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Effect of Joule Heating on Steady MHD Flow of Low Prandtl Fluid on a Porous Stretching Sheet

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"***Abstract***"

Aim of this paper is to investigate the effect of Joule heating on steady MHD flow of low Prandtl viscous and conducting fluid on a porous stretching sheet. In this analysis, the governing equations (viz. continuity, momentum and energy equations) are transformed into a system of ordinary differential equations. The momentum equation is solved analytically and energy equation is solved numerically using fourth order Runge-Kutta method along with shooting method. The fluid velocity and temperature characteristics are discussed and presented through graphs. The effects of skin-friction coefficient (viscous drag) and Nusselt number (rate of heat transfer) are derived, discussed and presented through tables for various physical parameters.

1. Introduction

The fluid dynamics over a stretching sheet is important in many practical applications such as extrusion of plastic sheet, paper production, glass blowing, metal spinning and drawing plastic films. Crane [1] was the first to consider the boundary layer flow caused by a stretching sheet, which moves with a velocity varying linearly with the distance from a fixed point. The heat transfer aspect of this problem was investigated by Carragher and Crane [2], under the conditions when the ambient fluid is proportional to a power of the distance from a fixed point. Fluid flows and heat transfer characteristics on stretching sheet with variable temperature condition have been investigated by Grubka and Bobba [3]. Elbashbeshy [4] discussed the heat transfer over a stretching sheet with variable heat flux. Elbashbeshy and Bajid [5] analyzed the stretching problem with internal heat generation and suction or injection in porous medium.

Effect of variable thermal conductivity and heat source/sink on MHD flow near a stagnation point on a linear stretching sheet was investigated by Sharma and Singh [6]. In this similar context Tamanna Sultana et al. [7] studied heat transfer in a porous medium over a stretching surface with internal heat generation and suction or injection in the presence of radiation.

The low Prandtl fluid flows have become important due to industrial applications, for instance it is used to deal with the problem of cooling of nuclear reactor by fluid having very low Prandtl number [Michiyoshi et al. (8), Fumizawa (9)]. Liquid metals have small Prandtl number of order 0.001~0.1 [e.g. Pr = 0.01 is for Bismuth, Pr = 0.023 is for Mercury etc.] and generally used as coolant because of very high thermal conductivity. They have ability to transport heat even if small temperature difference exists between the surface and the fluid. Due to this reason, liquid metals are used as coolant in nuclear reactors for the disposal of waste heat [Sharma and Singh (10)]. Kay (11), Arunachalam and Rajappa (12), Chaim (13) etc. presented low Prandtl fluid flow taking various geometries.

An analysis of thermal boundary layer in an electrically conducting fluid over a linearly stretching sheet in the presence of the constant transverse magnetic field with suction or blowing at the sheet was carried out by Chaim [14]. In this paper the viscous and Joule dissipation and internal heat generation was taken into account in the energy equation. The problem of viscous dissipation, Joule heating and heat source/sink on non-Darcy natural convection flow over an isoflux permeable sphere in a porous medium is numerically analyzed by Yih [15].

In this present study, the effect of Joule heating is investigated on steady MHD flow on a porous stretching sheet with a fluid of low Prandtl number.

2. Formulation of the Problem

Consider steady two-dimensional laminar viscous incompressible fluid over a porous stretching sheet in the presence of transverse magnetic field. The stretching sheet has a uniform temperature T_w and linear velocity $u_w x$. Stretching sheet is placed in the plane $y = 0$ and *x*-axis is taken along the sheet. Two equal and opposite forces are applied along *x*- axis hence the sheet is stretched linearly keeping the origin fixed as demonstrated in figure 1.

The governing equations of continuity, momentum and energy of laminar boundary layer are given as

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \quad , \tag{1}$$

$$\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} \right) = \frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho}u \quad , \tag{2}$$

$$\rho C_p \left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} \right) = \kappa \frac{\partial^2 T}{\partial y^2} + \sigma B_0^2 u^2 \tag{3}$$

where *u*, *v* are the velocity components along *x*- and *y*- axes, respectively, μ is the coefficient of viscosity, ρ is the density, σ is electrical conductivity, B_0 is magnetic field intensity, C_p is specific heat at constant pressure and κ is the thermal conductivity.
The corresponding boundary conditions are given by

$$u = u_w x \; , v = \pm v_w \; , T = T_w \quad , \text{ at } y = 0$$
$$u = 0 \quad , \quad T = T_\infty \quad , \qquad \text{at } y \to \infty \tag{4}$$

Positive and negative value of v_w indicates the blowing and suction, respectively and obviously $v_w = 0$ is corresponding to non-porous sheet. Since, the low Prandtl fluid is taken into account the order of viscous dissipation term is considered less than the order of Joule heating i.e. $\frac{\mu}{\rho C_p}\left(\frac{\partial u}{\partial y}\right)^2 << \sigma B_0^2 u^2$.

