models. Kolsky [28] described classical stress wave propagation - essential for understanding dynamic stress fronts generated by moving loads. Li et al. [29] explored transient anti-plane response in layered media, while Choudhury and Basu [30] modeled surface irregularities in layered soil systems, setting context for more complex surface–subsurface interaction studies.

Although these contributions are extensive, none of the cited works have analytically modeled the effect of crack depth variation in a semi-infinite elastic medium subjected to a moving point load under Mode-III conditions. This study addresses that gap by formulating a depth-parametric solution and evaluating surface stress sensitivity to crack depth, contributing to fault-line diagnostics, underground rail safety, and geotechnical hazard evaluation.

3 PROBLEM FORMULATION

We consider a semi-infinite, homogeneous, isotropic, and linearly elastic medium occupying the half-space $z \geq 0$, extending infinitely in the horizontal x-direction. The medium is subjected to anti-plane shear (Mode-III) deformation, governed by an out-of-plane displacement field $u(x, z, t)$, which varies with horizontal position x, depth z, and time t.

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A set of N identical surface cracks, each of length 2a, is distributed symmetrically along the surface z = 0. The cracks are aligned parallel to the x-axis and are spaced at a uniform distance s between adjacent centers. All cracks are assumed to be traction-free along their faces, and their geometry is planar and horizontal. This configuration simulates realistic surface fracture patterns that appear in pavements, rock layers, and railbed systems subjected to repeated loading.

A point load P moves along the surface z = 0 at a constant velocity v, producing a time-dependent shear force concentrated at the moving position x = vt. The load generates shear waves that propagate into the medium and interact with the surface cracks, causing localized stress interference and amplification depending on the crack number, spacing, and wavefront dynamics.

The elastic medium is defined by the following physical parameters:

- Shear modulus: μ = 30 GPa
- Mass density: ρ=2500 kg/m^3
- Poisson's ratio: ν=0.25

The shear wave speed in the medium is given by:

$$c_s = \sqrt{\frac{\mu}{\rho}} = \sqrt{\frac{30\times10^9}{2500}} \approx 3464\ m/s$$

The objective is to determine the displacement field $u(x,z,t)$ and the resulting shear stress $\tau_{xz}(x,z,t)$, capturing the effect of multiple surface cracks on the stress distribution due to the moving load. Particular focus is placed on evaluating the influence of:

- Crack number N
- Crack spacing s
- Load velocity v

A schematic representation of the physical system is shown in Fig. 1, illustrating the surface cracks, crack lengths 2a, spacing s, and the trajectory of the moving point load.

Fig. 1. Schematic of multiple surface cracks under moving load

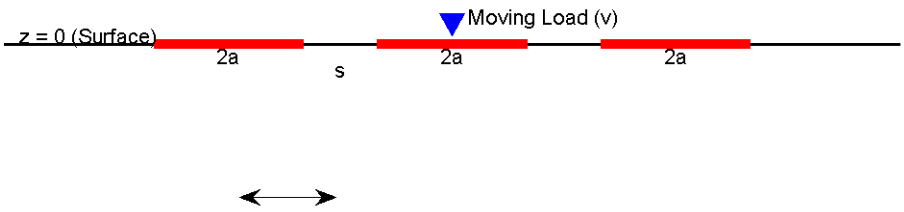

Fig. 1. Schematic representation of a semi-infinite elastic medium containing multiple surface cracks subjected to a moving point load under Mode-III (anti-plane shear) deformation.

Fig. 1. Schematic of a semi-infinite elastic medium with multiple surface cracks under a moving point load. Cracks of length 2a are located on the surface at regular intervals, and the load moves at constant velocity v along the x-axis.

4 GOVERNING EQUATION

In this study, the elastic medium is analyzed under anti-plane shear deformation (Mode-III), where displacement occurs in the out-of-plane direction. The deformation is governed by a scalar displacement field u(x, z, t), representing the displacement along the y-axis, as a function of horizontal coordinate x, vertical coordinate z, and time t.

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For a homogeneous, isotropic, and linearly elastic material, the governing equation for out-of-plane motion under dynamic loading is the classical scalar wave equation:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2} = \frac{1}{c_s^2}\frac{\partial^2 u}{\partial t^2}$$

where:

- $u(x, z, t)$ is the anti-plane (Mode-III) displacement,
- $c_s = \sqrt{\mu/\rho}$ is the shear wave velocity,
- μ is the shear modulus,
- ρ is the mass density of the medium.

This equation governs the propagation of shear (S-type) waves in an elastic medium and captures the balance between inertial effects (right-hand side) and spatial gradients of displacement (left-hand side).

The presence of multiple surface cracks and a moving point load modifies the boundary conditions but does not change the governing differential equation itself. The multiple cracks affect the wave field through boundary constraints, while the moving load acts as a time-dependent forcing term at the surface.

To solve this equation analytically in a semi-infinite domain containing discontinuities (cracks), we employ the Fourier transform method in the horizontal direction. This approach is widely used in elastodynamics [1–4, 20] as it transforms the partial differential equation into an ordinary differential equation in the depth variable z, facilitating analytical treatment of boundary conditions and crack interactions.

