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Effect of Mass Transfer and Mixed Convection on A Steady MHD Flow over A Porous Flat Plate

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***Abstract*−A steady mixed convection flow over a porous plate has been considered to investigate the combined effects of suction parameter, radiation parameter, Schmidt number, Prandtl number. The governing boundary layer equations are transformed into a non-dimensional form by group transformation and finally solved by using Runga-Kutta method with shooting technique. The numerical results have been depicted graphically to illustrate the influence of the mixed convection parameter and other various parameters along with Prandtl number on velocity, temperature and concentration profiles. The results for the skin-friction coefficient, Nusselt number and Sherwood number have also been analyzed. Good agreement is found between the numerical results of the present paper with published result for special case.**

Key Words: Mixed convection, variable viscosity, MHD flow, radiation, thermal conductivity, flat plate

I. INTRODUCTION

The problem of laminar hydrodynamic and thermal boundary layers over the flat plate in a uniform stream of fluid is a thoroughly researched problem in fluid mechanics. Hamad et al. [1], studied magnetic field effects of a nano-fluid past a vertical semi-infinite flat plate using group transformation. Reviews for the applications of group theory to differential equations can be found in the various researches done by [2-7]. The radiative flow of an electrically conducting fluid and heat and mass transfer situation arises in many practical applications, such as, in electrical power generation, solar power technology, space vehicle re-entry, nuclear reactors. It also occurs in many geophysical and engineering applications such as nuclear reactors, migration of moisture through air contained in fibrous insulations, nuclear waste disposal, dispersion of chemical pollutants through water-saturated soil and others as studied by Arasu et al. [7] and Chamakha et al. [9]. Radiation effect on boundary layer flow with and without applying a magnetic field has been investigated researchers [10-13]. Similarity representation of MHD flow with heat transfer taking into consideration variable viscosity and thermal conductivity by Seddeek et al. [14]. Mahanti et al. [15] investigated the effects of variable viscosity and thermal conductivity, which vary linearly on steady free convective flow of a viscous incompressible fluid along an isothermal vertical plate in the presence of heat sink. Recently, thermal convective surface boundary conditions were used by Aziz [16] and Makinde et al. [17]. They studied to solve different types of boundary layer equations. Recently, Hamad et al. [18] studied a steady laminar 2-D MHD viscous incompressible flow over a permeable flat plate with thermal convective boundary condition and radiation effects. The viscosity and thermal conductivity of fluid are assumed to vary linearly with temperature.

The objective of present investigation is to study mixed convection flow over a permeable porous plate. To find the solution, authors are using similarity and group method of transformation. The attempt has also been made to study the effects of radiation, suction and thermal convective parameters on the fluid flow and the rate of heat and mass transfer.

II. MATHEMATICAL FORMULATION OF THE PROBLEM

Consider the steady mixed convective flow of a viscous incompressible electrically conducting fluid past an infinite vertical porous plate in a porous medium of time independent permeability in presence of a transverse magnetic field B_0 as shown in the figure of physical model. Let $\bar{x}$-axis be along the plate in the direction of flow and $\bar{y}$-axis is normal to it. The velocity components along $\bar{x}$ and $\bar{y}$ axes are $\bar{u}$ and $\bar{v}$, T and C be the fluid temperature and concentration. Further μ, ρ, σ, k, R and M are the coefficient of viscosity, density, electric conductivity, thermal conductivity, radiation parameter and magnetic parameter of the fluid.

Alam et al. [19] considered and it has been assumed that the magnetic Reynolds number is much less than unity so that the induced magnetic field is neglected in comparison to the applied magnetic field B_0. The suction or injection are imposed on the permeable plate. The temperature of the plate surface is held uniform at T_w which is higher than the ambient temperature T_∞. The physical model has been given below:

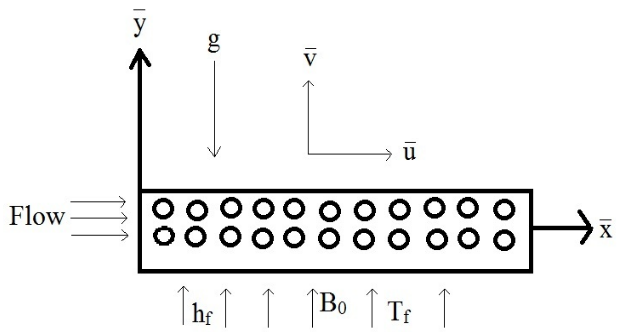

Fig. 1. Physical model

The species concentration at the plate surface is maintained uniform at C_w while the ambient fluid concentration is assumed to be C_∞. Further, all the fluid properties are assumed to be constant except that of the dynamic viscosity and thermal conductivity. The bottom surface of the plate is heated by convection from a hot fluid of temperature T_f it generates a heat transfer coefficient h_f as taken by Aziz [16].

