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Effect of Magnetic Field on Thermal Instability of Non-Newtonian Fluids in a Rotating Medium

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"Abstract"

Effect of magnetic field on the thermal instability of Walters' B′ Viscoelastic fluid in a rotating medium is considered. For stationary convection ,Walters' B′ viscoelastic fluid behaves like a Newtonian fluid. The rotation and magnetic field have a stabilizing effect where as the suspended particles has destabilizing effect on the system. Numerical computations are made and illustrated graphically.

Key Words *: Magnetic field , Thermal instability, Viscoelasticity, Suspended particle , Rotating medium.*

"1.Introduction"

The effect of rotation and magnetic field on thermal instability of Non-Newtonian fluid is considered. The importance of Non-Newtonian fluids in modern technology and industries is ever increasing and the investigations on such fluids are desirable. One such class of Non-Newtonian fluids is Walters'B' fluid. Here

,the effect of suspended particles, rotation and magnetic field on the Walters' B' viscoelastic fluid heated from below is considered.

A detailed account of the theoretical and experimental results of the onset of thermal instability in a fluid layer under varying assumptions of hydrodynamics has been given by Chandrasekhar(1981). Bhatia and Steiner(1972) have studied the problem of thermal instability of a Maxwellian viscoelastic fluid in the presence of rotation . Sharma and Aggarwal(2006) have studied the effect compressibility and suspended particles on thermal convection in a Walter's B' elastic- viscous fluid in hydromagnetics . Bhatia and Steiner(1973) analyzed the thermal instability of a Maxwellian viscoelastic fluid in the presence of a magnetic field . Aggarwal and Prakash(2009) have discussed the effect of suspended particles and rotation on thermal instability of ferromagnetic fluids .Sharma (1975) studied the stability of a layer of an electrically conducting Oldroydian fluid in the presence of a magnetic field .Sharma(1977) analyzed the thermal instability in compressible fluid in the presence of rotation and a magnetic fluid. Sharma(1999) et al have considered the thermosolutal instability of Walters' B' rotating in porous medium . Bhatia and Steiner(1973) have studied the problem of thermal instability in a visco-elastic fluid layer in Hydromagnetics

"2.Mathematical Formulation"

We consider an infinite horizontal layer of electrically conducting Walters'B' elastic-viscous fluid layer of thickness d permeated with suspended particles, bounded by the planes z=0 and z=d in the presence of rotation. This layer is heated from below so that , the temperature and density at the bottom surface z=0 are $T_{0,}\rho_0$ and at the upper surface z= d are T_d, ρ_d respectively and that a uniform adverse temperature $\beta \left(= \left|\frac{dT}{dz}\right|\right)$ is maintained. A uniform magnetic field $\vec{H}$ = (0,0,H) and gravity field $\vec{g}$ (0,0,-g) pervades the system. The equations of motion and continuity for Walters'B' viscoelastic fluid in the presence of suspended particles and magnetic field with rotation are

$$\frac{\partial \vec{q}}{\partial t} + (\vec{q}.\nabla)\vec{q} = -\frac{1}{\rho_0}\nabla p + \vec{g}\left(1+\frac{\delta\rho}{\rho_0}\right) + \left(\nu - \nu'\frac{\partial}{\partial t}\right)\nabla^2\vec{q} + \frac{\mu_e}{4\pi\rho_0}(\nabla X \vec{H})X\vec{H} + \frac{KN_0}{\rho_0}(\vec{q}_d - q + 2q \times \Omega \qquad (1)$$

$$\nabla.\vec{q} = 0 \qquad (2)$$

where p, ρ , T ,$\vec{q}$ (u,v,w), $\vec{q}_d$ ($\vec{x}$, t) , N($\vec{x}$, t), ν,ν' denote fluid pressure, density, temperature, fluid velocity, suspended particles velocity, suspended particles number density, kinematic viscosity and kinematic viscoelasticity respectively. Here $\vec{g}$(0,0,-g) is acceleration due to gravity, $\vec{x}$ (x,y,z) and K=6$\pi\mu\eta'$, η' being particle radius, is the stokes' drag coefficient.