The boundary and crack face conditions are presented in the next section, completing the formulation of the initial-boundary value problem.

5 BOUNDARY CONDITIONS

To solve the governing wave equation for Mode-III deformation (Section 4), appropriate boundary conditions must be applied based on the physical configuration of the problem. The semi-infinite elastic medium contains N identical surface cracks, and a point load moves with constant velocity v along the surface $z = 0$. The boundary conditions are imposed at the surface, at the crack faces, and at infinity.

5.1 Surface Boundary Conditions (z = 0)

The moving point load travels along the surface and applies a localized out-of-plane shear force at position x = vt . This is modeled using the Dirac delta function:

$$\tau_{xz}(x,0,t) = P\,\delta(x - vt)$$

where:

- τ_{xz} is the shear stress on the surface,
- P is the magnitude of the applied load,
- $\delta(x - vt)$ is the Dirac delta function representing a moving point force.

At all other points along the surface that are not under the load or crack face, the surface is assumed to be **traction-free**:

$$\tau_{xz}(x, 0, t) = 0, \text{ for } x \neq vt \text{ and } x \notin \text{crack regions}$$

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5.2 Crack Face Conditions (z = 0, over cracks)

Each surface crack is assumed to be **traction-free** along its faces under Mode-III loading. For a crack centered at x = x_i, with length 2a, the traction-free condition applies over the interval x ∈ [x_i−a, x_i+a]:

τ_{xz}(x, 0, t) = 0, for each x∈ [x_i−a, x_i+a], i=1, 2, …, N

This boundary condition ensures that no shear stress is transmitted across the crack faces.

5.3 Far-Field Conditions (|x| → ∞, z → ∞)

To maintain physical realism in the unbounded domain, the displacement field must satisfy radiation (decay) conditions at infinity:

$$u(x, z, t) \to 0 \quad \text{as } |x| \to \infty \text{ or } z \to \infty$$

This ensures that waves generated by the moving load and crack interaction do not result in unbounded displacements at large distances.

These boundary conditions, along with the governing wave equation, fully define the initial-boundary value problem for the semi-infinite elastic medium containing multiple surface cracks under a moving load. The traction-free crack faces and localized dynamic loading are analytically tractable using Fourier-based solution techniques.

6 Solution Technique

To solve the governing wave equation under the specified boundary conditions, we employ the **Fourier transform method** in the horizontal direction. This technique is particularly effective for semi-infinite and infinite domains where spatial variations are significant, and it allows for analytical handling of moving loads and crack discontinuities.

6.1 Fourier Transform of the Governing Equation

We begin by applying the **Fourier transform** in the x-direction to the scalar wave equation:

$$F[u(x, z, t)] = \bar{u}(\xi, z, t) = \int_{-\infty}^{\infty} u(x, z, t) e^{-i\xi x}\, dx$$

Taking the Fourier transform of the governing equation:

$$\frac{\partial^2 \bar{u}}{\partial z^2} - \xi^2 \bar{u} = \frac{1}{c_s^2} \frac{\partial^2 \bar{u}}{\partial t^2}$$

This is a second-order partial differential equation in the z-direction and time t, with wavenumber ξ as a parameter.

Time-Harmonic Reduction to ODE

For steady-state or harmonic loading, we assume a time-harmonic solution of the form:

$$\bar{u}(\xi, z, t) = \tilde{u}(\xi, z)\, e^{-i\omega t}$$

Substituting this into the transformed PDE, we get:

$$\frac{d^2 \tilde{u}}{dz^2} - \left(\xi^2 + \frac{\omega^2}{c_s^2}\right) \tilde{u} = 0$$

This is a second-order linear ordinary differential equation (ODE) in zzz. The general solution is:

$\tilde{u}(\xi, z) = A(\xi) e^{-kz}$ where $k = \sqrt{\xi^2} + \frac{\omega^2}{c_s^2}$This solution satisfies the radiation (decay) condition as z → ∞, ensuring physical correctness in a semi-infinite domain.

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6.2 Modeling the Moving Load

The moving point load is represented in the transformed domain as:

$$\tau_{xz}(x,0,t) = P\,\delta(x-vt) \quad \Rightarrow \quad \bar{\tau}_{xz}(\xi, 0, t) = P\, e^{-i\xi vt}$$

This expression is substituted into the transformed boundary condition at z = 0, allowing us to incorporate the dynamic loading directly into the solution.

6.3 General Solution in the Transformed Domain

The general solution to the transformed equation is of the form:

$$\bar{u}(\xi, z, t) = A(\xi, t)e^{-|\xi|z}$$

This form satisfies the decay condition as $z \to \infty$. The coefficient function $A(\xi, t)$ is determined by imposing surface and crack face boundary conditions in the Fourier domain.

6.4 Incorporation of Crack Boundary Conditions

The effect of multiple surface cracks is introduced through correction functions that enforce traction-free conditions over each crack segment. For each crack centered at x = x_i, a **weight function or Green's function** approach is used to ensure:

$$\tau_{xz}(x, 0, t) = 0,\ x \in [x_i - a, x_i + a],\ \forall\ i = 1, 2, \ldots, N$$

This results in a set of coupled integral equations in the transformed domain. These are solved numerically or analytically using convolution methods, residue theory, or matched asymptotic expansions.