Under the above assumptions, the governing equations for the problem can be written as Kays et al. [20].

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \qquad (1)$$

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = -\frac{1}{\rho}\frac{\partial P}{\partial \bar{x}} + \nu\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + \frac{1}{\rho}\frac{\partial \mu}{\partial T}\frac{\partial T}{\partial \bar{y}}\frac{\partial \bar{u}}{\partial \bar{y}} - \frac{\sigma B_0^2}{\rho}\bar{u} - \frac{\mu \bar{u}}{\rho K} + g\beta(C - C_\infty) \qquad (2)$$

$$\bar{u}\frac{\partial T}{\partial \bar{x}} + \bar{v}\frac{\partial T}{\partial \bar{y}} = \frac{\partial}{\partial \bar{y}}\left(k(T)\frac{\partial T}{\partial \bar{y}}\right) - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial \bar{y}} \qquad (3)$$

$$\bar{u}\frac{\partial C}{\partial \bar{x}} + \bar{v}\frac{\partial C}{\partial \bar{y}} = D_m\frac{\partial^2 C}{\partial \bar{y}^2} \qquad (4)$$

The boundary conditions are given by

$$\bar{u} = 0, \bar{v} = -v_w, C = C_w, -k\frac{\partial T}{\partial \bar{y}} = h_f[T_f - T_w] \quad \text{at } \bar{y} = 0$$
$$\bar{u} \to u_e(\bar{x}), T \to T_\infty, C \to C_\infty \quad \text{as } \bar{y} \to \infty \qquad (5)$$

In the free stream flow, $\bar{u} = u_e(\bar{x})$ and hence momentum equation (2) becomes

$$\bar{u}_e\frac{d\bar{u}_e}{d\bar{x}} = -\frac{1}{\rho}\frac{\partial p}{\partial \bar{x}} - \frac{\sigma B_0^2}{\rho}\bar{u}_e(\bar{x}) \qquad (6)$$

Using equations (2) and (6), equation of momentum becomes

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = \bar{u}_e\frac{d\bar{u}_e}{d\bar{x}} + \nu\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + \frac{1}{\rho}\frac{\partial \mu}{\partial T}\frac{\partial T}{\partial \bar{y}}\frac{\partial \bar{u}}{\partial \bar{y}} - \frac{\sigma B_0^2}{\rho}(\bar{u} - \bar{u}_e) - \frac{\mu \bar{u}}{\rho K} + g\beta(C - C_\infty) \qquad (7)$$

Assumed the viscosity and thermal conductivity as linearly temperature dependent [19]:

$$\mu(t) = \mu_\infty[1 + b_0(T_f - T)],$$
$$k(t) = k_\infty[1 + c(T - T_\infty)]$$

Where, μ_∞ and k_∞ are the constant undisturbed viscosity and thermal conductivity, $b_0 > 0$, c are constants depend on fluid.

Using Rosseland's approximation for radiation from [21], we obtained

$$q_r = -\frac{4\sigma_1}{3k_1}\frac{\partial T^4}{\partial \bar{y}} \qquad (8)$$

Where σ_1 is the Stefan–Boltzman constant and k_1 is the absorption coefficient. It is assumed that the temperature variation within the flow is such that T^4 may be expanded in a Taylor series about T and neglecting higher order terms, we get

$$T^4 \cong 4TT_\infty^3 - 3T_\infty^4 \qquad (9)$$

Equations (8) and (9) give

$$\frac{\partial q_r}{\partial \bar{y}} = -\frac{16\sigma_1 T_\infty^3}{3k_1}\frac{\partial^2 T}{\partial \bar{y}^2} \qquad (10)$$

Using equations (8) and (10), the energy equation (3) becomes

$$\bar{u}\frac{\partial T}{\partial \bar{x}} + \bar{v}\frac{\partial T}{\partial \bar{y}} = \frac{\kappa_\infty}{\rho c_p}\frac{\partial}{\partial \bar{y}}\left(\frac{\partial T}{\partial \bar{y}}(1 + S\theta)\right) + \frac{16\sigma_1 T_\infty^3}{3\kappa_1 c_p \rho}\frac{\partial^2 T}{\partial \bar{y}^2} \qquad (11)$$

Here $S = c(T_f - T_\infty)$ is thermal conductivity parameter.