Since the force exerted by the fluid on the particle is equal and opposite to that exerted by the particle on the fluid, there must be an extra force term, equal in magnitude but opposite in sign, in the equation of motion for the particles. The effect due to pressure, gravity, Darcy's force and magnetic field on the particles are small and so are ignored. If mN is the mass of particles per unit volume, then the equations of motion and continuity for the particles, under the above assumptions are

$$mN\left\{\frac{\partial \vec{q}_d}{\partial t} + (\vec{q}_d.\nabla)\vec{q}_d\right\} = KN(\vec{q} - \vec{q}_d) \qquad (3)$$

$$\frac{\partial N}{\partial t} + \nabla.(N\vec{q}_d) = 0 \qquad (4)$$

If $C_v, C_{pt}\, T\ and\ q'$ denote the heat capacity of fluid at constant volume, heat capacity of the particle, temperature and effective thermal conductivity of the pure fluid respectively. The equation of heat conduction gives

$$\rho_0 C_v\left(\frac{\partial}{\partial t} + \vec{q}.\nabla\right)T + mNC_{pt}\left(\frac{\partial}{\partial t} + \vec{q}_d.\nabla\right)T = q'\nabla^2 T \qquad (5)$$

The Maxwell's equation yield

$$\frac{\partial \vec{H}}{\partial t} = (\vec{H}.\nabla)\vec{q} + \eta\nabla^2\vec{H} \qquad (6)$$

$$\nabla.\vec{H} = 0 \qquad (7)$$

The equation of state for the fluid is

$$\rho = \rho_0\left[1 - \alpha(T - T_0)\right] \qquad (8)$$

where α is the coefficient of thermal expansion and the subscript zero refers to values at the reference level z=0.The kinematic viscosity v, kinematic viscoelasticity v', electrical resistivity η and coefficient of thermal expansion α are all assumed to be constants.

"3.Perturbation equations"

The basic motionless solution is

$\vec{q}$ = (0,0,0), $\vec{q}_d$ = (0,0,0), T = T_0 -βz

$$\rho = \rho_0(1+\alpha\beta z), N = N_0 = constant \quad (9)$$

Assume small perturbations around the basic solution and let $\delta p, \delta\rho, \theta$ *and* $\vec{h}$ (h_x, h_y, h_z) denote respectively the perturbations in fluid pressure P, density ρ, temperature T and magnetic field $\vec{H}$. The change in density $\delta\rho$ caused mainly by the perturbation θ in temperature is given by

$$\delta\rho = -\alpha\rho_0\,\theta \quad (10)$$

Then the linearized perturbation equations of Walters'B' viscoelastic fluid become

$$\frac{\partial\vec{q}}{\partial t} = -\frac{1}{\rho_0}\nabla\delta p + \vec{g}\frac{\delta\rho}{\rho_0} + \left(v - v'\frac{\partial}{\partial t}\right)\nabla^2\vec{q} + \frac{\mu_e}{4\mu\rho_0}(\nabla\times\vec{h})\times\vec{H} + \frac{KN_0}{\rho_0}(\vec{q_d} - \vec{q}) + 2(\vec{q}\times\vec{\Omega}) \quad (11)$$

$$\nabla.\vec{q} = 0 \quad (12)$$

$$mN_0\frac{\partial\vec{q_d}}{\partial t} = KN_0\,(\vec{q} - \vec{q_d}) \quad (13)$$

$$\frac{\partial\vec{h}}{\partial t} = (\vec{H}.\nabla)\,\vec{q} + \eta\nabla^2\vec{h} \quad (14)$$

$$(1+h_1)\frac{\partial\theta}{\partial t} = \beta(w + h_1 s) + K\nabla^2\theta \quad (15)$$

$$\nabla.\vec{h} = 0 \quad (16)$$

where η stands for electrical resistivity ,K = $\frac{q}{\rho_0 c_v}$ and

$h_1 = \frac{mN_0 C_{pt}}{\rho_0 C_v}$

Eliminating $\overrightarrow{q_d}$ in equation (11) with the help of equation (13) ,writing the scalar components of resulting equations and eliminating u,v,$h_x h_{y,}$ and δp between them by using equation (12) and equation (16), we obtain

$\left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)$ $\Big[\frac{\partial}{\partial t}\ \nabla^2 w - g\alpha\left(\frac{\partial^{2\theta}}{\partial x^2}+\right.$ $\partial 2\theta\partial y2-\ \mu eH4\pi\rho 0\ \partial\partial z\,\nabla 2hz+\ 2\Omega$ $\partial\xi\partial z$