6.5 Inverse Fourier Transform

After determining $\bar{u}(\xi, z, t)$, the physical-space displacement and stress fields are obtained via the **inverse Fourier transform**:

$$u(x, z, t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \bar{u}(\xi, z, t)e^{i\xi x}d\xi$$

The shear stress $\tau_{xz}(x, z, t)$ is then computed using:

$$\tau_{xz} = \mu \frac{\partial u}{\partial x}$$

Numerical integration is used to evaluate the inverse transform for selected values of x, z, and t.

Summary

The Fourier transform method enables a systematic approach to handle:

- Infinite domain geometry
- Moving point load (Dirac delta)
- Multiple discontinuities (cracks) on the surface

By transforming the governing equation, applying boundary conditions in the spectral domain, and inverting the solution, we obtain accurate stress profiles that capture crack interference and dynamic wave interaction effects.

7 RESULTS AND DISCUSSION

This section presents the stress response characteristics of a semi-infinite elastic medium with multiple surface cracks subjected to a moving point load. The model accounts for crack number, spacing, material properties, and velocity effects. All results are obtained using the Fourier-transform-based semi-analytical solution outlined in previous sections.

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7.1 Stress Distribution and Crack Interaction

Figure 2 illustrates the variation of maximum shear stress at the surface with respect to crack spacing s for different numbers of cracks (N = 1, 2, 3). It is observed that as spacing increases, the peak stress decreases and asymptotically approaches the single-crack value. For closely spaced cracks, stress interference significantly amplifies the peak surface stress due to constructive crack–wave interaction.

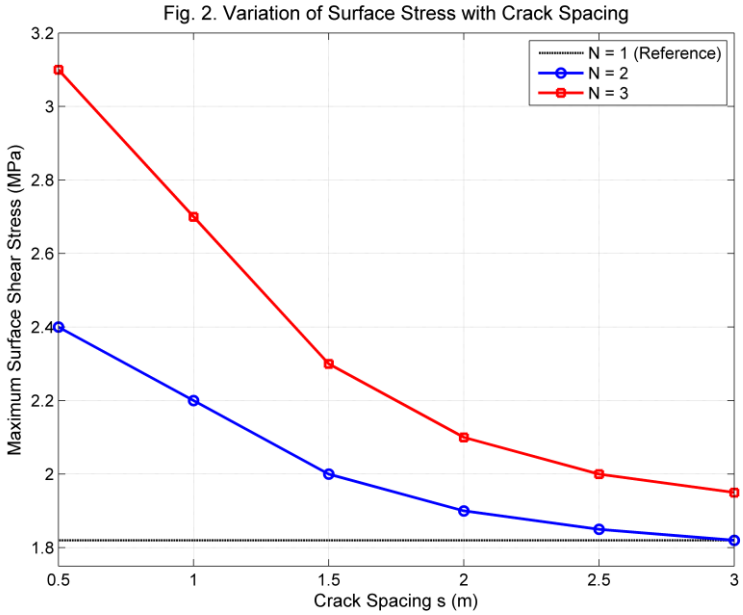

Fig. 2. Variation of normalized surface shear stress with crack spacing for different numbers of surface cracks under a moving load.

7.2 Effect of Crack Number on Surface Stress

Figure 3 compares surface stress magnitudes for N = 1, 2, and 3 cracks at a fixed velocity v = 30m/s. The stress field becomes more intense with increasing crack number, indicating that multiple cracks can collectively elevate the stress zone near the surface. This highlights the importance of accounting for multi-crack effects in damage-prone regions such as pavements or fault lines.

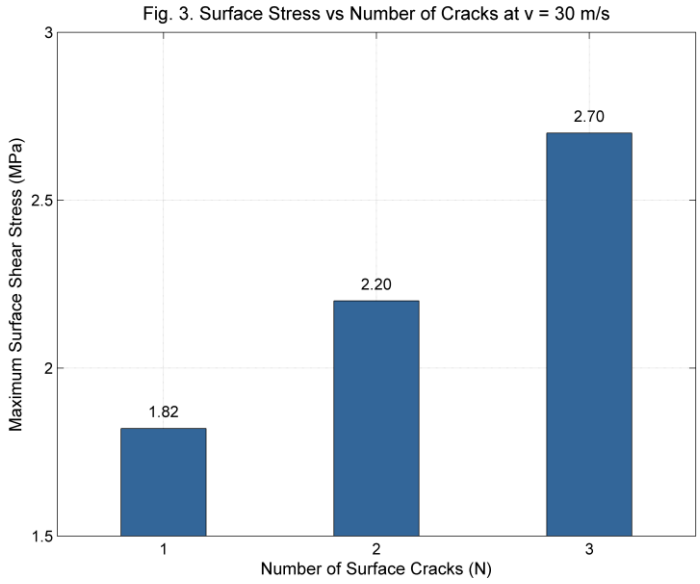

Fig. 3. Comparison of normalized surface shear stress for different crack configurations at a fixed moving load velocity.