Now, the following dimensionless variables have been introduced as considered by Hamad et al. [18]:

$$x = \frac{\bar{x}}{l},\ y = \frac{\bar{y}\sqrt{Re}}{l},\ u = \frac{\bar{u}}{u_\infty},\ \bar{v} = \frac{v\sqrt{Re}}{u_\infty},\ u_e = \frac{\bar{u}_e}{u_\infty},$$
$$\theta = \frac{T - T_\infty}{T_f - T_\infty},\ \varphi = \frac{C - C_\infty}{C_w - C_\infty}$$

$$u = \frac{\partial \psi}{\partial y},\ v = -\frac{\partial \psi}{\partial x} \qquad (12)$$

Where $Re = u_\infty l/\nu$ is the Reynolds number, ψ is the stream function, l being the characteristic length and u_∞ is reference velocity.

Hence, equations (7), (11) and (3) reduce in the following form:

$$\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\partial y}-\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2}-\frac{\partial^3\psi}{\partial y^3}+A\frac{\partial\theta}{\partial y}\frac{\partial^2\psi}{\partial y^2}-u_e\frac{du_e}{dx} +M\left(\frac{\partial\psi}{\partial y}-u_e\right)+C'\frac{\partial\psi}{\partial y}-\lambda\varphi=0 \quad (13)$$

$$\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y}-\frac{1}{Pr}\left(1+S\theta+\frac{4R}{3}\right)\frac{\partial^2\theta}{\partial y^2} -\frac{1}{Pr}S\left(\frac{\partial\theta}{\partial y}\right)^2=0 \quad (14)$$

$$\frac{\partial\psi}{\partial y}\frac{\partial\varphi}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial\varphi}{\partial y}-\frac{1}{Sc}\frac{\partial^2\varphi}{\partial y^2}=0 \quad (15)$$

Subject to boundary conditions,

$$\frac{\partial\psi}{\partial y}=0,\ \frac{\partial\psi}{\partial x}=-\frac{v_w}{u_\infty\sqrt{Re}},\ \varphi=1,\ \frac{\partial\theta}{\partial y}=-\frac{lh_f}{\sqrt{Re}}\left(\frac{1-\theta(0)}{1+S\theta(0)}\right) \quad \text{at } y=0 \quad (16)$$

$$\frac{\partial\psi}{\partial y}\to u_e,\ \theta\to 0,\ \varphi\to 0 \quad \text{as } y\to\infty$$

Where

$$M=\frac{\sigma B_0^2 l}{\rho u_\infty},\ Gr=\frac{gl^3\beta(C_w-C_\infty)}{k'\upsilon^2},\ Pr=\frac{\mu_\infty c_p}{k_\infty},$$
$$Sc=\frac{\nu}{D_m},\ R=\frac{4\sigma_1 T_\infty^3}{k_1 k_\infty}$$

$$A=b(T_f-T_\infty),\ \lambda=\frac{Gr}{Re^{5/2}},\ k'=\sqrt{\frac{\upsilon}{U_\infty}},\ c'=\frac{\mu l}{\rho K U_\infty} \quad (17)$$

The application of group transformations has been considered to find similarity reduction of equations (13), (14) and (15). Consider the following group transformations

$$x^\#=x\Omega^{\alpha_1},\ y^\#=y\Omega^{\alpha_2},\ \psi^\#=\psi\Omega^{\alpha_3},\ \theta^\#=\theta,\ \varphi^\#=\varphi \quad (18)$$

Where α_1, α_2, α_3 are constants and Ω is the parameter of point transformation. Now finding the relation among α's such that

$$\Delta_j(x^\#,y^\#,\psi^\#,\theta^\#,\varphi^\#,.....,\frac{\partial^3\psi^\#}{\partial y^{\#3}})=H_j(x,y,\psi,\theta,\varphi,.....,\frac{\partial^3\psi}{\partial y^3})$$
$$\varphi_j(x,y,\psi,\theta,\varphi,.....,\frac{\partial^3\psi}{\partial y^3})(j=1,2,3)$$

Δ_1, Δ_2 and Δ_3 are conformally invariant under the group transformation (18), [2].

