International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 2 Issue 6, June - 2013

Effect of Bond Length, Bond Clearance And Torsional Stiffness of Adherends On Torque Transmission Capability of Adhesively Bonded Cylindrical Joint

Walame M V[1] , Ahuja B B[2]
[1,2]*Department of Production Engineering, College of Engineering, Pune-411005*

Abstract

The increasing demand for light weight, high quality and more cost effective product has led adhesive bonding to emerge as one of the primary ways of fastening structural members. Adhesive joints have been previously been designed empirically but now a day's data is available to design adhesive joints in an optimum way. The fixing of cylindrical components subjected to torque is a common requirement in industrial manufacture and as an alternative to keyed fastening of mechanical members there is a need to design and optimize the adhesively bonded cylindrical joint facilitating keyless fastening. The present paper aims at the development of analytical model for adhesively bonded cylindrical joint subjected to torsion loading for parametric study of the joint. The analysis is based on classical torsion theory and constitutive, equilibrium and compatibility equations of theory of elasticity are used to obtain stress field in the adhesive layer. The model developed is used to analyse the effect of geometrical and material parameters on the torque transmission capability of bonded cylindrical joint and validated experimentally. The equations are also derived for optimum bond length and optimum bond clearance for maximum torque transmission capability.

Keywords: *Bonded joint, Torsion, Analytical solution, Stress distribution, Optimization*

1. Introduction

In automotive applications intelligent lightweight constructions can only be obtained by consistently using a material mix of steel, light metal and plastics so-called multi-material design. In such situation as traditional joining techniques have their well-known limitations and adhesive bonding is only the competitive joining technique which is rapidly gaining acceptance amongst major manufacturers because of specific advantages over conventional mechanical fastening techniques.

In recent years, with the development of high strength adhesive materials and with the progress in techniques of adhesive bonding, various kinds of adhesive bonded joints are now being used in the manufacturing of light structures. Some typical examples of adhesively bonded cylindrical joints subjected to torsion loading are shaft to shaft, gear to shaft, rotor to shaft, fan to shaft, pulley to shaft etc.

Adhesive joints have previously been designed empirically. For successful industrial application of adhesive joints, reliable and easy to use methods of designing adhesive joints are required. Such methods would not only provide confidence in the use of adhesives, but would also enable improved and optimum joint designs.

The increased application of adhesive bonding was accompanied by development of mathematical and numerical methods to analyse and predict the behaviour of joints, but at present also this is still an open problem. An exact solution for the stress field of these joints is difficult to obtain due to geometric complexities and material nonlinearity of the adhesive. This has resulted in a wide range of solutions developed each with different assumptions and simplifications.

2. Literature Survey

D. Chen and S. Cheng [1] analysed the stress distribution in adhesive bonded tubular lap joints subjected to torsion. The analysis was based on the elasticity theory in conjunction with variational principle of complimentary energy, with two adherends may be having different materials and different thickness. The closed form solution so obtained was used to determine the stress intensities in adhesive layer and stress concentration factor.

Choon T. Chon [2] analysed the stress distribution of tubular lap joint in torsion whose adherends were of composite materials and obtained a closed form solution. The stress concentrations at and near the end was studied as function of various parameters such as wrap angles, overlap length and thickness of adhesive layer.

N. Pugno and G. Surace [3] analysed problem of torsion in adhesive bonded tubular joint. The stress field in the adhesive layer was obtained based on the elasticity theory and pure torsion theory. A special type of tubular joint was proposed by optimizing the tubular joint for torsional strength.

Kozo Ikegami [4] investigated the strength of adhesive shaft joint under combined axial tensile and torsional load analytically and experimentally to study effect of overlap length and cross sectional ratio. The joint considered was consisting of two steel shafts and a coupling bonded with adhesive. The stress strain distributions were computed by the elastic Finite Element Method with the assumption of symmetric three dimensional conditions. For strength evaluation Von-Mises conditions were applied to the outer and inner adherends and to the adhesive layer.

