Effect of Chemical Reaction on MDH Oscillatory flow Through a Vertical Porous Plate with Heat Generation

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Abstract

The effect of chemical reaction on MHD oscillatory flow through a vertical porous plate with heat generation was studied.The dimensionless governing equations for this model were solved by a closed analytical form. The influence of various parameters on the velocity,temperature and concentration fields as well as the Coefficient of skin-friction number were presented graphically and qualitatively.

Keywords: Heat generation, Unsteady, Porous medium, MHD, and skin friction

1 INTRODUCTION

Applications of combined heat and mass transfer flow with chemical reaction play important role in the design of chemical processing equipament,formation and dispersion of fog, distribution of temperature and moisture over agricultural fields and groves of fruits trees damage of crops due to freezing, food processing and cooling of towers. Investigation of periodic flow through a porous medium is important from practical point of view because fluid oscillations may be expected in many magnetohydrodynamics devices and natural phenomena, where fluid flow is generated due to oscillating pressure gradient or due to vibrating walls.Consequently,A.J. Chamkha(2003) studied the MHD flow of a numerical of uniformly stretched vertical permeable surface in the presence of heat generation/ absorption and a chemical reaction. A.S. Idowu et al.(2013), Heat and mass transfer of magnetohydrodynamic (MHD) and dissipative fluid flow pass a moving vertical porous plate with variable suction. F.M. Hady et al. (2006) researched on the problem of free convection flow along a vertical wavy surface embedded in electrically conducting fluid saturated porous media in the presence of internal heat generation or absorption effect.Kim and Fedorov (2004) studied Transient mixed radiative convection flow of a micro polar fluid past a moving semi-infinite vertical porous plate while K.Vajravelu and Hadjinicolaou(1993) studied the heat transfer characteristics in the laminar boundary layer of a viscous fliud over a streching sheet with viscous dissipation or frictional heating and internal heat generation.

The study of heat generation or absorption effects in moving fluids is important in view of several physical problems such as fluids undergoing exothermic or endothermic or transfer chemical reactions. M.A. Hossain et.al.(2004) investigated the problem of natural convection flown along a vertical wavy surface with uniform surface temperature in the presence of heat generation/ absorption. In this direction M.A. Alam et al.(2006) studied the problem of free convection heat and mass transfer flow past an inclined semi-infinite heated surface of an electrically conducting and steady viscous incompressible fluid in the presence of a magnetic field and heat generation. Md Abdus and Mohammed M.R.(2006) considered the thermal radiation interraction with unsteady MHD flow past a vertical porous plate immersed in a porous medium. The importance of radiation in the fluid led Muthucumaraswamy and Chandrakala (2006) to study radiative heat and mass transfer effect on moving isothermal vertical plate in the presence of chemical reaction. Muthucumaraswamy and Senthih (2004) considered a Heat and Mass transfer effect on moving vertical plate in the presence of thermal radiation.

In many chemical engineering processes, the chemical reaction do occur between a mass and fluid in which plate is moving. These processes take place in numerous industrial application such as polymer production, manufacturing of ceramics or glassware and food processing. In the light of the fact that, the combination of heat and mass transfer problems with chemical reaction are of importance in many processes, and have, therefore, received a considerable amount of attention in recent years. In processes such as drying, evaporation at the surface of a water body,energy transfer in a wet cooling tower and the flow in a desert cooler, heat and mass transfer occur simultaneously. Possible applications of this type of flow can be found in many industries. For example, in the power industry, among the methods of generating electricity is one in which electrical energy is extracted directly from a m the oving conducting fluid. Naving Kumar and Sandeep Gupta (2008) investigated the effect of variable permeability on unsteady two-dimensional free convective flow through a porous bounded by a vertical porous sourface. P.R. Sharma, Navin and Pooja (2011) have studied the Influence of chemical reaction on un steady MHD free convective flow and mass transfer through viscous incompressible fluid past a heated vertical plate immersed in porous medium in the presence of heat source. R. Muthucumaraswamy and Ganesan(2001) studied the effect of the chemical reaction and injection on flow characteristics in an unsteady upward motion of an isothermal plate. R.A Mohammed (2009) studied double-diffusive convection-radiation interaction on unsteady MHD flow over a vertical moving porous plate with heat generation and soret effects. Shaik Abzal, G.V, Ramana Reddy and S.Vijayakumar Varma.(2011), had investigated the Unsteady MHD free convection flow and mass transfer near a moving vertical plate in the presence of thermal radiation.

Base on these investigation, work has been reported in the field. In particular, the study of heat and mass transfer, heat radiation is of considerable importance in chemical and hydrometallorgical industries.Mass transfer process are evaporation of water from a pound to the atmosphere the diffusion of chemical impurities in lakes, rivers and ocean from natural or artificial sources. Magnetohydrodynamic mixed convection heat transfer flow in porous plate and non-porous media is of considerable interest in the technical field due to its frequent occurence in industrial technology and geothermal application, high temperature plasma application to nuclear fusion energy conversion, liquid metal fluid and MHD power generation systems combined heat mass transfer in natural convective flows on moving vertical porous plate.V.Srinvasa Rao and L.Anand Babu.(2010), studied the finite element anlysis of radiation and mass transfer flow past semi-infinite moving vertical plate with viscous dissipation.

In view of these we studied the influnce of chemical reaction on MHD oscillatory flow through a vertical porous plate with heat generation. The expressions are obtained for velocity,temperature and concentration analytically. The effects of various emerging parameters on the velocity and temperature are discussed through graphs in detail.

2 MATHEMATICAL ANALYSIS

Consider unsteady two-dimensional hydromagnetic laminar, incompressible, viscous, electrically conducting and heat source past a semi-infinite vertical moving heated porous plate embedded in a porous medium and subjected to a uniform transverse magnetic field in the presence of themal diffusion, and thermal radiation effects. According to the coordinate system, the x-axis is taken along the plate in upward direction and y-axis is normal to the plate. The fluid is assumed to be a gray, absorbing-emitting but non-scattering medium. It is assumed that there is no applied voltage of which implies the absence of an electric field. The transversely applied magnetic field and magnetic Reynolds number are very small and hence the induced magnetic field is negligible. Viscous terms are taken into account the constant permeability porous medium. The MHD term is derived from an order-of-magnitude analysis of the full Navier-stokes equation. It is assumed here that the hole size of the porous plate is significantly larger than a characteristic microscopic length scale of the porous medium. The fluid properties are assumed to be constants except that the influence of density variation with temperature has been considerd in the body-force. Since the plate is semi-infinite in length, therefore all physical quantities are functions of y and t only. Hence, by the usual boundary layer approximations, the governing equations for unsteady flow of a viscous incompressible fluid through a porous medium are:

Continuity equation

$$\frac{\partial u^*}{\partial y^*} + \frac{\partial v^*}{\partial y^*} = 0 \quad (1)$$

Linear momentum equation

$$\frac{\partial u^*}{\partial t^*} = \nu\frac{\partial^2 u^*}{\partial y^{*2}} + g\beta(T^* - T^*_\infty) + g\beta^*(C^* - C^*_\infty) - \frac{\sigma B_o^2}{\rho}u^* - \frac{\nu}{K^*}u^* \quad (2)$$

Energy equation

$$\rho C_p\frac{\partial T^*}{\partial t^*} = k\frac{\partial^2 T^*}{\partial y^{*2}} - \frac{\partial q_r}{\partial y^*} - Q_o(T^* - T^*_\infty) \quad (3)$$

Diffusion equation

$$\frac{\partial \phi^*}{\partial t^*} + v^*\frac{\partial \phi^*}{\partial y^*} = D\frac{\partial^2 \phi^*}{\partial y^{*2}} - k_r^{*2}(C^* - C^*_\infty) \quad (4)$$

The boundary conditions for the velocity and temperature fields are

$$Fort \leq 0 : u^* = 0, T^* = T^*_\infty, C^* = C^*_\infty, \quad \forall \quad y^*$$

$$Fort \geq 0 : u^* = U_0, T^* = T^*_w, C^* = C^*_w, \forall \quad y^* = 0$$
$$and \quad u^* \to 0, T^* \to T^*_\infty, T^* \to T^*_\infty, \quad as \quad y^* \to \infty \quad (5)$$

Where x and y are dimensions coordinates, u^* and v^* are dimensionless velocities, t^* is dimensionless time, T^* is the dimensional temperature, g- the acceleration due to gravity, β - the volumetric coefficient of thermal expansion, β^* is the volumetric coefficient of thermal expansion with concentration, ρ - the density of the fluid, C_p is the specific heat at constant pressure, D is the species diffusion coefficient, K^* is the permeability of the porous medium,k_r^{*2},is the chemical reaction, $Q0$ is the heat generation/absorption constant, B_0- magnetic induction, ν- the kinematic viscosity, α is the thermal diffusivity, U_o is the scale of free stream velocity, T^*_w- wall dimensional temperature, T^*_∞- the free stream temperature far away from the plate, $megat^*$ -the angular velocity.

where σ^* is the Stefan-Boltzmann constant. It should be noted that by using the Rosseland approximation the present analysis is limited to optically thick fluids. If temperature differences within the flow are sufficient small.

$$\frac{\partial q_r}{\partial y^*} = 4\left(T^* - T^*_\infty\right)\int_0^\infty K_{\lambda\zeta}\left(\frac{de_{b\lambda}}{dT^*}\right)d\lambda = 4I_1\left(T^* - T^*_\infty\right)$$

Where$K_{\lambda\zeta}$ is the absorption coefficient,$e_{b\lambda}$ is planck and the subscript ζ refers to values at the wall.

3 METHOD OF SOLUTION

In order to write the governing equations and the boundary conditions in dimensionless form,the following non-dimensional quantities are introduced

$$u = \frac{u^*}{U_o}, \quad y = \frac{U_o y^*}{\nu}, \quad t = \frac{U_o^2 t^*}{4\nu}, \quad n = \frac{\nu n^*}{U_o}, \quad \theta = \frac{T^* - T^*_\infty}{T^*_w - T^*_\infty},$$

$P_r = \frac{\nu\rho C_p}{k} = \frac{\nu}{\alpha}, \quad Gr = \frac{g\beta\nu(T_w - T_\infty)}{U_o^3}, \quad Gm = \frac{g\beta\nu(C_w - C_\infty)}{U_o^3}, \quad Ec = \frac{U_o^2}{C_p(T_w - T_\infty)}, \quad M = \frac{\sigma B_o^2 u\nu}{\rho U_o^2}, \quad \eta = \frac{\nu Q_o}{U_o^2 \rho C_p} \quad R = \frac{4I_1\nu}{kU_0^2} \quad Sc = \frac{\nu}{D}$ (6)

into governing equations(1)-(4) reduce to:

$$\frac{1}{4}\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + Gr\theta + Gm - \left(M + \frac{1}{k}\right)u(7)$$

$$\frac{1}{4}\frac{\partial \theta}{\partial t} = \frac{1}{Pr}\left(\frac{\partial^2 \theta}{\partial y^2} - (R^2 + \eta)\right)\theta \tag{8}$$

$$\frac{1}{4}\frac{\partial C}{\partial y} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} - k_r^2 \tag{9}$$

Where u and v are dimensionless velocities, t is dimensioT_w^*- wall dimensional temperature, T_∞^*- the free stream temperature far away from the plate, ω -the angular velocity, θ is dimensionless temperature function, U_0 is the scale of free stream velocity, R_e is the Reynolds number, Pr is prandtl number,U is velocity,n is the frequency, M is the Hartmann number,K is the permeability parameter, Gr is thermal Grashot number, η the heat source parameter,and Ec is Eckert number, A is a real positive constant of suction velocity parameter, $< \epsilon$, and $\epsilon A < 1$ are small less than unity, i.e $\epsilon A << 1$, V_0 is a scale of suction velocity normal to the plate
The boundary conditions(5) are given by the following dimensionless form.

$$u = 0, \theta = 0, C_0 = 0 \ \forall \quad y > 0, \ t \leq 0$$

$$\begin{aligned} &u = 1, \theta = 1, C = 1 \quad at \quad y = 0, \ t > 0 \\ &u = 0, \theta = 0, C = 0 \quad as \quad y^* \to \infty \end{aligned} \tag{10}$$

In order to reduce the above system of partial differential equations to a system of ordinary differential equations in dimensionless form, the velocities, momemtum, temperature,and mass are represented [17] as:

$$u(y,t) = u_0(y)e^{i\omega t}..., \tag{11}$$

$$\theta(y,t) = \theta_0(y)e^{i\omega t}..., \tag{12}$$

$$C(y,t) = \theta_0(y)e^{i\omega t}..., \tag{13}$$

On substituting equations(11)-(13) into equations(7)-(9) and neglecting the coefficient of like powers of ϵ, we get the following set of differential equations.