By equation (13), we have

$$\frac{\partial\psi}{\partial y}\frac{\partial^3\psi}{\partial x\partial y^2}-\frac{\partial\psi}{\partial x}\frac{\partial^3\psi}{\partial y^3}-\frac{\partial^4\psi}{\partial y^4}+A\frac{\partial^2\theta}{\partial y^2}\frac{\partial^2\psi}{\partial y^2}+\frac{\partial\theta}{\partial y}\frac{\partial^3\psi}{\partial y^3} +\frac{\partial^2\psi}{\partial y^2}(M+c')-\lambda\frac{\partial\varphi}{\partial y}=0 \quad (19)$$

By using above group transformation in equation (19), we get the following relation

$$\alpha_1+3\alpha_2-2\alpha_3=4\alpha_2-\alpha_3=2\alpha_2-\alpha_3=\alpha_2-\alpha_3 \quad (20)$$

On solving the equation, we get $\alpha_1=\alpha_3,\ \alpha_2=0$

Similarly equations (14), (15) and (16) are also giving $\alpha_1=\alpha_3,\ \alpha_2=0$, so these equations show invariant under the group transformation (18).

Now the characteristic equations are

$$\frac{dx}{x}=\frac{dy}{0}=\frac{d\psi}{\psi}=\frac{d\theta}{0}=\frac{d\varphi}{0} \quad (21)$$

Which give the following similarity transformations:

$$\eta=y,\ \psi=xf(\eta),\ \theta=\theta(\eta),\ \text{and}\ \varphi=\varphi(\eta) \quad (22)$$

Using these transformations, the momentum, energy and mass equations become

$$f'''=(A\theta'-f)f''+M(f'-1)+f'^2+c'f'-\lambda\varphi \quad (23)$$

$$\theta''=\frac{-1}{1+S\theta+4R/3}\left(S\theta'^2+Prf\theta'\right) \quad (24)$$

$$\varphi''=-Scf\varphi' \quad (25)$$

Subject to the boundary conditions

$$f=f_w,\ f'=0,\ \theta=\frac{1+b\theta'}{1-Sb\theta'},\ \varphi=1 \quad \text{at } \eta=0$$
$$f'\to 1,\ \theta\to 0,\ \varphi\to 0 \quad \text{as } \eta\to\infty \quad (26)$$

The physical quantities of interest are the Skin friction coefficient C_f, Nusselt number Nu and Sherwood number Sh, which are defined as

$$C_f=\frac{\mu}{\rho\bar{u}_e^2}\left(\frac{\partial\bar{u}}{\partial\bar{y}}\right)_{\bar{y}=0},\ Nu=\frac{-\bar{x}}{T_f-T_\infty}\left(\frac{\partial T}{\partial\bar{y}}\right)_{\bar{y}=0}, \quad (27)$$
$$Sh=\frac{-\bar{x}}{C_w-C_\infty}\left(\frac{\partial C}{\partial\bar{y}}\right)_{\bar{y}=0}$$

III. METHOD OF SOLUTION

The system of ordinary differential equations (23), (24) and (25) subject to the boundary conditions (26) have been solved numerically using Runga-Kutta method with shooting technique. The computations were carried out using step size of $\Delta\eta = 0.01$ selected to be satisfactory for a convergence criterion of 10^{-6} in all cases.

The physical quantities skin friction coefficient C_f, Nusselt number Nu and Sherwood number Sh indicate the wall shear stress, rate of heat transfer and rate of mass transfer respectively and these are proportional to the numerical values of $f''(0)$, $-\theta'(0)$ and $-\varphi'(0)$ respectively.

IV. RESULTS AND DISCUSSION

The numerical results have been computed and represented in the form of the dimensionless velocity, temperature, concentration, wall heat transfer, the rate of heat and mass transfer. Prandtl number Pr = 0.7 for air at 1 atmospheric pressure, Schmidt number Sc = 0.22 for Hydrogen, Sc = 0.67 for water vapour, Sc = 0.78 for Ammonia were taken. The values for the skin friction coefficient, Nusselt number and Sherwood number have been tabulated below:

Table 1. Effect on Skin friction coefficient C_f, Nusselt number Nu and Sherwood number Sh for $f_w = 0.5$, Pr = 0.7, M = 0.5, R = 1, Sc = 0.1, A = 0.1, S = 1, λ = 0.6, a = 1, b = 0.3 and c' =0.2.