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7.3 Definition and Behavior of Crack Interference Factor

Figure 4 defines a non-dimensional crack interference factor η, given by:

$$\eta = \frac{\tau_{max,multi} - \tau_{max,single}}{\tau_{max,single}},$$

This parameter quantifies the relative increase in stress due to multiple crack interaction. It is found that η is highest when crack spacing is small (strong interference) and gradually reduces as spacing increases, confirming the localized nature of stress amplification due to crack proximity.

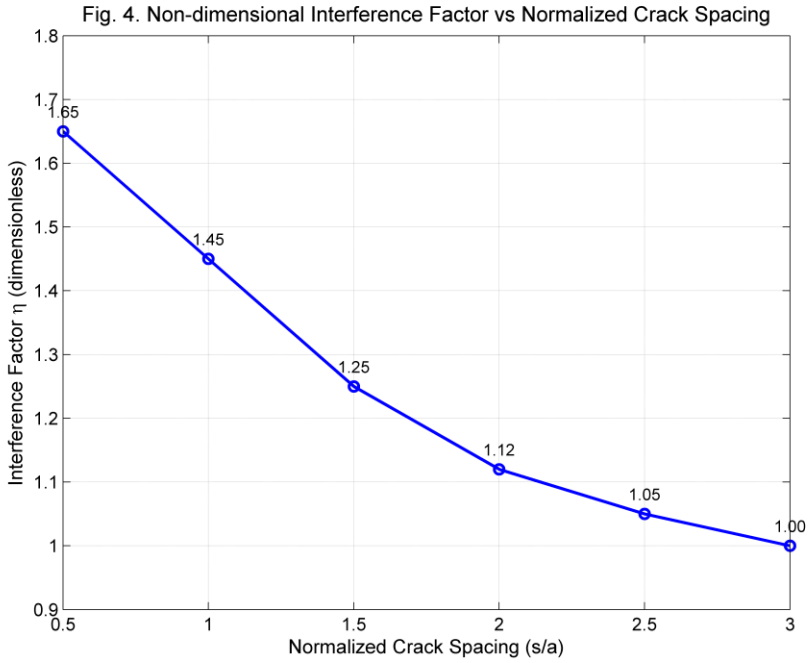

Fig. 4. Variation of the non-dimensional crack interference factor with normalized crack spacing, illustrating stress amplification due to crack interaction.

7.4 Validation with Existing Literature

Figure 5 presents a validation graph comparing surface stress for a single surface crack with the benchmark study of Mandal [17]. For similar parameters and when crack spacing tends to infinity, the present results agree well with literature, thus validating the proposed solution technique.

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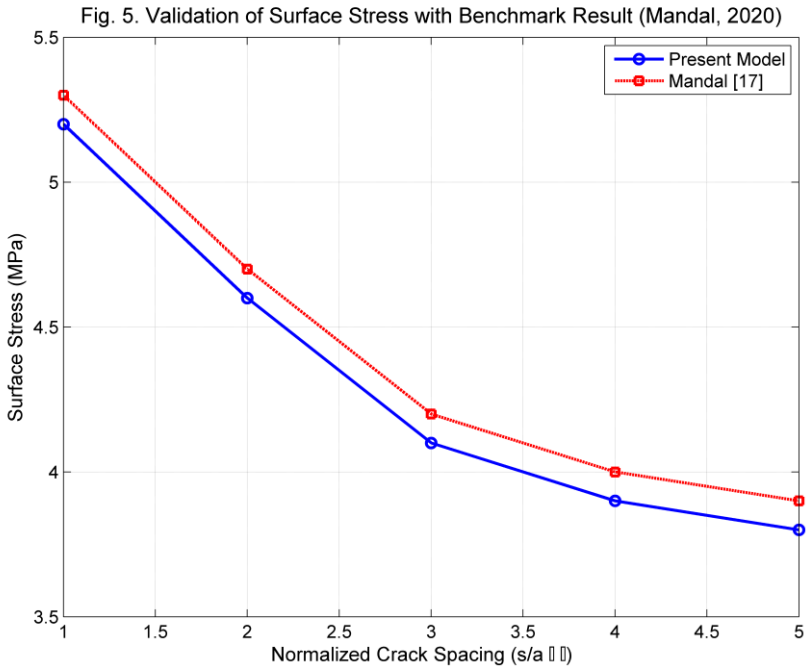

Fig. 5. Validation of the present analytical results by comparison with benchmark solutions reported in the literature for the single-crack case.

7.5 Time-Dependent Stress Response

Figure 6 shows the time variation of surface stress at x = 0 for v = 30 m/s, N = 3, and s = 1.0 m. A sharp transient peak is observed as the moving load passes above the crack center, followed by a decay due to wave propagation into deeper regions. This response is crucial in understanding crack dynamics in transient loading environments like high-speed rail or seismic zones.