$$u_0'' - \left(M + \frac{1}{k} + \frac{i\omega}{4}\right)u_0 = -Gr\theta_0(y) - GmC_0(y) \tag{14}$$

$$\theta_0'' - \left(\frac{Pri\omega}{4} + (R^2 + F)\right)\theta_0 = 0 \tag{15}$$

$$C_0'' - \left(\frac{Sciw}{4} + k_r^2\right)C_0 = 0 \tag{16}$$

and the corresponding boundary conditions reduced to:

y=0: u=$e^{i\omega t}$, $\theta_0 = e^{i\omega t}$, $C_0 = e^{i\omega t}$
$y \to \infty$: $u = 0$, $\theta_0 = 0$, $C = 0$, (17)

The solutions of equations (14)-(16) subject to the boundary conditions (11)-(13) and (17) are respectively:

$$U_0 = e^{-i\omega t}((e^{-Ny}(1 - L_1) - L_2) + L_1 e^{-Ay} + L_2 e^{-Py}) \tag{18}$$

$$\theta_0 = e^{-i\omega t}e^{-Ay} \tag{19}$$

$$C_0 = e^{-i\omega t}e^{-Ny} \tag{20}$$

where

A $= \sqrt{\left(\frac{Pri\omega}{4} + (R^2 + F)\right)}$
N= $\sqrt{\left(M + \frac{1}{K} + \frac{i\omega}{4}\right)}$
P=$\sqrt{\left(Sc(\frac{i\omega}{4} + k_r^2)\right)}$
$L_1 = \frac{-G_r}{A^2 - \left(M + \frac{1}{K} + \frac{i\omega}{4}\right)}$

$L_2 = \frac{-G_m}{P^2 - \left(M + \frac{1}{K} + \frac{i\omega}{4}\right)}$

In view of the above solutions, the velocity, temperature and concentration distributions in the boundary layer become

$$u(y,t) = (e^{-Ny}(1 - L_1 - L_2) + L_1 e^{-Ay} + L_2 e^{-Py} \tag{21}$$

$$\theta(y,t) = e^{-Ay} \tag{22}$$

$$c(y,t) = e^{-Ny} \tag{23}$$

Skin-friction Coefficient is expressed as follows:

$$C_f = -\left(\frac{\partial u}{\partial y}\right)_{y=o} = N(1 - L_1 - L_2) + AL_1 + PL_2 \quad (24)$$

4 Results and Discusssions:

The formulation of the problem that accounts for the influence of permeability parameter and heat source dissipation on the flow of an incompressible,through a moving vertical porous plate with heat source in the presence of transverse magnetic field applied normal to the plate was accomplished.The governing equations of the flow problem were solved by a closed analytical form.The expressions for velocity, temperature, concentration and skin-friction number were obtained. In order to get physical insight of the problem, The above physical quantities are computed numerically for different values of the governing parameters viz., Permeability parameter K, Heat source η,Prandtl numberP_r, magnetic parameter M,Eckert number E_c,thermal Grashof number G_r, mass Grashof number G_m, chemical reaction k_r^2. The numerical calculations of these results are presented through graphs and tables. With convection that the real parts of complex quatities are involked for numerical discussion.
In order to assess the accuracy of this method, we have compared our results with accepted data for the velocity,temperature and concentration profile for the case of cooling of the porous plate as computed by Idowu et al.(Idowu et al. 2013). The result of these comparisons are found to be in very good agreement.

Fig. 1. We observed from fig.1. that as velocity profiles for different values of the permeability M. Clearly, as M increases the peak value of velocity tends to increase.

The effect of heat generation η on the velocity and temperature profiles are showing in Fig.10 and 11 respectively. From this figures it is clearly seen that an increase in heat generation leads to decrease in the velocity and temperature fields.

The velocity profiles for different values of thermal Grashof number Gr and solutal Grashof number Gm are described in fig.6 and 7. It is observed that an increasing in Gr or Gm leads to a rise in the values of velocity. Here the thermal Grashof number represent the effect of the free convection currents. Physically, $Gr > 0$ means heating of fluid of cooling of the boundary surface, $Gr < 0$ means cooling of the fluid of heating of the boundary surface and $Gr = 0$ corresponds to the absence of free convection current. In addition, the curves show that the peak value of velocity increases rapidly near the wall of the porous plate as Grshof number increases, and then decays to the relevant free stream velocity.

The velocity and temperature profiles across the boundary layer for different values of prandtl number Pr are plotted in fig.2 and 11. The numerical results shows that the effect of increasing values of prandtl number results in a decreasing fluid velocity and temperature. From figure 11, it is observes that an increase in the prandtl number results a decrease of thermal boundary layer thickness and in general lower average temperature within the boundary layer. The reason is that smaller values of prandtl numbers are equivalent to increase in the thermal conductivity of the fluid and therefore, heat is able to diffus away from the heated surface more rapidly for higher values of prandtl number as the thermal boundary layer is thicker and rate of heat transfer is reduced.

From figure 8. It is observed that an increased in the chemical reaction,contributes to the decrease in the velocity of the fluid medium.

Fig.3. represent the Schmidt number Sc. The effect of increasing values of Sc results in a decreasing velocity distribution across the boundary layer. The concentration across the boundary layer for various values of Schmidt number Sc. It is shown from fig.13. that an increasing in Sc result in a decreasing the concentration distribution, because the smaller values of Sc are equivalent the chemical molecular diffusivity.

The influence radiation on the velocity and temperature profiles across the boundary layer are presented in Fig.4 and 12. We see that the velocity and temperature distribution across the boundary layer decreases with increasing in radiation R.

The effect of frequency on velocity, temperature and concentration profiles across the bpundary layer are presented in Fig. 9, 14 and 15. We see that the velocity, temperature and concentration across the boundary layer decreases with increasing in frequency ω

Fig.16. When the Schmidt number is increase, the skin friction also increases.

In Fig.17. It is seen that as the prandtl number increased,the skin friction decreases.

Fig.18. As the magnetic field increases the skin friction reduces quite significantly.

5 CONCLUSIONS

In the present paper, an atempt is made to invesigate the effects of permeability parameter and heat source on an unsteady MHD two-dimensional free convection Heat transfer flow of a visous incompressible and dissipative fluid flow past a moving vertical porous plate with variable suction. The dimensionless governing equations are solved analytically by using perturbation method. The conclusions of the study are as follows:

1. The velocity decreases with an increase in the permeability parameter or Grashof number.
2. The velocity increases with an increase in the thermal Grashof number and mass Grashof number.
3. The velocity as well as temperature with an decrease in the Heat source,frequency and Prandtl number.
4. The velocity increases with an increase in the magnetic parameter.
5. The velocity decreases with an increase in the chemical reaction parameter.
6. The velocity decreases with an increase in the Schmidt number parameter.
7. The velocity decreases with an increase in the radiation parameter.
8. The skin-friction coefficient increases with an increase in the Permeability parameter,Heat source, Eckert number, Grashof number,Prandtl number and Magnetic parameter.
9. The Nusselt number increases with an increase in the Prandtl number and Eckert number.