parameter	values	$f''(0)$	$-\theta'(0)$	$-\varphi'(0)$
Pr	6.8	0.715250	-1.153453	-0.199205
	10	0.720591	-1.595283	-0.199205
S	0.3	0.698607	-0.291478	-0.199205
	0.5	0.697801	-0.264784	-0.199204
	0.7	0.696513	-0.229748	-0.199204
f_w	0.1	0.610156	-0.244192	-0.134265
	0.5	0.485973	-0.235062	-0.135410
	1	0.361994	-0.231405	-0.136201
Sc	0.22	0.692411	-0.219478	-0.299199
	0.67	0.687241	-0.219479	-0.436542
	0.78	0.683141	-0.219479	-0.636919
R	5	0.694815	-0.156904	-0.198921
	10	0.694117	-0.135690	-0.198921
M	0.1	0.306965	-0.196202	-0.175980
	0.9	0.962371	-0.230002	-0.212143
λ	0.7	0.687862	-0.224897	-0.199821
	1	0.696528	-0.220453	-0.192688
	1.2	0.698463	-0.219784	-0.188926

Figure 2 exhibits the effect of physical parameters on velocity f', temperature θ and concentration φ. It is seen that the suction has a significant effect on the boundary layer thicknesses. It can be observed that the velocity f' rises with suction parameter whereas temperature θ and concentration φ fall with rising f_w. It is also noticed that the thickness of momentum, thermal and concentration boundary layer reduce with an increase in f_w. The variation of velocity f' and temperature θ for different values of the radiation parameter R have been depicted in Figure 3. It reveals that the velocity f' and temperature θ increase with an increase in radiation parameter R. This is because rises in R have the tendency to increase the conduction effects and to increase temperature at each point away from the surface. Therefore, higher value of radiation parameter implies higher surface heat flux. It is also observed that momentum boundary layer thickness decreases while the thermal boundary layer thickness increases with the increasing values of R.

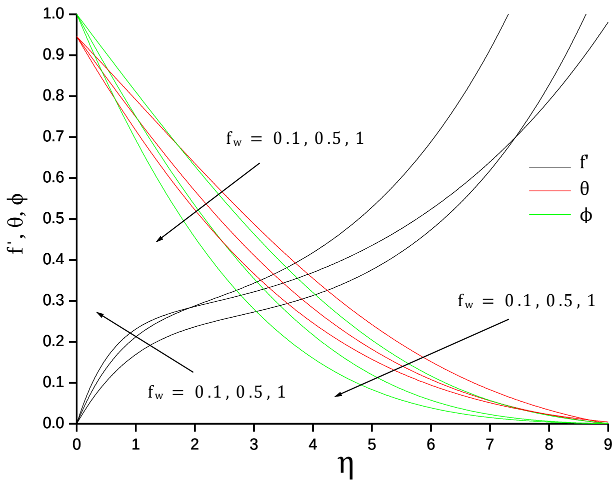

Fig. 2. Effect of suction parameter f_w on velocity f', temperature θ, concentration φ, for Pr = 0.7, M = 0.1, R = 1 Sc = 0.1, A = 0.1, S = 0.5, λ = 0.6, a = 1, b = 0.1 and c' =0.2.

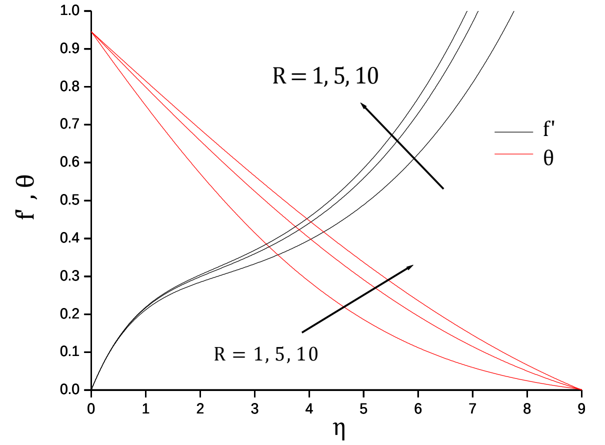

Fig. 3. Effect of radiation parameter R on velocity f', temperature θ, for f_w = 0.5, Pr = 0.7, M = 0.1, Sc = 0.1, A = 1, S = 1, λ = 0.6, a = 1, b = 0.5, and c' =0.2.