Tezcan Sekercioglu, Alper Gulsoz, Hikmat Rende [5] investigated experimentally effect of different surface roughness values on bonding strength for both static and dynamic loading conditions. For experimentation adherends of structural steel and anaerobic adhesive Loctite 638 were used.

Tezcan Sekercioglu [6] investigated experimentally the effect of parameters namely interference fit, bonding clearance, surface roughness, adherend materials and temperature on bonding strength of adhesive. For shear strength estimation of adhesively bonded tubular joints the nonlinear models (GASSEM) were developed using Genetic Algorithm (GA) approach.

The present paper aims at the development of analytical model for adhesively bonded cylindrical joint subjected to torsion loading for parametric study and optimization of the joint parameters for maximum torsional strength.

3. Analytical Solution

The following assumptions are made in the current study-

- Two shafts and adhesive layer forming the cylindrical joint are governed by an isotropic linear elastic law.
- Deformations considered are very small and classical Torsion theory is used.
- The thickness of adhesive layer is very small as compared to the dimensions of shaft and the variation of stress across the thickness of adhesive layer is neglected.
- The torsion load carried by thin adhesive layer is ignored. The external torsion load is assumed to be resisted by shafts only.

Consider two shafts bonded adhesively by a thin adhesive layer with 2*c* as bond length, as shown in fig.(1) subjected to torsion moment T.

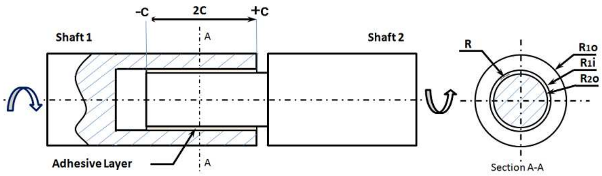

Figure 1: Shafts bonded adhesively by thin adhesive layer

Torque transfer takes place gradually from shaft 1 to shaft 2 and at any cross section torque T_1 and torque T_2 are produced in shaft 1 and shaft 2 respectively. The external torsion load is assumed to be resisted by shafts only and hence the sum of the moments absorbed by the two shafts must be equivalent to the applied torsional moment T for every cross section. Here torsion load carried by thin adhesive layer is ignored.

$$T = T_1(z) + T_2(z) \qquad [1]$$

The torsional moment *Ti* (*z*) at any section *z* of shaft *i* can be expressed as function of z.

$$T_1(z) = T * f(z) \qquad [2]$$

$$T_2(z) = T * (1 - f(z)) \qquad [3]$$

With Boundary conditions

$$\begin{aligned} T_1(-c) &= T \quad , \quad T_1(+c) = 0 \\ T_2(-c) &= 0 \quad , \quad T_2(+c) = T \end{aligned} \qquad [4]$$

At any cross section if the rotations of shaft 1 and shaft 2 are identical then there is no relative displacement between them at that cross section. But if at any cross section if the rotations of shaft 1 and shaft 2 are different from each other, a relative rotation occurs resulting in circumferential relative displacement at the bond layer.

$$\theta(z) = \theta_2(z) - \theta_1(z) \qquad [5]$$

From the Torsional moments absorbed by the two shafts at the joint, the rotations $\theta_1(z)$ and $\theta_2(z)$ of the cross sections of shaft 1 and shaft 2 respectively can be obtained from the compatibility and the circumferential relative displacements at the bond layer can be written as –

$$\theta(z) = \int_{-c}^{z} \frac{T_2(z)}{G_2 J_2}\, dz - \int_{-c}^{z} \frac{T_1(z)}{G_1 J_1}\, dz + \Delta\theta_i \qquad [6]$$

Where G_i = Shear modulus of shaft i , J_i = polar moment of inertia of shaft i and $\Delta\theta_i$ is the difference in the absolute rotations of shaft 1 and shaft 2 at the initial section $(z = -c)$.