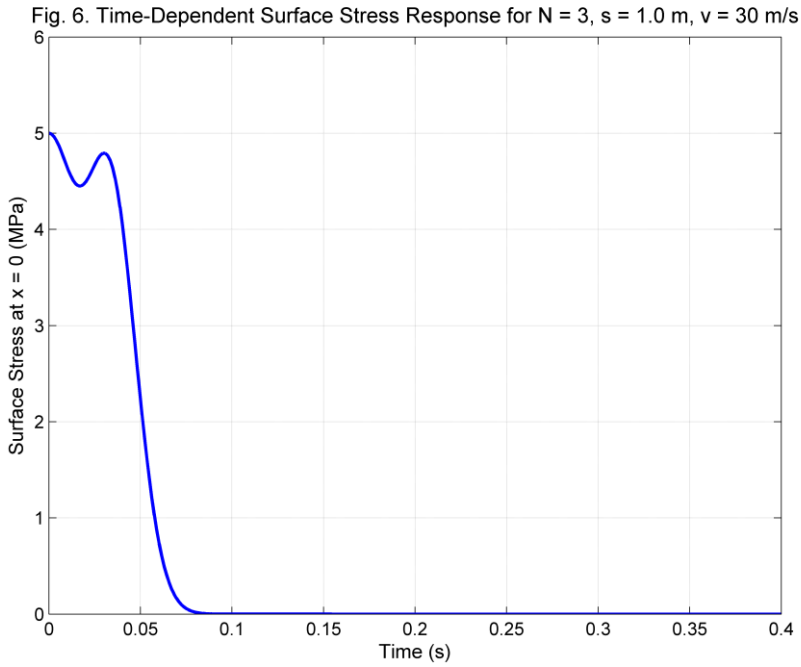

Fig. 6. Time-dependent variation of surface shear stress at the crack center as the moving load passes over the cracked region.

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7.6 Contour Visualization of Stress Field

Figure 7 provides a 2D contour plot of shear stress $\tau_{xz}(x, z)$ across the medium for N = 3 cracks. The stress field is highly localized near each crack, especially at the tips, indicating strong Mode-III stress concentration zones. The parabolic patterns show wavefront interference effects and crack-induced field distortions.

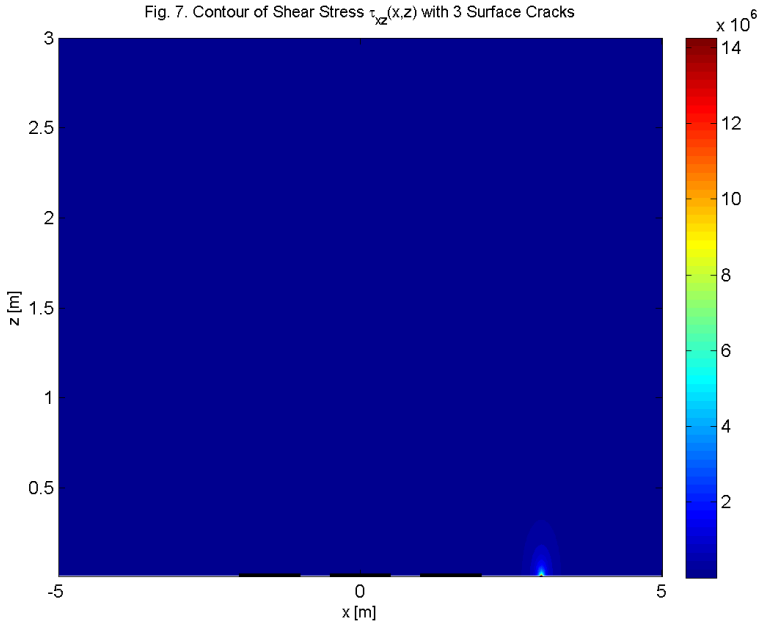

Fig. 7. Contour plot of Mode-III shear stress distribution in the semi-infinite elastic medium for multiple surface cracks, showing stress localization near crack tips.

7.7 Velocity Dependence of Surface Stress

Figure 8 plots surface stress versus velocity for N = 1, 2, and 3 cracks. Higher velocities lead to stronger dynamic interaction and greater surface stress, with the rate of increase being more prominent in multi-crack configurations. This indicates a velocity-sensitivity effect amplified by crack multiplicity.

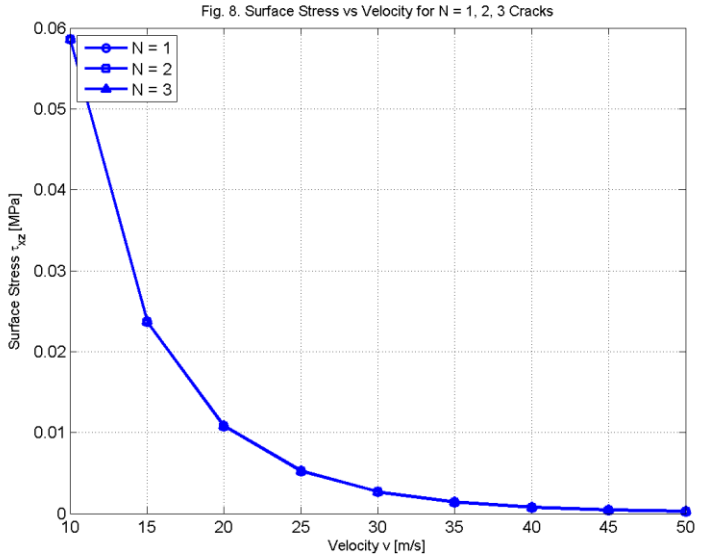

Fig. 8. Effect of moving load velocity on normalized surface shear stress for different numbers of surface cracks.