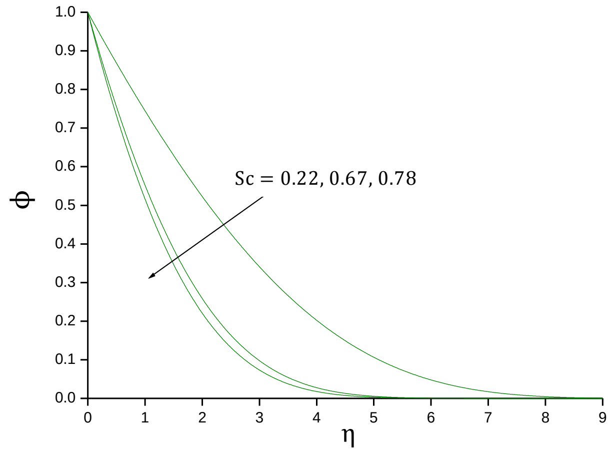

Fig. 4. Effect of Schmidt number Sc on concentration φ for f_w = 0.5, Pr = 0.7, M = 0.1, R = 1, A = 0.1, S = 1, λ = 0.7, a = 1, b = 0.3, and c' =0.2.

The effect of Schmidt number on concentration is represented through figure 4. It has been observed that as Schmidt number increases, the mass transfer rate increases and concentration decreases. There is a little change in temperature θ and concentration φ in case of moderate changes in Schmidt number Sc.

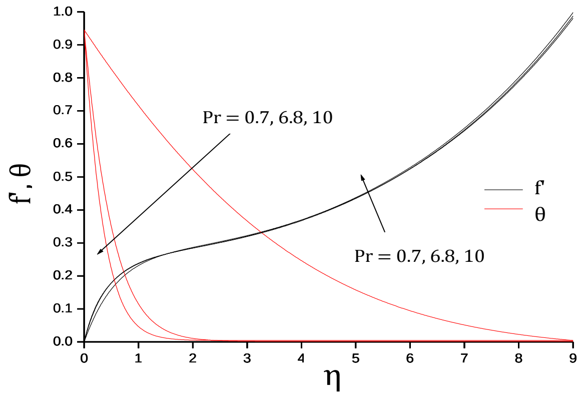

Fig. 5. Effect of Prandtl number Pr on velocity f', temperature θ, for f_w = 0.5, R = 1, M = 0.9, Sc = 0.78, A = 0.5 S = 1, λ = 0.6, a = 1, b = 0.2 and c' =0.3.

Figure 5 shows the variation of velocity f' and temperature θ for the variation of Pr. It is observed that θ decreases with an increase in Pr. It is observed that at higher Pr, the fluid has a thinner thermal boundary layer and this increase the wall temperature gradient θ'(0). It can also be observed that Pr reduces the velocity f' and thicken the corresponding boundary layer.

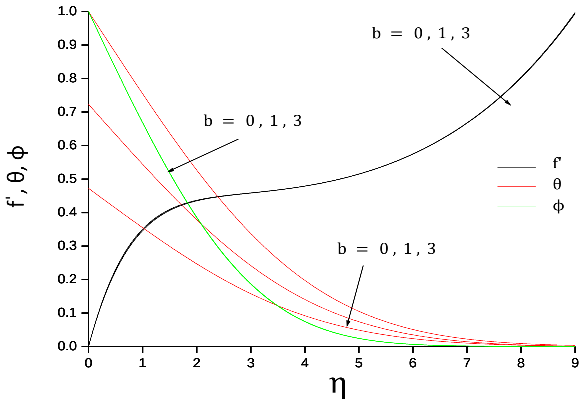

Fig. 6. Effect of convective heat transfer parameter b on velocity f', temperature θ, concentration φ, for f_w = 0.5 Pr = 0.7, M = 0.6, R = 1, Sc = 0.78, λ = 0.6, A =0.5, S = 0.3, a = 1 and c' =0.2.

To show the variations of thermal convective parameter b on the field variables velocity f', temperature θ and concentration φ respective we have drawn figure 6. This figure shows that velocity f' concentration φ and temperature θ reduce with increasing value of b.

The authors also attempted the case study the effect of injection parameter. The results were also seen with the good agreement as done by Hamad et al. [18].