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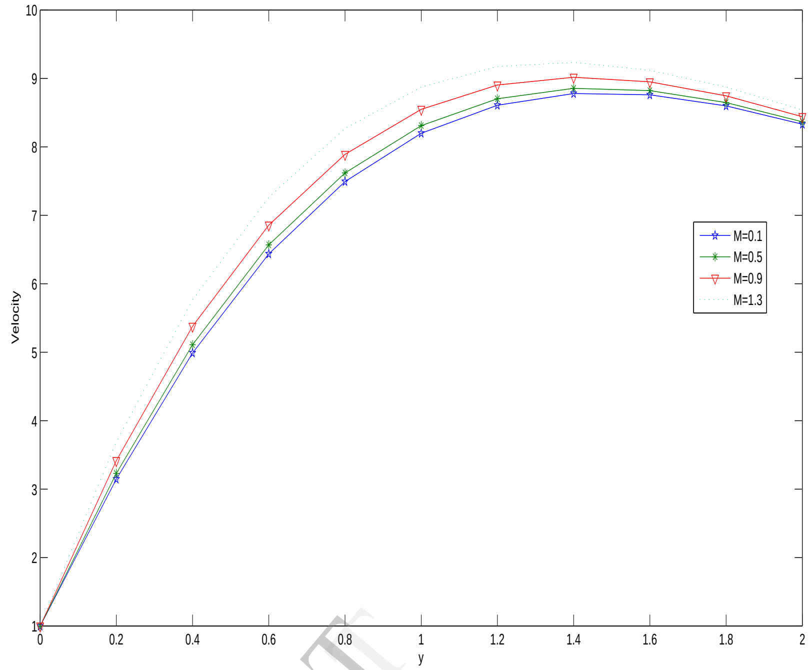