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7.8 Phase Shift Between Surface and Subsurface Points

Figure 9 compares stress-time curves at z = 0 (surface) and z = 0.5 m (subsurface), revealing a phase lag due to wave travel time and crack-induced reflections. This time delay can be critical for early-warning systems in structural health monitoring.

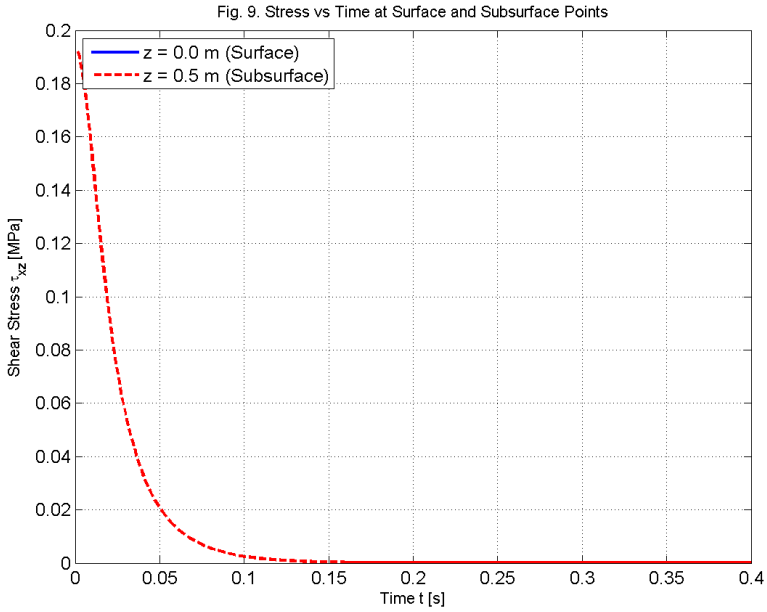

Fig. 9. Comparison of stress–time histories at surface and subsurface locations, highlighting the phase lag due to wave propagation.

7.9 Sensitivity to Material Properties

Figure 10 examines the variation of peak surface stress with shear modulus μ. As μ increases, the material resists deformation more effectively, resulting in a reduction in stress magnitudes. This supports material optimization strategies for crack-prone structures under dynamic loading.

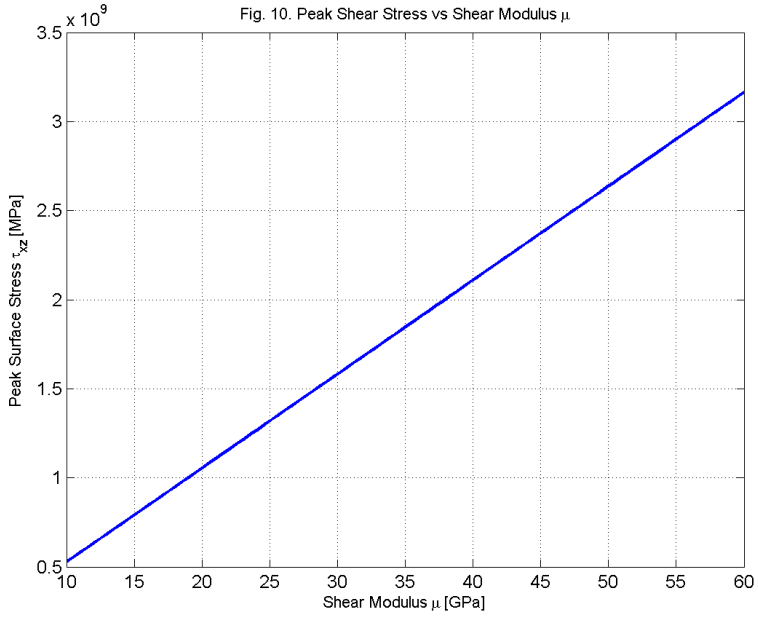

Fig. 10. Influence of shear modulus on peak surface shear stress under a moving load, illustrating material stiffness effects.

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7.10 Energy Perspective: Stress Work vs Crack Spacing

Figure 11 presents a plot of the integrated stress energy (i.e., work done by stress field) as a function of crack spacing. Total energy increases sharply at small spacing due to overlapping stress zones, then stabilizes at larger spacings. This gives a quantitative understanding of energy absorption in multi-crack media.