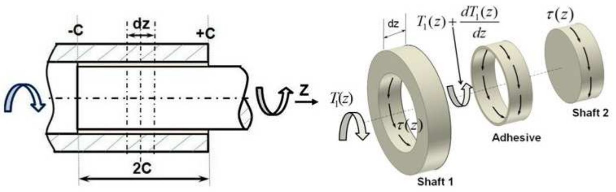

Figure 2: Bonded cylindrical joint under torsion

Consider an differential element of length dz $(-c \le z \le +c)$ belonging to the shaft 1 as shown in fig.(2) and imposing rotational equilibrium the stress field in the adhesive layer equivalent to applied torsional moment can be written as –

$$\tau(z) = \frac{-1}{2\pi R^2}\frac{dT_1(z)}{dz} \qquad [7]$$

Where R is the mean radius of adhesive surface and $T_1(z)$ is torsional moment in shaft 1 at section 'z'.

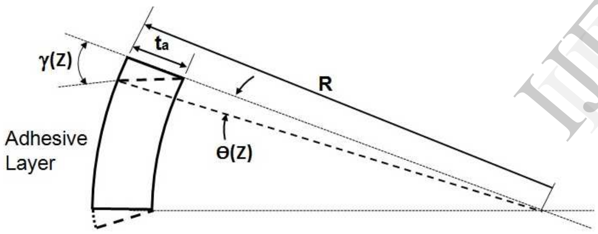

Fig. 3: Shearing strain in adhesive layer

The strain field γ (z) in the adhesive layer can be obtained from the stress field equation (7).

$$\gamma(z) = \frac{-1}{2\pi R^2 G_a}\frac{dT_1(z)}{dz} = \frac{R\ \theta(z)}{t_a} \qquad [8]$$

Where t_a is adhesive layer thickness and G_a Shear modulus of adhesive.

Substituting values of $\theta(z)$, $\tau(z)$ from equation (6) and (7), the above equation can be written as-

$$\frac{dT_1(z)}{dz} = \frac{-2\pi R^3 G_a}{t_a}\left(\int_{-c}^{z}\frac{T_2(z)}{G_2J_2}dz - \int_{-c}^{z}\frac{T_1(z)}{G_1J_1}dz + \Delta\theta_i\right) \qquad [10]$$

Differentiating equation (10) w.r.t. z and Substituting values of $T_1(z)$, $T_2(z)$ from equations (2) and (3) the following equation can be written

$$\frac{d^2 f(z)}{dz^2} - K\left(\frac{G_1J_1 + G_2J_2}{G_1J_1G_2J_2}\right) f(z) = \frac{-K}{G_2J_2} \qquad [11]$$

Where $K = \dfrac{2\pi R^3 G_a}{t_a}$

The solution of equation (11) may be considered to be of the form

$$f(z) = C_1 Cosh(Az) + C_2 Sinh(Az) + \beta \qquad [12]$$

By applying boundary conditions from equation (4), the two unknown coefficients in the general solution can be derived as-

$$C_1 = \frac{1-2\beta}{2Cosh(Ac)} \quad , \quad C_2 = \frac{-1}{2Sinh(Ac)} \qquad [13]$$

Where

$$A = \sqrt{\left(\frac{G_1J_1 + G_2J_2}{G_1J_1G_2J_2}\right)K} \quad , \quad \beta = \frac{G_1J_1}{G_1J_1 + G_2J_2} \qquad [14]$$

Substituting value of f(z) from equation (12) in to equations (2) and (3) the variation of torque in shaft 1 and shaft 2 along the bond length can be obtained as –

$$T_1(z) = T*(C_1 Cosh(Az) + C_2 Sinh(Az) + \beta) \qquad [15]$$

$$T_2(z) = T*\{1-(C_1 Cosh(Az) + C_2 Sinh(Az) + \beta)\} \qquad [16]$$

Differentiating equation (15) and substituting the value in the equation (7) and after rearrangement the equation for stress field in adhesive layer can be obtained as

$$\tau(z) = \frac{-T\,\alpha\, R\, G_a}{A t_a}\left(C_1 Sinh(Az) + C_2 Cosh(Az)\right) \qquad [17]$$

$$where \quad \alpha = \left(\frac{G_1J_1 + G_2J_2}{G_1J_1G_2J_2}\right)$$

4. Torque transmission capability

Torque transmission capability of adhesively bonded cylindrical joint can be obtained by rearranging equation (17) for torsional moment

$$T = -\frac{\tau_a\, A t_a}{\alpha\, R\, G_a}\left(C_1 Sinh(Az) + C_2\, Cosh(Az)\right)^{-1} \qquad [18]$$