$+\frac{mN_0}{\rho_0}\ \frac{\partial}{\partial t}\nabla^2 w\ =\left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)\Big(v-$ $v'\partial\partial t\nabla 4w$ (17)

$\left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)\left[H_1\frac{\partial}{\partial t}-k\nabla^2\right]\theta =$ $\beta\left[\frac{m}{K}\frac{\partial}{\partial t}+H_1\right]w$ (18)

$\left(\frac{\partial}{\partial t}-\eta\nabla^2\right)h_z = H\frac{\partial w}{\partial z}$ (19)

"4.Dispersion relation"

We now analyze the disturbances into normal modes, assuming that the perturbation quantities are of the form

$[w,\theta,\ \xi,\zeta,h_z] =$ $[W(z),\Theta(z),Z(z),X(z),K(z)]exp\big(ik_x x+$ $ky\,y+nt$ (20)

where k_x, k_y are wave numbers along x and y directions respectively, k($=\sqrt{k_x^2+k_y^2}$) is the resultant wave number of the disturbances and n is the growth rate. Using expression (20) equations (17) – (19), in non-dimensional form, become

$(D^2-a^2)\Big[\sigma-(1-F\sigma)(D^2-$ $a2+M\sigma 1+J1\sigma$W- $\mu eHd4\pi\rho 0v$ $(D^2-a^2)\ DK$ $+\frac{gad^2}{v}a^2\Theta +$ $2\frac{\boldsymbol{\Omega d^3}}{\boldsymbol{v}}\ DZ = 0$

(21)

$(1+J_1\sigma)[D^2-a^2-$ $H1p1\sigma\Theta\boldsymbol{=-\beta d2K}H1+J1\sigma W$

(22)

$(D^2-a^2-p_2\sigma)K =$ $-\frac{Hd}{\eta}\ DW$ (23)

$$\Big[(1-F\sigma)(D^2-a^2)-$$

$\sigma 1+M1+J1\sigma Z=\mu eHd4\pi\rho 0v$

$$DX-\frac{2\Omega d}{v}DW$$

(24)

$$(D^2-a^2-p_2\sigma)X=-\frac{Hd}{\eta}\,DZ \quad (25)$$

where we have expressed the co-ordinate x, y and z in the new unit of length d, time t in the new unit of length $\frac{d^2}{K}$ and put a=kd , $\sigma=\frac{nd^2}{v}$, J=$\frac{m}{k}$, $J_1=\frac{Jv}{d^2}$,

M=$\frac{mN_0}{\rho_0}$, $P_1=\frac{v}{k}$ is the prandtl number, $p_2=\frac{v}{\eta}$ is the magnetic prandtl number, F=$\frac{V}{d^2}$ is the dimensionless kinematic viscoelasticity, $H_1=1+h_1$ and D=$\frac{d}{dz}$ eliminating Θ, Z, K *and* X from equation (21) to (25), we obtain

$$(D^2-a^2)\left[\sigma\left(1+\frac{M}{1+J_1\sigma}\right)+(\sigma F-1)(D^2-a^2)\right]W$$

$$-\frac{R\,a^2(H_1+J_1\sigma)}{(1+J_1\sigma)(D^2-a^2-H_1p_1\sigma)}W$$

$$-T_A\left[\frac{(1+J_1\sigma)(D^2-a^2-p_2\sigma)}{(1-F\sigma)(1+J_1\sigma)(D^2-a^2)(D^2-a^2-p_2\sigma)}-\right.$$

$$\left.\sigma(1+J_1\sigma)-M\sigma+Q(1+J_1\sigma)D^2\right]D^2W$$

$$+Q\frac{(D^2-a^2)}{(D^2-a^2-p_2\sigma)}D^2W=0 \qquad (26)$$

where R=$\frac{g\alpha\beta d^4}{vK}$ is the Rayleigh number and Q=$\frac{\mu_E H^2 d^2}{4\pi\rho_0 v\eta}$ is the Chandrasekhar number.