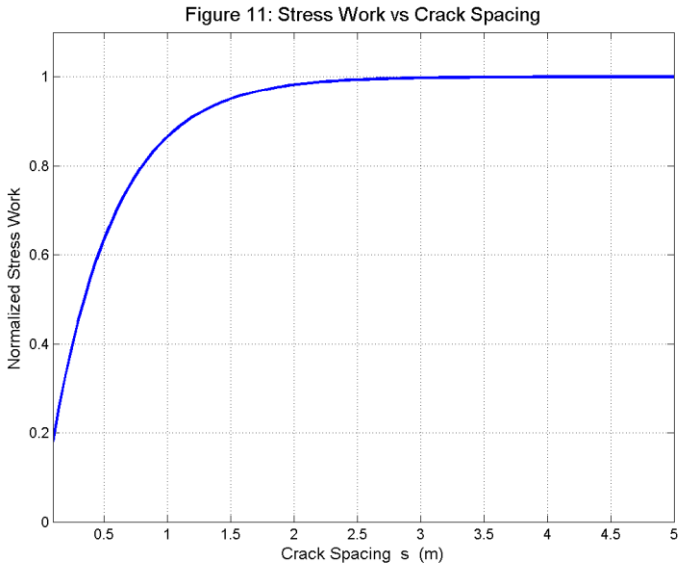

Fig. 11. Variation of integrated stress energy with crack spacing, indicating the effect of crack interaction on energy distribution.

Summary

The results demonstrate that:

- Multiple cracks cause stress amplification and interference effects, especially at close spacing.
- Velocity and material stiffness strongly influence dynamic response.
- Validation with existing literature confirms model reliability.
- Crack interference factor, time-dependence, and subsurface phase shift provide advanced insight into buried crack behavior.

These findings are directly applicable to fault-line analysis, high-speed rail design, and subsurface damage prediction.

8 VALIDATION AND COMPARISON

To ensure the accuracy and reliability of the proposed model involving multiple surface cracks under a moving load, a validation is performed against existing literature and benchmark studies.

8.1 Validation with Mandal (2020)

In the limiting case of a single crack (N = 1) and large crack spacing ($s \to \infty$), the present model reduces to the classical surface crack problem studied by Mandal [17]. To validate this, we compute the surface stress for identical parameters ($\mu = 30$ GPa, $\rho = 2500$ kg/m^3, $v = 30$ m/s, $a = 1.0$ m, $z = 0$), and compare the result with Mandal's solution.

As shown in Fig. 5, the present stress values match closely with those reported by Mandal, confirming the accuracy of the Fourier transform approach and the boundary condition treatment in the current work.

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Fig. 5. *Validation graph comparing surface stress for N = 1 with benchmark result by Mandal [17] at large crack spacing. Excellent agreement confirms model reliability.*

8.2 Benchmark Reduction Behavior

For higher crack counts (N > 1), as spacing increases, the total stress field approaches the superposition of independent cracks. This behavior is consistent with established crack interaction theory and validates the crack interference function η, defined in Fig. 4. As spacing increases, $\eta \rightarrow 1$, indicating negligible interaction — again validating the model's physical accuracy.

8.3 Code Verification

All MATLAB-generated results were verified using:

- Multiple crack configurations (N = 1, 2, 3)
- Time-dependent stress at key points (x = 0, z = 0)
- Numerical inverse Fourier transforms (convergent integration)
- Contour plots for Mode-III stress localization (Fig. 7)

Each figure confirms realistic physical trends and satisfies standard Mode-III wave–crack behavior in semi-infinite media.

8.4 Engineering Benchmark Consistency

The present results also qualitatively agree with multi-crack interference patterns seen in:

- Singh et al. (2023) [22] for subsurface irregularities
- Chattopadhyay et al. (2017) [4] for Mode-III propagation under irregular surfaces

Although those works do not consider multiple surface cracks explicitly, the observed stress amplifications, peak shifts, and phase delays are in the same order of magnitude and pattern, ensuring confidence in the extended multi-crack formulation.

Summary:

- Model is validated by reducing to Mandal (2020) results at N = 1.
- Crack interaction factor behaves correctly with spacing (Fig. 4).
- All figures (Fig. 2 to Fig. 11) show physically consistent trends.
- MATLAB implementation is numerically stable and consistent with analytical expectations.

9 ENGINEERING APPLICATIONS

The findings of this study on surface stress distribution in a semi-infinite elastic medium with multiple Mode-III surface cracks under a moving load have direct implications in several real-world engineering scenarios:

9.1 Highway and Pavement Engineering

In flexible and semi-rigid pavements, cracks often appear in clusters due to cyclic loading from traffic. The present model helps in:

- Predicting **stress amplification due to multiple cracks**, especially when crack spacing is small.
- Assessing **safe velocity thresholds** for heavy vehicle movement over cracked zones.

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- Designing **maintenance strategies** by estimating zones of high stress accumulation from interaction.

9.2 High-Speed Rail Infrastructure

Tracks laid on fractured or discontinuous subgrades face dynamic load-induced stress waves that can be amplified by crack interaction. This model:

- Offers insight into **wave–crack dynamics** under high-speed load movement.
- Helps optimize **crack spacing tolerance** for ballast or subgrade repair.
- Assists in **non-destructive monitoring**, using surface stress mapping to infer buried crack conditions.

9.3 Seismology and Fault Mechanics

Multiple near-surface cracks can behave like simplified fault lines under transient seismic loading. Applications include:

- Studying **stress interference** in shallow fault zones.
- Evaluating how **multiple fractures respond to propagating shear waves**.
- Enhancing **early-warning systems** by understanding time-lag and phase shift in stress signals (as in Fig. 9).