Figure 1: Effect of M on velocity

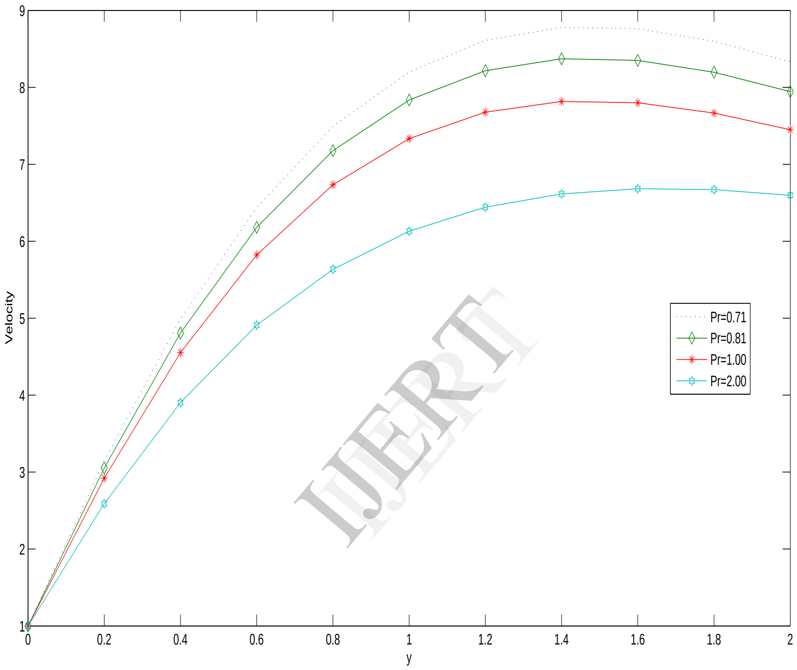

Figure 2: Effect of Pr on velocity

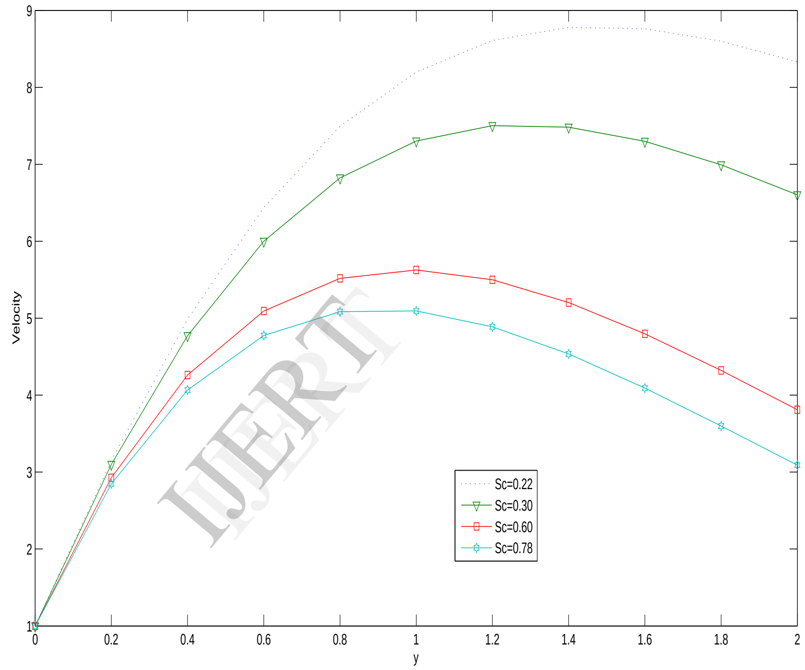

Figure 3: Effect of Sc on velocity

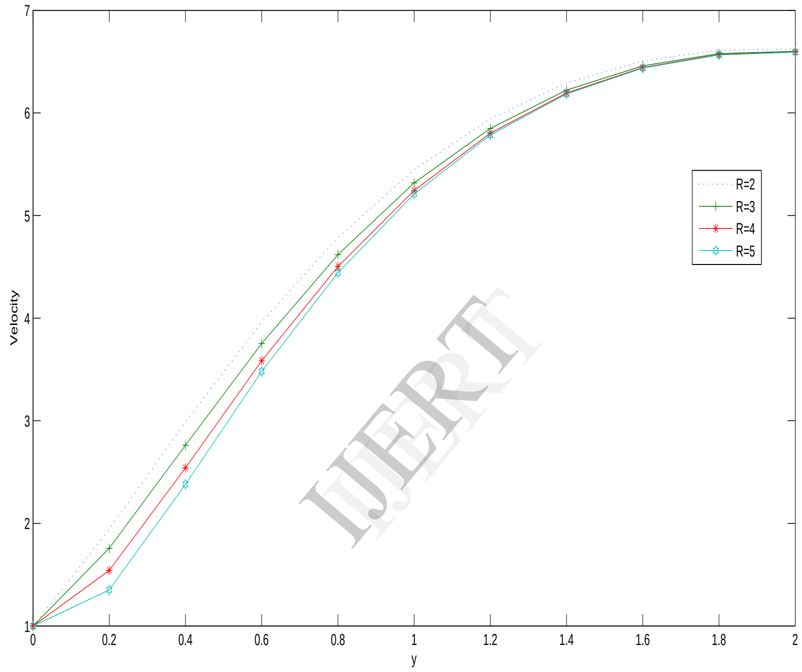

Figure 4: Effect of R on velocity

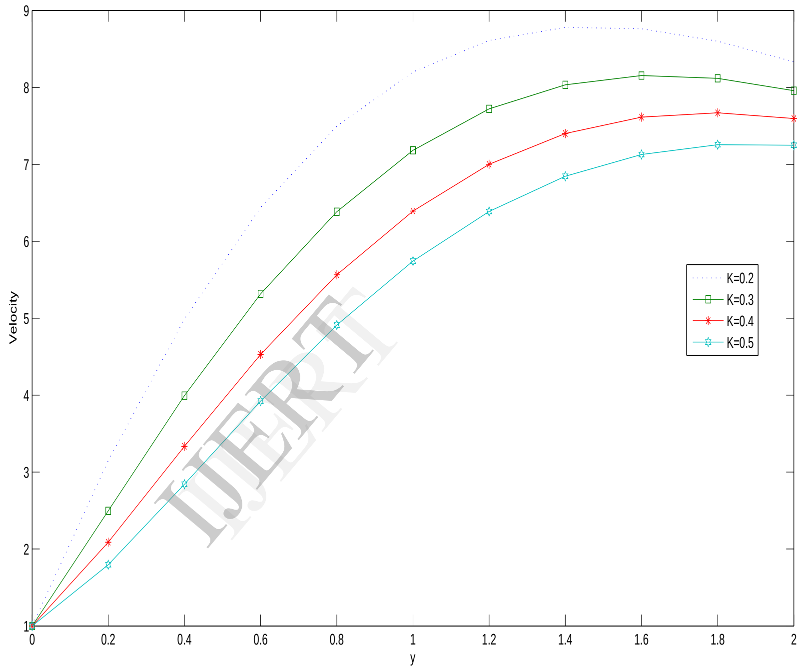

Figure 5: Effect of K on velocity