From equation (17) it is observed that the maximum interface shear stress occurs at the ends of the joint then the maximum or failure torque for the joint can be obtained by substituting value of shear strength of adhesive (τ_a) and value of z as +c or –c .

$$T_{max} = -\frac{\tau_a\, A t_a}{\alpha\, R\, G_a}\left(C_1 Sinh(\pm Ac) + C_2\, Cosh(\pm Ac)\right)^{-1} \qquad [19]$$

5. Optimum Bond Length ($c_{optimum}$)

The theoretical maximum torque transmission capacity $(T_{max})_{th}$ of the adhesively bonded shafts occur when bond length tends to infinity (i.e. $2c\rightarrow\infty$)

$$(T_{max})_{th} = \lim_{c\rightarrow\infty} \frac{\tau_a A t_a}{\alpha R G_a}\left(\frac{-1}{C_1 Sinh(Ac) + C_2 Cosh(Ac)}\right) \quad [20]$$

Inserting values of C_1, C_2 from equation (13) and solving the equation $(T_{max})_{th}$ can be obtained as -

$$(T_{max})_{th} = \frac{\tau_a A t_a}{\alpha \beta R G_a} \quad [21]$$

The optimum bond length can be defined as the length of the bond at which 99% of the theoretical maximum torque transmission capacity $(T_{max})_{th}$ is achieved.

Considering the case $G_1J_1>G_2J_2$, where the maximum shear stress occurs at z=+c and from equations (19) and (21) the following equation can be written as –

$$\frac{\tau_a A t_a}{\alpha R G_a}\left(\frac{-1}{C_1 Sinh(Ac) + C_2 Cosh(Ac)}\right) = \frac{0.99\,\tau_a A t_a}{\alpha \beta R G_a} \quad [22]$$

Inserting values of C_1, C_2 from equation (13) and solving for optimum bond length ($c_{optimum}$)

$$c_{optimum} = \frac{1}{A} Tanh^{-1}\left(\frac{-2\beta \pm \sqrt{(2\beta)^2 + (3.9204)*(1-2\beta)}}{(1.98)*(1-2\beta)}\right) \quad [23]$$

6. Optimum Bond Clearance $(t_a)_{optimum}$

Considering the case $G_1J_1>G_2J_2$, where the maximum shear stress occurs at z=+c, from equation (19) maximum torque transmission capacity can be written as-

$$T_{max} = -\frac{\tau_a A t_a}{\alpha R G_a}\left(C_1 Sinh(Ac) + C_2 Cosh(Ac)\right)^{-1} \quad [24]$$

Inserting values of C_1, C_2 from equation (13) and rearranging the equation (24)

$$T_{max} = -\sqrt{\frac{2\pi R \tau_a^2}{G_a \alpha}}\;\frac{\left(\sqrt{t_a}\right) Tanh\left(c\sqrt{\frac{2\pi R^3 G_a \alpha}{t_a}}\right)}{\left(\left(\frac{1-2\beta}{2}\right) Tanh^2\left(c\sqrt{\frac{2\pi R^3 G_a \alpha}{t_a}}\right) - \frac{1}{2}\right)} \quad [25]$$

The above problem is of maximization and by differentiating equation (25) w.r.t. bond clearance (t_a) and equating to zero we can get the optimum value of bond clearance (t_a) at which torque transmission capacity is maximum. Numerical differentiation technique or graphical method can be adopted for solving above equation.