3. Method of Solution

The continuity equation (1) is satisfied by introducing the stream function

$$\psi(x, y) = \sqrt{\frac{\mu u_w}{\rho}} x f(\eta) \quad \text{such that}$$

$$u = \frac{\partial \psi}{\partial y} \quad , \quad v = -\frac{\partial \psi}{\partial x} \quad . \tag{5}$$

The momentum and energy equations can be transformed into ordinary differential equations by taking the similarity variable and dimensionless temperature parameter respectively as

$$\eta = y\sqrt{\frac{u_w \rho}{\mu}} \quad \text{and} \quad \theta = \frac{T - T_\infty}{T_w - T_\infty} \tag{6}$$

Using eq. (6), the equations (2) and (3) along with the boundary conditions are reduced to

$$f^{///} + ff^{//} - (f^{/})^2 - M^2 f^{/} = 0 \, , \tag{7}$$

$$\theta^{//} + \Pr f\theta^{/} + M^2 Ec \Pr (f^{/})^2 = 0 \, , \tag{8}$$

where *f* is non-dimensional stream function. Prime denotes the differentiation with respect to η.

Corresponding boundary conditions are reduced to

$$f = f_w \, , \; f^{/} = 1, \, \theta = 1 \quad \text{at } \; \eta = 0 \, ,$$
$$f^{/} \to 0, \, \theta \to 0 \qquad \text{at } \; \eta \to \infty \, , \tag{9}$$

where $f_w = \left(\pm v_w \sqrt{\frac{\rho}{\mu u_0}} \right)$ is injection and suction velocity at the plate for $f_w < 0$ and $f_w > 0$, respectively.

The governing boundary layer equation (7) along with the boundary conditions (9) admits a solution [Ahmad and Mubeen (16)] of the form given by

$$f(\eta) = f_w + \frac{1}{r} - \frac{1}{r}e^{-r\eta} \; , \tag{10}$$

where

$$r = \frac{f_w + \sqrt{f_w^2 + 4(M^2 + 1)}}{2} . \tag{11}$$

Further the energy equation (8) with the boundary conditions (9) is solved using Runge-Kutta fourth order method along with shooting technique [Conte and Boor (17) and Sharma and Singh (6)].

4. Skin-friction Coefficient

The shear stress at the sheet is given by

$$\tau_w = \mu \left(\frac{\partial u}{\partial y} \right)_{y=0} . \tag{12}$$

The skin-friction coefficient at the sheet is defined as

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho u_w^2} = 2x^{3/2}\,\mathrm{Re}^{-\frac{1}{2}} f^{//}(0) \qquad (13)$$

$$\Rightarrow \qquad C_f \propto f^{//}(0).$$

5. Nusselt Number

The local heat flux is given as

$$q_w(x) = \kappa\left(\frac{\partial T}{\partial y}\right)_{y=0}. \qquad (14)$$

Now the Nusselt Number at the sheet is defined as

$$Nu_x = \frac{q_w(x)}{\kappa(T_w - T_\infty)} = -(\mathrm{Re}_x)^{1/2}\theta^{/}(0) \qquad (15)$$

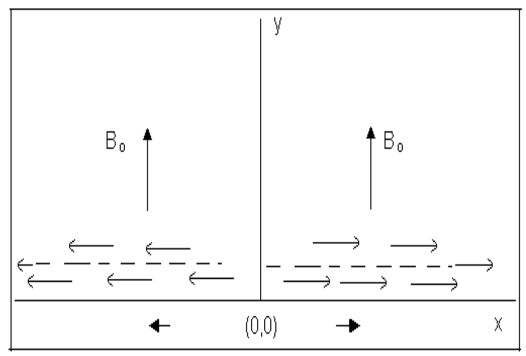
$$\Rightarrow \qquad Nu_x \propto -\theta^{/}(0).$$