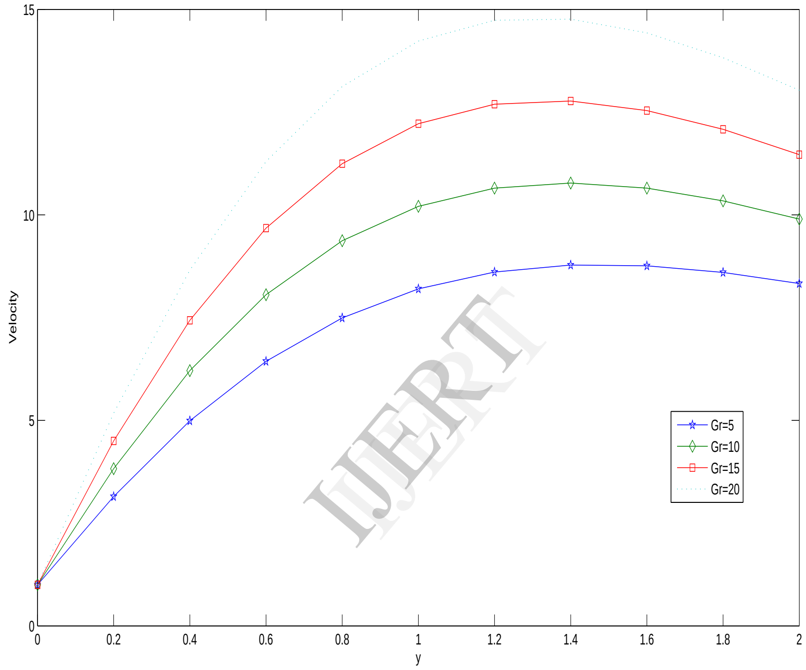

Figure 6: Effect of Gr on velocity

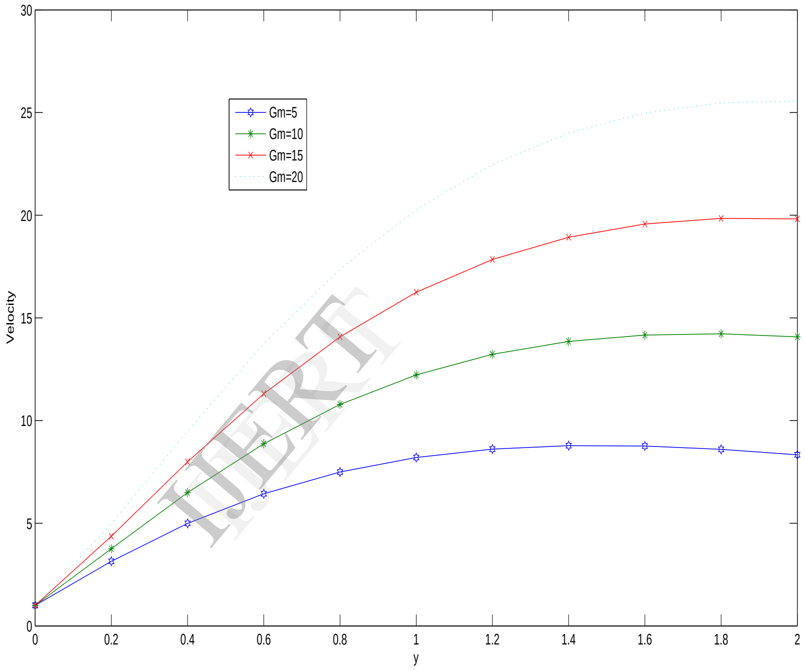

Figure 7: Effect of Gm on velocity

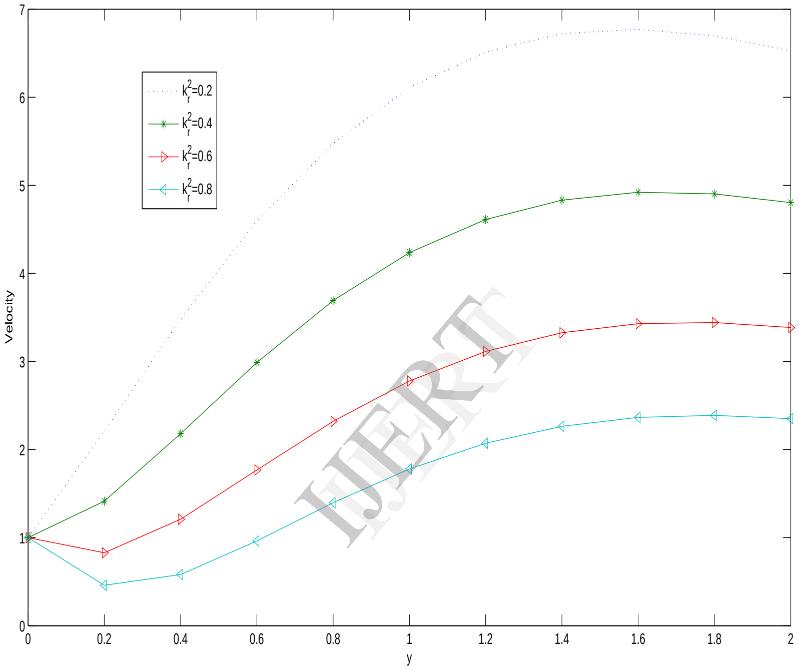

Figure 8: Effect of k_r^2 on velocity

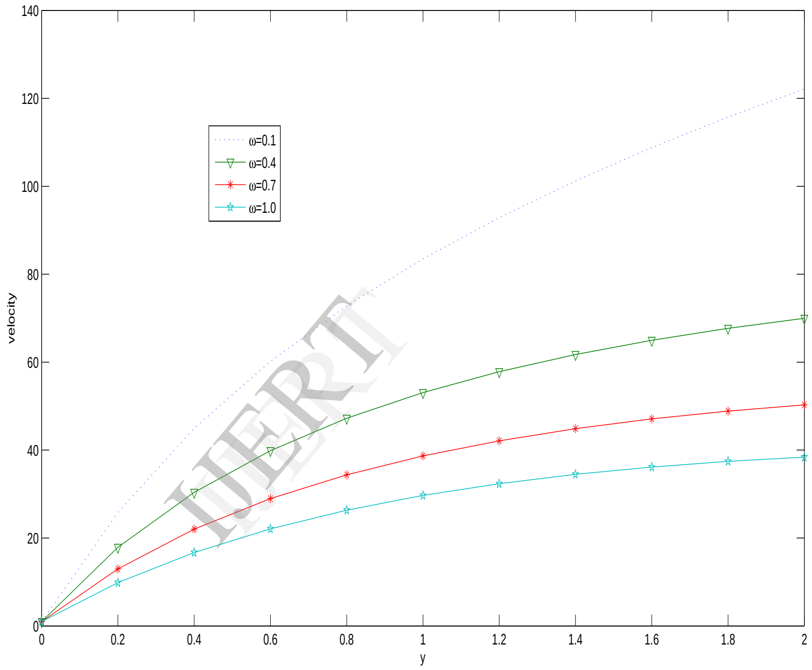

Figure 9: Effect of ω on velocity

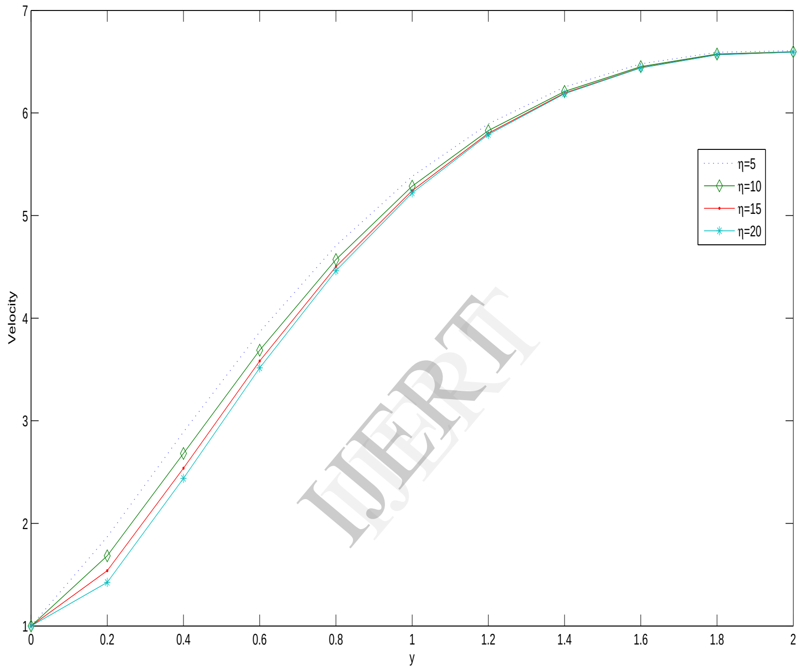