Free – Free boundary conditions are

W=D^2 W =0, DZ=0, $\boldsymbol{\Theta}$ =0 at Z=0,1 and DX=0,K=0 (27)

Using the above boundary conditions given in (27), it can be shown with the help of equations (21)- (25) that all the even order derivatives of W must vanish for z=0 and z=1 hence the proper solution W characterizing the lowest mode is

$$W=W_0\sin\pi z \qquad (28)$$

where W_0 is a constant, substituting the proper solution (28) in equation (26), we obtain the dispersion relation,

$$R_1 = \frac{Q_1\frac{(1+X)}{(1+X+ip_2\sigma_1)} - (1+x)\left[i\sigma_1\left[1+\frac{M}{1+i\pi^2 J_1\sigma_1}\right] + iF_1\sigma_1(1+x) - (1+x)\right]W}{x\frac{(H_1+i\pi^2 J_1\sigma_1)}{(1+i\pi^2 J_1\sigma_1)(1+x+IH_1p_1\sigma_1)}W}$$

$$+ \frac{TA_1\left[\frac{(1+x+ip_2\sigma_1)}{(1-iF_1\sigma_1)(1+X)(1+X+ip_2\sigma_1) - \frac{i\sigma_1}{\pi^2} + Q_1}\right]}{x\frac{(H_1+i\pi^2 J_1\sigma_1)}{(1+i\pi^2 j_1\sigma_1)(1+x+iH_1p_1\sigma_1)}W} \quad (29)$$

where $x=\frac{a^2}{\pi^2}$, $i\sigma_1 = \frac{\sigma}{\pi^2}$, $R_1 = \frac{R}{\pi^4}$, $T_A = \frac{T_A}{\pi^4}$, $Q_1 = \frac{Q}{\pi^2}$, $F_1 = \pi^2$F .

"5.Stationary Convection"

When the instability sets in as stationary convection, the marginal state will be characterized by σ =0. Putting σ =o, the dispersion relation (7.29) reduces to

$$R_1 = \left(\frac{1+x}{xH_1}\right)\left[(1+x)^2 + Q_1 + T_A\left(\frac{(1+x)}{Q_1+(1+x)^2}\right)\right] \quad (30)$$

To study the effects of magnetic field, suspended particles and rotation, we examined the natures of $\frac{dR_1}{dQ_1}$, $\frac{dR_1}{dH_1}$, $\frac{dR_1}{dT_A}$.

Equation (7.30) yields,

$$\frac{dR_1}{dQ_1} = \left(\frac{1+x}{xH_1}\right)\left[T_A\frac{(1+x)}{\{Q_1+(1+x)^2\}^2}\right] \quad (31)$$

$$\frac{dR_1}{dQ_1} = -\left(\frac{1+x}{xH_1^2}\right)\left[(1+x)^2 + \right.$$

Q1+TA1+xQ1+1+x2

(32)

$$\frac{dR_1}{dT_A} = \frac{(1+x)}{xH_1}\left[\frac{(1+x)}{(1+x)^2+Q_1}\right]$$

(33)

It is clear from equation (31) - (33) that for stationary convection the magnetic field and rotation postpone the onset of convection where as the suspended particles hasten the onset of convection.

TABLE -1

Variation of R_1 with Q_1 for a fixed value of H_1= 50

and T_A = 20.

Sl.No	Q_1	R_1			
		x=6	x=8	x=10	x=12
1	100	3.4935	4.0947	4.8839	5.8582
2	200	5.6361	6.3369	7.0770	8.0225
3	300	8.1410	8.5830	9.2734	10.1893
4	400	10.4689	10.8300	11.4712	12.3572
5	500	12.7976	13.0794	13.6697	14.5257
6	600	15.1267	15.3284	15.8687	16.6946