7. Effect of variation of GJ ratio

Variation of shear stress in adhesive layer

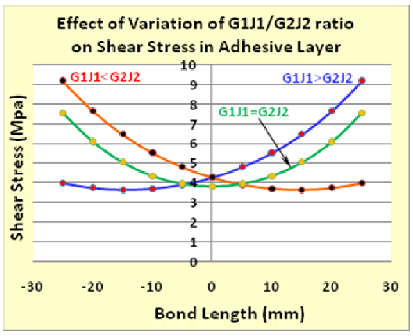

Fig. 4: Effect of variation of G_1J_1/G_2J_2 ratio's on variation of shear stress in adhesive layer

From the graph of Shear stress in adhesive layer Vs Bond length obtained in fig.4 it is observed that

- The maximum interface shear stress occurs at the ends of the joint.
- Depending upon the ratio of G_1J_1 /G_2J_2 in the bond area the maximum interface shear stress can occur at right end, left end or simultaneously at both ends.
- The maximum interface shear stress occurs at the end of stiffer shaft.
- The maximum value of shear stress in the case of $G_1J_1=G_2J_2$ is found to be minimum as compared to maximum values of shear stresses in the cases $G1J1>G_2J_2$ and $G_1J_1 < G_2J_2$.

8. Stress Concentration Factor (SCF)

- From Fig. 5 it can be observed that Stress concentration factor (SCF) is minimum when $G_2J_2 = G_1J_1$.
- Shear stress in adhesive layer depends upon the rate of change of torque in shaft 1 and shaft 2 along bond length and torque variation is steeper at the end of stiffer shaft when $G_1J_1 \neq G_2J_2$ as compared to rate of change of torque in case of $G_2J_2 = G_1J_1$. And hence the value of SCF is higher when $G_1J_1 \neq G_2J_2$.

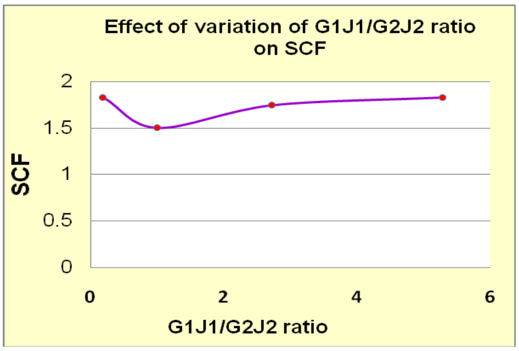

Fig. 5: Effect of variation of G_1J_1 /G_2J_2 ratio on Stress Concentration Factor

9. Experimental Investigations

Specimen geometries used to investigate the failure torque of adhesively bonded shafts is as shown in figure (6).

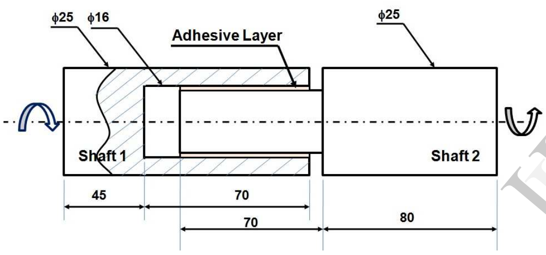

Fig. 6: Specimen dimensions

Materials MS (IS 226), Aluminium (IS H30) and their combinations were used for shafts with adhesive Loctite 620 and Loctite 638. The properties of the adhesives from the manufacturer's catalogue are as given below-
Loctite 620 : Shear Strength as per ISO 10123 : Steel pins and collars ≥17.20 N/mm^2
Loctite 638 : Shear Strength as per ISO 10123 : Steel pins and collars ≥ 13.5 N/mm^2
Both adhesives are single component Anaerobic Adhesives which cure at room temperature when deprived of contact with oxygen. The adhesive was cured at room temperature for minimum 24 Hrs. for steel shaft and minimum 72 Hrs. for Aluminium shaft as recommended in manufacturers catalogue. A 'V' block arrangement was used to ensure alignment of shafts prior to adhesive curing. The adhesive used is thixotropic in nature which ensured uniform spreading of adhesive and no spilling or leakage.