REFERENCES

[1] M.A.A. Hamad, I. Pop and A.I.M. Isamil, "Magentic field effects on free convection flow of a nanofluid past a vertical semi-infinite flat", *Nonlinear Anal. Real World Appl.* 2011, 12 (3), pp. 1338–1346.

[2] A.G. Hansen, "Similarity Analysis of Boundary Layer Problems in Engineering", *Prentice Hall, Englewood Cliffs, N. J*, 1964.

[3] W.F. Ames, "Nonlinear Partial Differential Equations in Engineering", *Academic Press, New York*, 1972.

[4] T.Y. Na, "Computational Methods in Engineering Boundary Value Problem", *Academic Press, New York*, 1979.

[5] L. Dresner, "Similarity solutions of non-linear partial differential equations", *In: Pitman Advanced Publishing Program*, 1983.

[6] B.J. Cantwell, "Introduction to Symmetry Analysis", *Cambridge University Press*, 2003.

[7] P.P. Arasu, P. Loganathan, R. Kandasamy and I. Muhaimin, "Lie group analysis for thermal-diffusion and diffusion-thermo effects on free convective flow over a porous stretching surface with variable stream conditions in the presence of thermophoresis particle deposition", *Nonlinear Anal. Hybrid Syst.* 2011, 5, pp. 20–31.

[8] M.A.A. Mahmoud, "Thermal radiation effect on unsteady MHD free convection flow past a vertical plate with temperature-dependent viscosity", *Can. J. Chem. Eng.* 2009, 87, pp. 47–52.

[9] A.J. Chamkha, "Coupled heat and mass transfer by natural convection about a truncated cone in the presence of magnetic field and radiation effects", *Numer. Heat Transfer A: Appl.* 2001, 39, pp. 511–530.

[10] M.A.A. Mahmoud, "Variable viscosity effects on hydromagnetic boundary layer flow along a continuously moving vertical plate in the presence of radiation", *Appl. Math. Sci.* 2007, 1, pp.799–814.

[11] R.C. Bataller, "Radiation effects on Blasius flow", *Appl. Math. Comput*. 2008, 198, pp. 333–338.

[12] K.L. Hsiao, "Mixed convection with radiation effect over a nonlinearly stretching sheet", *World Acad. Sci. Eng. Technol.* 2010, 62, pp. 242–338.

[13] R.N. Jat and S. Chaudhary, "Radiation effects on the MHD flow near the stagnation point of a stretching sheet", *ZAMP*, 2010, 33, pp. 2–5.

[14] M.A. Seddeek, A.A. Afify and M.A. Hanaya, "Similarity solutions for steady MHD Falkner–Skan flow and heat transfer over a wedge by considering the effect of variable viscosity and thermal conductivity", *Applic. Appl. Math.* 2009, 4, pp. 301–313.

[15] N.C. Mahanti and P. Gaur, "Effects of varying viscosity and thermal conductivity on steady-free convective flow and heat transfer along an isothermalvertical plate in the presence of heat sink", *J. Appl. Fluid Mech*, 2009, 2, pp. 23–28.

[16] A. Aziz, "A similarity solution for laminar thermal boundary layer over flat plate with convective surface boundary condition", *Commun. Nonlinear Sci. Numer. Simul.* 2009, 14, pp. 1064–1068.

[17] O.D. Makinde, "On MHD heat and mass transfer over a moving vertical plate with a convective surface boundary condition", *Can. J. Chem. Eng*. 2010, 9999, pp. 1–8.

[18] M.A.A. Hamad, J.U. Uddin and A.I.M. Ismail, "Radiation effects on heat and mass transfer in MHD stagnation-point flow over a permeable flat plate with thermal convective surface boundary condition, temperature dependent viscosity and thermal conductivity", *Nuclear Engineering and Design*, 2012, 242, pp. 194-200.

[19] M.S. Alam, M.M. Rahman and M.A. Sattar, "Effects of variable suction and thermophoresis on steady MHD combined free-forced convective heat and mass transfer flow over a semi-infinite permeable inclined plate in the presence of thermal radiation", *Int. J. Therm. Sci*. 2008, 47, pp. 758–765.

[20] W.M. Kays and M.E. Crawford, "Convective heat and mass transfer", *4th ed. McGraw Hill, New York*, 2005.

[21] M.Q. Brewster, "Thermal Radiative Transfer and Properties", *John Wiley and Sons Inc., New York*, 1992.