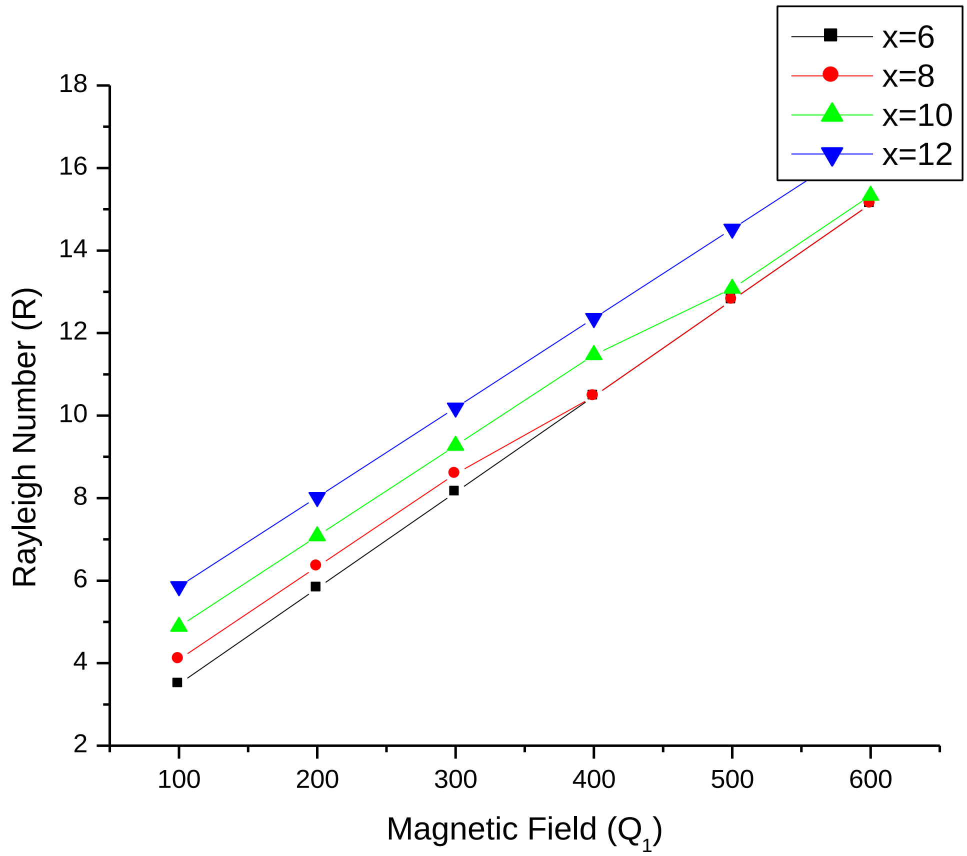

Fig.1 : Variation of R_1 with Q_1 for a fixed value of H_1= 50 and $T_A = 20$.

TABLE -2

Variation of R_1 with T_A for a fixed value of H_1 =20 and Q_1=60.

Sl. No	T_A	R_1				
		x=2	x=4	x=6	x=8	x=10
1	100	5.5010	6.0658	8.9755	11.1626	13.6318
2	200	5.8271	7.3816	11.5963	14.3940	17.3086
3	300	6.1533	8.6973	14.2171	17.6253	20.9853
4	400	6.4793	10.0131	16.8380	20.8560	24.6621
5	500	6.8054	11.3289	19.4588	24.0881	28.3389
6	600	7.1315	12.6447	22.0796	27.3195	32.0157

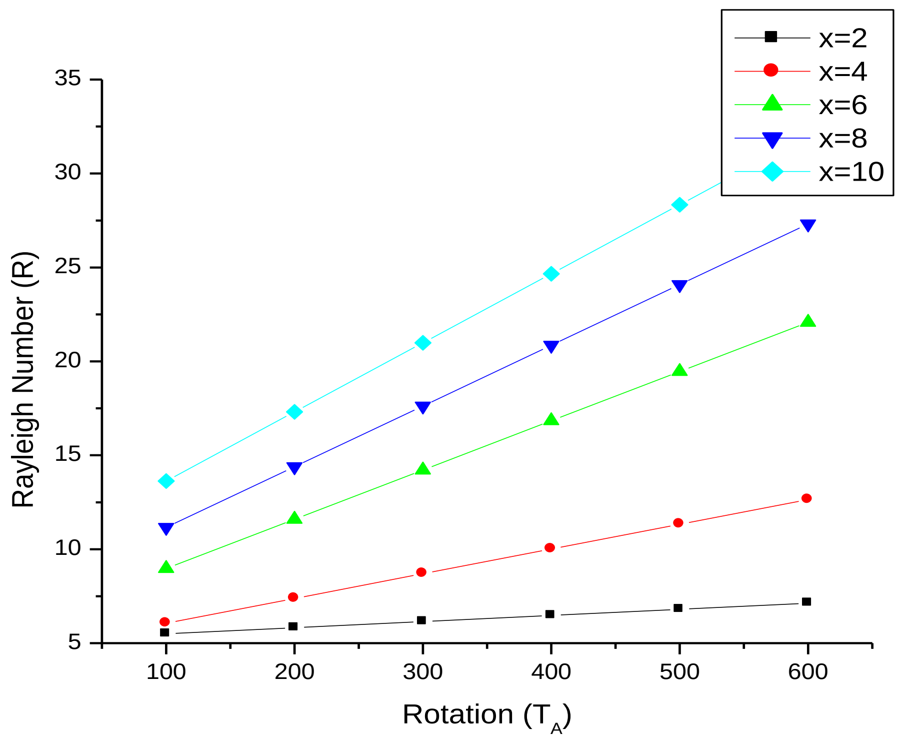

Fig .2 : Variation of R_1 with T_A for a fixed value of H_1 =20 and Q_1=60.