6. Particular Case

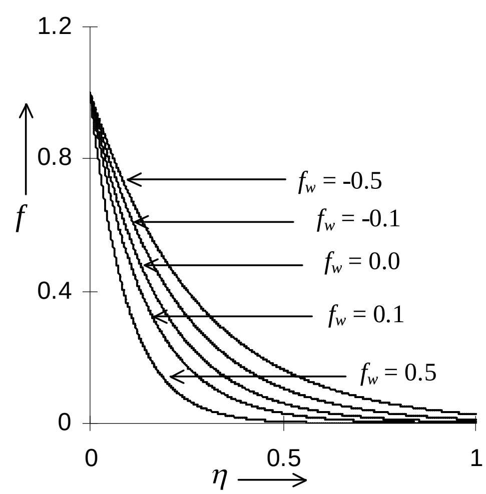
In the absence magnetic field (i.e. M = 0), the results of present paper are reduced to those obtained by Chaim [13]. It is seen from Table 1 that the numerical results $\theta^{/}(0)$ of present paper are in good agreement.

7. Results and Discussion

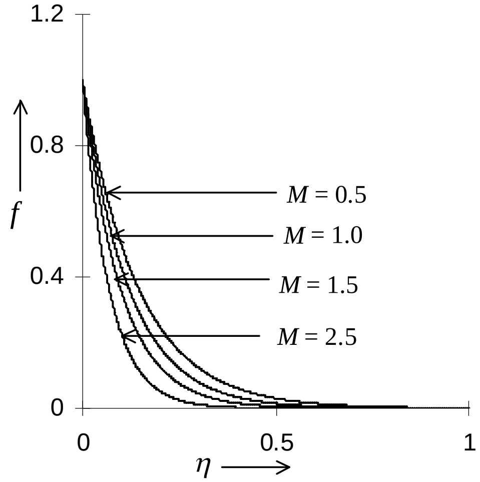
The energy equation (8) with the boundary conditions (9) is solved using Runge-Kutta fourth order method along with shooting technique for different values of the parameters M, Pr and f_w taking step size 0.001.
The numerical calculations are presented in the form of tables and graphs for various values of physical parameters.
Figure 2 shows the velocity profiles for different values of suction and injection parameter f_w with M = 0.5. It can be seen easily that the velocity profiles decrease with the increase of f_w. The Hartmann number (M) shows an importance of magnetic field sets in Lorentz force, which results in retarding force on the velocity field and therefore as M increases the velocity profiles decrease. As observed from figures 3 and 4 it is also noted that the velocity profiles decrease, while taking increment in M for f_w = 0.5 (figure 3, in case of suction) and for f_w =-0.5 (figure 4, in case of injection). Figures 5 and 6 show the effect of the Hartmann number on temperature profiles at Pr=0.01 for f_w = 0.5 (figure 5, in case of suction) and for f_w =-0.5 (figure 6, in case of injection), respectively and it is observed that temperature profiles increase with the increase in Hartmann number. Because the magnetic field retards the velocity of fluid and therefore temperature of the fluid near the sheet is higher. It is observed from figure 7 that with the increase in Prandtl number temperature profiles decreases.
The skin-friction coefficient and Nusselt number are presented by equations (13) and (15), which are directly proportional to $f^{//}(0)$ and $-\theta^{/}(0)$, respectively. The effect of physical parameters on these two are presented in the table 2 and table 3.
It is observed from table2, for the both cases (suction and injection) the value of $f^{//}(0)$ decreases as the value of Hartmann number increases. It is noted from table 3 the numerical values of $-\theta^{/}(0)$ decrease as f_w increases. Further in case of suction, the value of $-\theta^{/}(0)$ decreases as the value of Prandtl number Pr increases and reverse result is found in case of injection. Also when Hartmann number increases the values of $-\theta^{/}(0)$ increase.



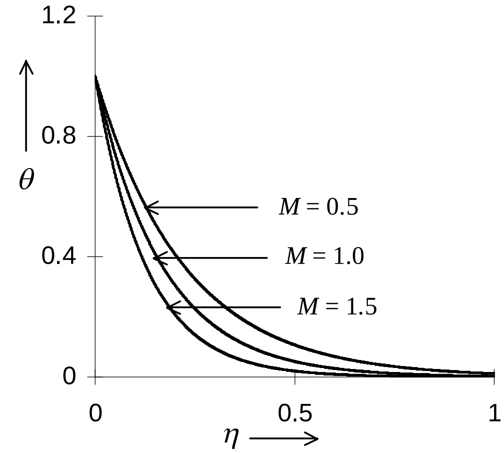
"Figure 1. Schematic diagram of the problem".