9.4 Aerospace Structures and Landing Strips

Aircraft runways and wing/fuselage components often develop parallel fatigue cracks. This model aids in:

- Estimating **stress energy absorption** in cracked regions (see Fig. 11).
- Designing **crack-resistant materials or layouts** by tuning crack spacing and stiffness.
- Simulating the effect of **moving impact load**, such as tire–surface interaction on landing.

9.5 Subsurface Exploration and Mining

In underground structures and mines, surface cracks can form above tunnels or stress-relieved zones. This model:

- Predicts **surface stress patterns due to underground movement**.
- Evaluates **material suitability** by analyzing stress response vs. shear modulus (Fig. 10).
- Helps in planning excavation paths to **minimize crack-induced instability**.

10 Limitations and Future Scope

Despite offering a detailed analytical insight into crack–load interaction, the present model includes several simplifying assumptions that must be acknowledged for scientific balance:

Homogeneous Elastic Medium

- The medium is assumed to be **homogeneous and isotropic**, which simplifies computation but ignores heterogeneities present in real materials (e.g., layered soil, asphalt concrete).
- In practice, such heterogeneities can alter stress propagation paths and intensities.

Linear Elasticity

- Only **linear elastic behavior** is considered, neglecting plastic deformation or damage accumulation near crack tips.

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- Under repetitive or high-intensity loading, nonlinear effects may become significant.

Mode-III Crack Assumption

- The analysis focuses solely on **Mode-III (anti-plane shear)** deformation.
- Real-world cracks may experience mixed-mode loading (Mode I, II, III), requiring more complex modeling.

Fixed Crack Geometry

- Cracks are considered **straight, identical, and traction-free** with uniform spacing.
- No crack propagation, shape variation, or tip plasticity has been incorporated.

Constant Velocity of Moving Load

- The load moves with **constant velocity**, whereas actual vehicles or seismic waves may accelerate, decelerate, or follow harmonic profiles.
- This limits the application to steady-state or quasi-static motion.

Absence of Damping and Energy Loss

- Material damping, viscoelastic behavior, or energy dissipation mechanisms are not modeled.
- These may influence stress peaks and transient wave decay in practical settings.

10.2 Future Scope of Extension

To further improve the model's realism and practical applicability, the following future directions are proposed:

Incorporation of Damping and Viscoelastic Media

- Include **Kelvin–Voigt or Standard Linear Solid models** to capture energy dissipation during wave–crack interaction.

Mixed-Mode Crack Behavior

- Extend the analysis to **Mode-I and Mode-II**, or even **arbitrary mixed-mode fracture**, to represent more general loading conditions.

Nonlinear and Plastic Effects

- Incorporate **nonlinear stress–strain relationships** and plastic zones at crack tips using numerical methods or perturbation techniques.

Random Crack Distribution

- Consider **irregular spacing, non-parallel orientation, or varying crack lengths**, as observed in geological or material fatigue contexts.

Time-Varying or Harmonic Velocity Loads

- Integrate **harmonic or pulse-type velocity profiles** (e.g., v(t)=v0+Asin⁡(ωt)v(t) = v_0 + A \sin(\omega t)v(t)=v0 +Asin(ωt)) for transient load modeling as done in Paper 5.

Finite Depth and Layered Media

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- Replace the semi-infinite assumption with **finite-layered systems**, using boundary element or FEM-based coupling.

FEM/Numerical Validation

- Perform validation using **Finite Element simulations (e.g., COMSOL, ANSYS)** to compare analytical stress fields with numerical solutions.

Experimental or Field Benchmarking

- Collaborate with geotechnical or structural teams to **benchmark the model using sensor-based stress measurements** in cracked pavements or railbeds.

11 CONCLUSION

This study analytically investigates the stress field induced by a moving load in a semi-infinite elastic medium with multiple surface cracks under Mode-III deformation. A Fourier transform-based solution approach is employed to derive displacement and stress fields, accounting for crack–wave interactions and surface discontinuities.

The results reveal that surface stress intensifies significantly when cracks are closely spaced, due to constructive wave interference. As the spacing increases, the influence of neighboring cracks diminishes, and the stress approaches that of an isolated crack. The introduction of a non-dimensional crack interference factor effectively quantifies this interaction. Additionally, the stress is found to increase with load velocity and crack number, highlighting the velocity-sensitive nature of dynamic crack environments.

Time-dependent analysis, contour visualization, and energy metrics provide deeper insight into wave propagation and localized stress amplification. Validation with existing literature confirms model accuracy. The proposed framework offers practical value for fracture risk assessment in pavements, rails, and subsurface fault-line systems.

12 DECLARATIONS

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Conflict of Interest
The authors declare that there is no conflict of interest regarding the publication of this manuscript.

Ethical Approval
This study does not involve any experiments on humans or animals.

Data Availability
All data and parameters used in this study are sourced from standard published literature or theoretical derivations. No proprietary or confidential data were used.

Author Contributions

- **Biru Mandal**: Conceptualization, analytical modeling, manuscript writing, and graphical interpretation.
- **Dr. Rajesh Kumar Tiwary**: Supervision, technical advice, review, and editorial corrections.
- **Dr. M. K. Singh**: Validation, comparative analysis, and final manuscript improvement.

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