TABLE: 3

Variation of R_1 with H_1 for a fixed value $T_{A_1} = 20$, and $Q_1 = 50$

Sl No	H_1	R_1				
		x=2	**x=4**	**x=6**	**x=8**	**x=10**
1	100	0.9000	0.95417	1.1715	1.4892	1.8952
2	200	0.4501	0.4779	0.5858	0.7446	0.9475
3	300	0.3008	0.3180	0.3905	0.4964	0.6318
4	400	0.2250	0.2385	0.2929	0.3723	0.4738
5	500	0.1800	0.1908	0.2343	0.2978	0.3790
6	600	0.1500	0.1590	0.1952	0.2482	0.3159

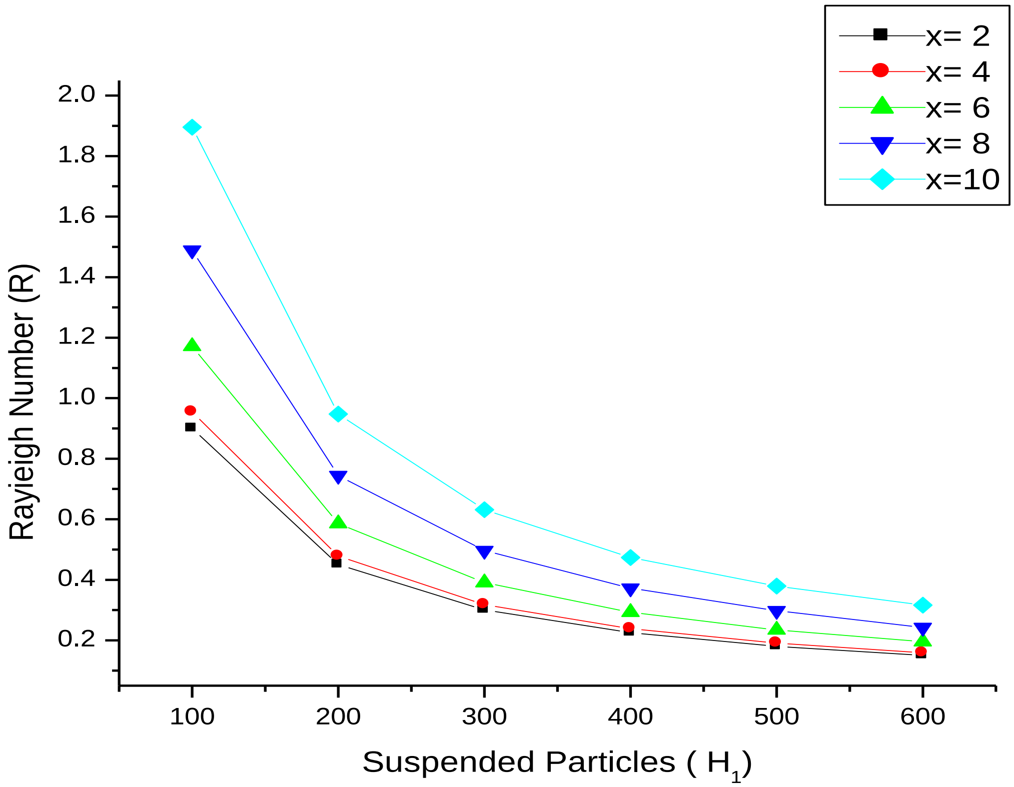

Fig 3 : Variation of R_1 with H_1 for a fixed value $T_{A_1} = 20$, and $Q_1 = 50$.

"6.Conclusion"

The Walters' B' fluid is one such important Non- Newtonian fluid. For stationary convection, Walters' B' viscoelastic fluid behaves like a Newtonian fluid. It is also found that, rotation and magnetic field postpones the onset of convection whereas the suspended particles hasten the onset of convection. The rotation and magnetic field have stabilizing effect whereas the suspended particles has destabilizing effect on the system.

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