Figure 10: Effect of η on velocity

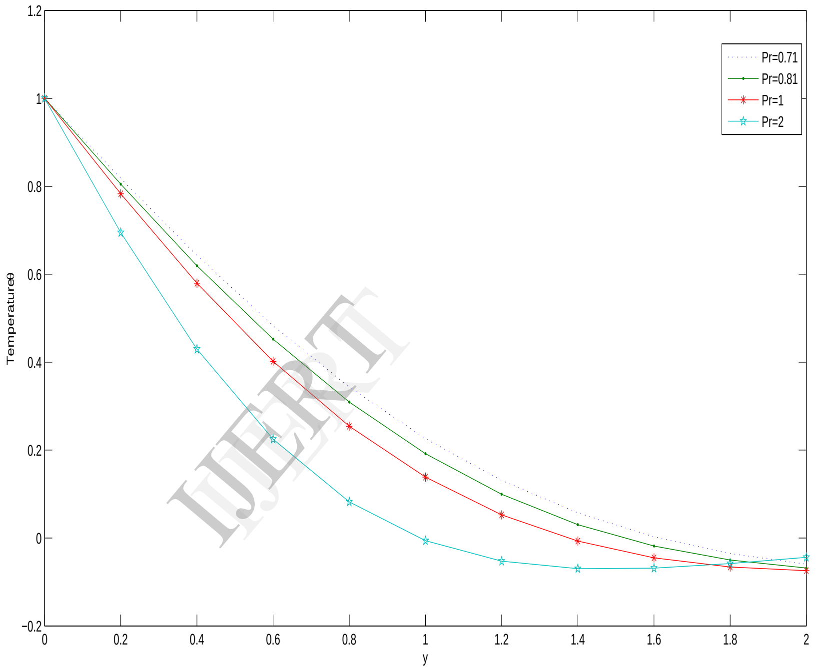

Figure 11: Effect of Pr on temperature

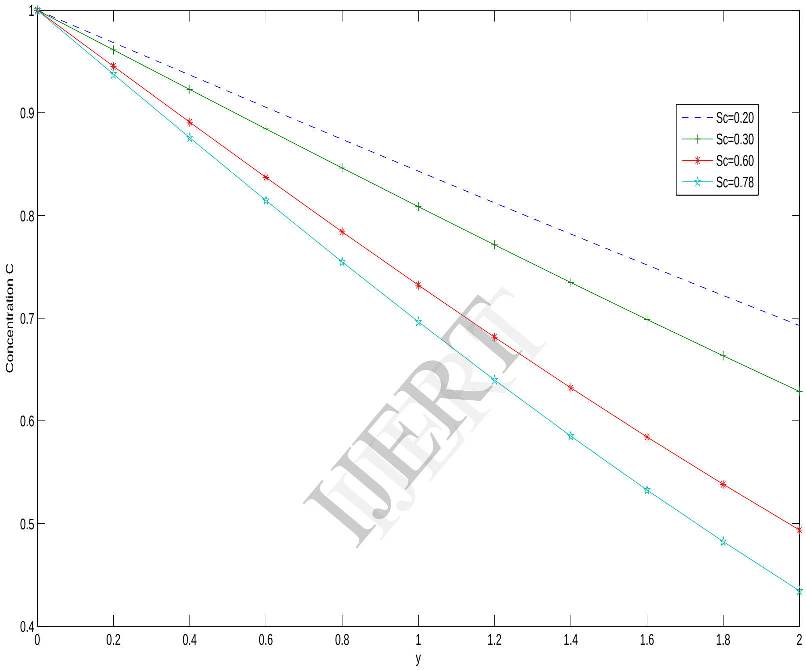

Figure 12: Effect of Sc on concentration

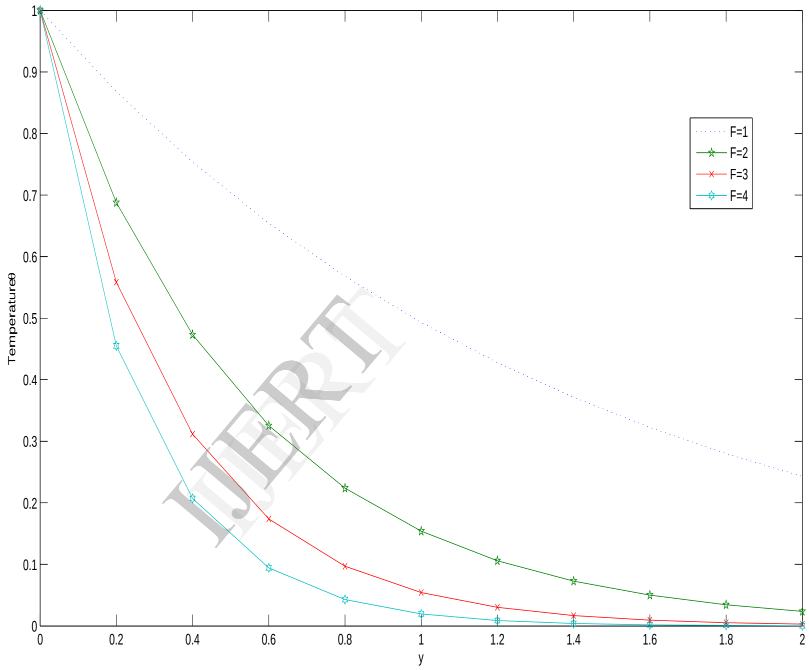

Figure 13: Effect of R on temperature

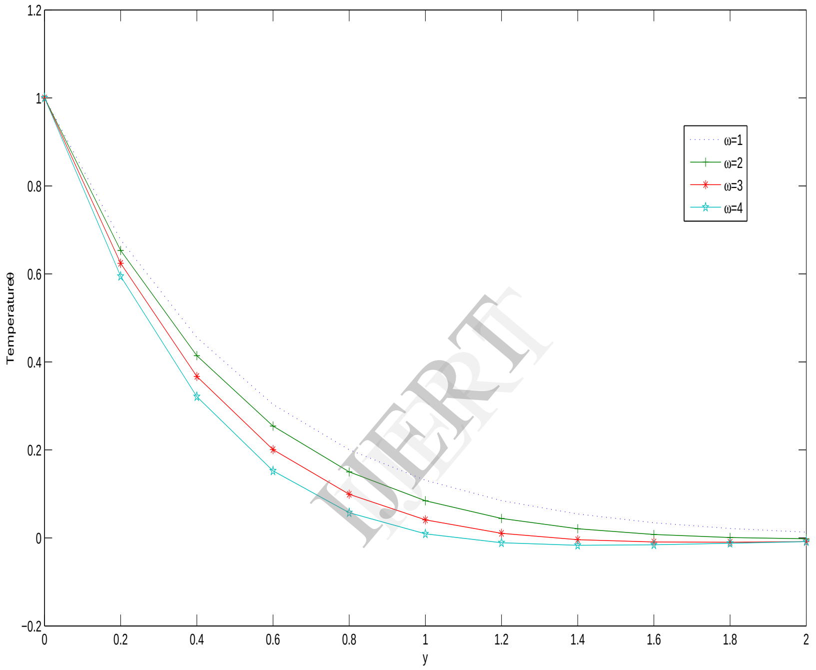