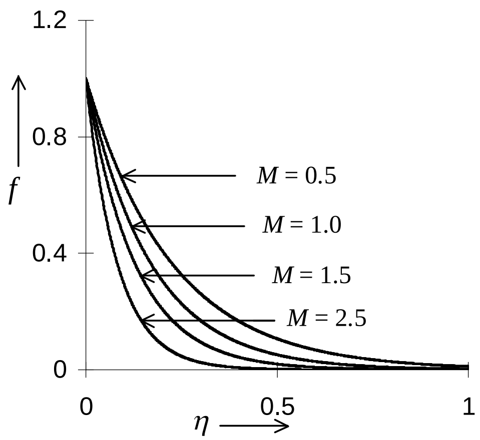
"Figure 2. Velocity distribution when M = 0.5"



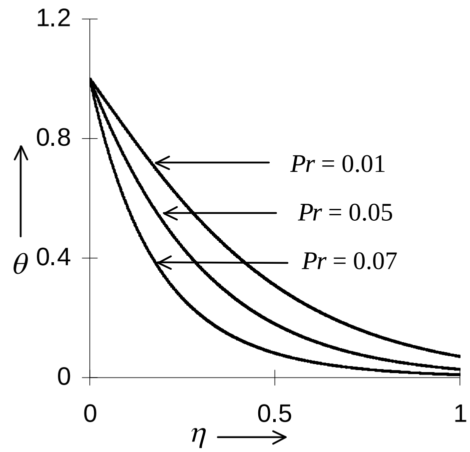
"Figure 3. Velocity distribution when $f_w = 0.5$".



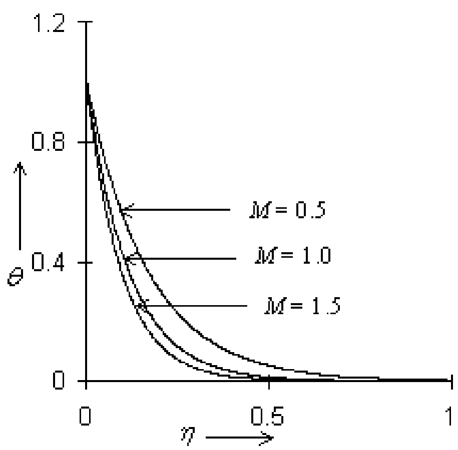
"Figure 6. Temperature distribution when $f_w = -0.5$".



"Figure 4. Velocity distribution when $f_w = -0.5$".



"Figure 7. Temperature distribution when $M = 0.5$".



"Figure 5. Temperature distribution when $f_w = 0.5$".

Table 1. Numerical values of $\theta^{/}(0)$ for various values of *Pr* are compared with the results obtained by Chaim (1998)

Pr	f_w	$-\theta^{/}(0)$	
		Chaim [13]	Present work
0.023	0.0	0.0224886	0.02302313
0.023	1.5	0.0457422	0.04576912
0.1	0.0	0.0912924	0.0913340
0.1	1.5	0.1955034	0.19554759
1.0	0.0	0.5819767	0.5836673
1.0	1.5	1.760407	1.76268901

Table 2. Numerical values of $f^{//}(0)$ for various values of M and f_w.

	M	$f^{//}(0)$
f_w =0.5	0.5	-1.395644
	1.0	-1.686141
	1.5	-2.070027
	2.5	-2.954163
f_w = 1.0	0.5	-1.724745
	1.0	-2.000
	1.5	-2.320829
	2.5	-3.238613
f_w = 1.5	0.5	-2.096291
	1.0	-2.350781
	1.5	-2.702562
	2.5	-3.545085
f_w = -0.5	0.5	-0.895644
	1.0	-1.186141
	1.5	-1.570027
	2.5	-2.454163
f_w = -1	0.5	-0.724745
	1.0	-1.000
	1.5	-1.370829
	2.5	-2.238613
f_w = -1.5	0.5	-0.596291
	1.0	-0.850781
	1.5	-1.202562
	2.5	-2.045085

Table 3. Numerical values of $\theta^{/}(0)$ for various values of physical parameters

	Pr	M	$-\theta^{/}(0)$
f_w =0.5	.01	0.5	0.4050120
	.01	1.0	0.4038703
	.01	1.5	0.4024835
f_w = 1.0	.01	0.5	0.4073560
	.01	1.0	0.4063797
	.01	1.5	0.4051136
	.07	0.5	0.4481621
f_w = -0.5	.01	0.5	0.4003308
	.01	1.0	0.3988151
	.01	1.5	0.3971187
f_w = -1	.01	0.5	0.3979683
	.01	1.0	0.3962657
	.01	1.5	0.3944089
	.07	0.5	0.3812571

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