Figure 14: Effect of ω on temperature

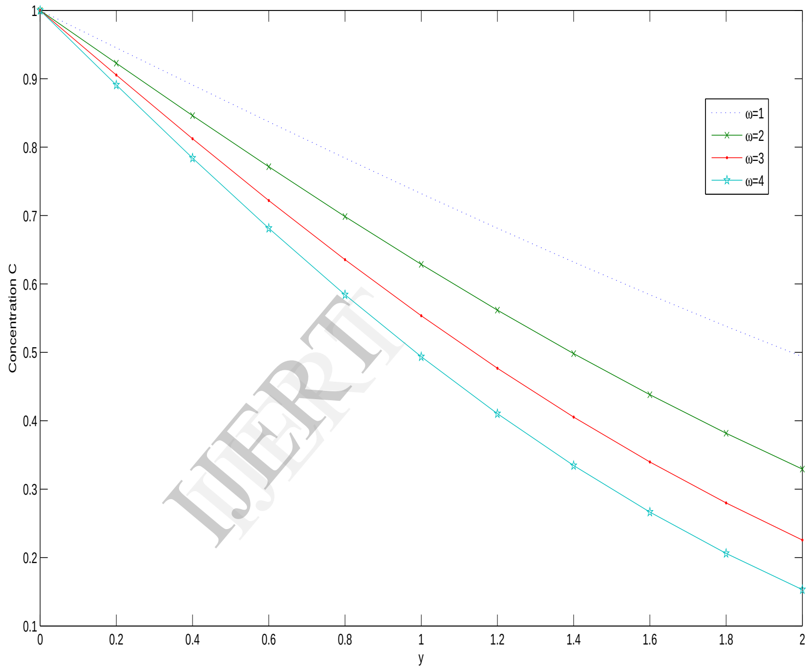

Figure 15: Effect of ω on concentration

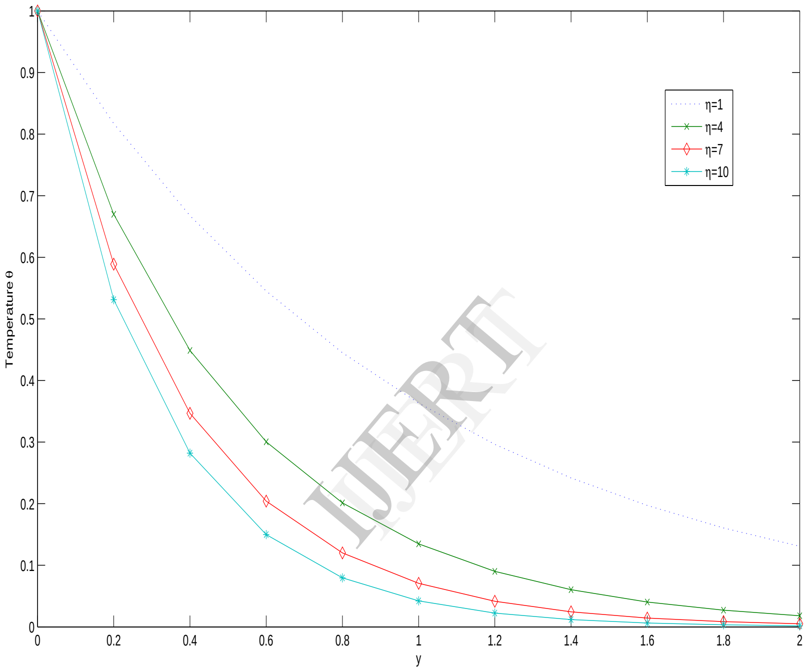

Figure 16: Effect of η on velocity

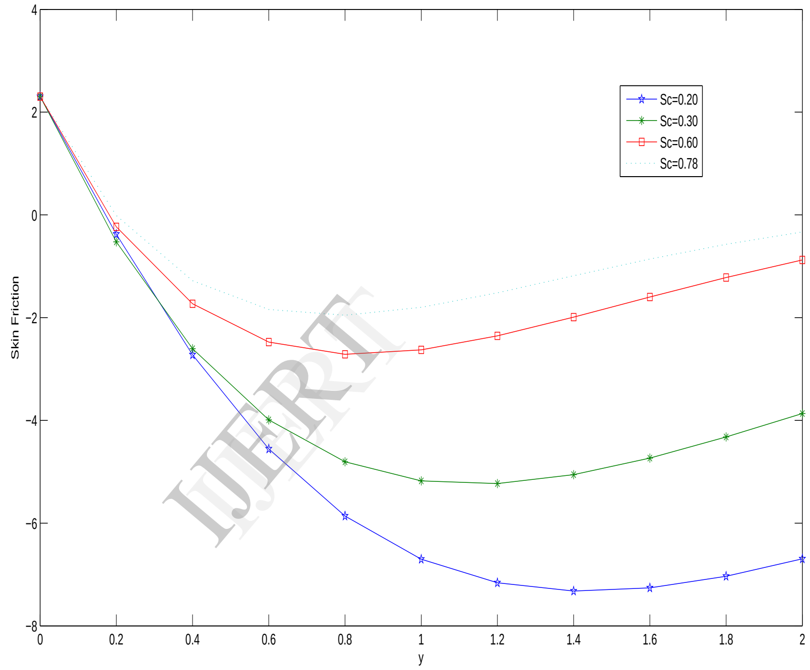

Figure 17: Effect of Sc on skin friction

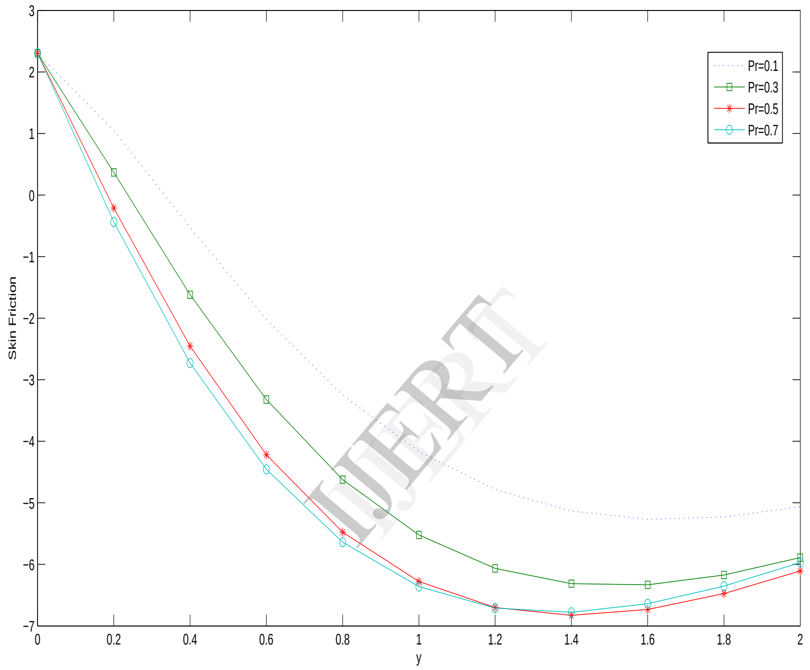

Figure 18: Effect of Pr on skin friction