The failure torque for the adhesive joint was measured by using KISTLER Multicomponent Dynamometer (Type 9257B) capable of measuring 6-Components of Forces and Moments Fx, Fy, Fz, Mx, My, Mz . Experimentation was carried out for five clearances and five bond lengths with the shaft materials and adhesives mentioned above.

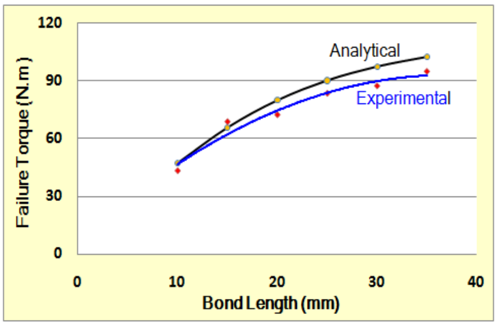

Fig. 7: Analytical Calculations and Experimental values of failure torque for Steel-Steel shafts with Bond Clearance 0.11mm constant and LOCTITE 620 Adhesive

- As the bond length increases % error between experimental and analytical values increases this may be because of the reason that with the increase in length the number of defects also increases which reduces the torque transmission capability.
- Another reason for difference in experimental and analytical values may be because that the adhesive layer is more likely to exhibit nonlinear behaviour where as in the analytical solution linear behaviour is assumed.
- Both experimental and analytical values obtained indicate that as bond length increases failure torque increases but after certain length it remains almost constant and hence increasing bond length beyond this limit will not be beneficial in torque transmission capacity of the joint.

4. Conclusions

i. The maximum interface shear stress occurs at the ends of the joint and occurs at the end of shaft having higher torsional stiffness.
ii. As the bond length increases the maximum shear stress induced in adhesive layer decreases and torque

transmission capability increases. But when bond length **($c_{optimum}$)** is reached, the stress remains almost constant and maximum torque transmission capacity is achieved.

iii. As the bond clearance increases the maximum shear stress induced in adhesive layer decreases and torque transmission capability increases. But when bond clearance **($(t_a)_{optimum}$)** is reached, the stress remains almost constant and maximum torque transmission capacity is achieved.

iv. Stress concentration factor (SCF) is minimum when torsional stiffness of both shafts is same in bond area ($G_2J_2 = G_1J_1$) and maximum torque transmission capacity is achieved.

v. Experimental and analytical values for failure torque are in good correlation and although the derivations are based on isotropic materials a better perspective of the response of actual joint can be obtained by understanding how different parameters affect the torsion load capacity of shaft joint.

References

[1] D. Chen and S. Cheng. " Torsional Stress in Tubular Lap Joint". *Journal of Solids Structures, 29(7):*845–853, 1992

[2] Choon T Chon. "Analysis of tubular joint in torsion". *Journal of Composite Materials*, 16:268, 1982.

[3] N.Pugno and G. Surace. "Tubular bonded joint under torsion: theoretical analysis and optimization for uniform torsional strength". *Journal of Strain Analysis*, 36(1), 2001

[4] Kozo Ikegami. "Tensile and torsional strength of metal shaft joint connected adhesively with cylindrical coupling". *22nd DANUBIA-ADRIA Symposium on Experimental Methods in Solid Mechanics*, 2005.

[5] Tezcan Sekercioglu, Hikmat Rende, Alper Gulsoz, Cemal Meran "The effects of surface roughness on the strength of adhesively bonded cylindrical components". *Journal of Materials Processing Technology* (142) :82–86, 2003.

[6] Tezcan Sekercioglu. "Shear strength estimation of adhesively bonded cylindrical components under static loading using the genetic algorithm approach". *International Journal of Adhesion and Adhesives*, (25):352–357, 2005

[7] Zhenyu Ouyang and Guoqiang Li. "Interfacial debonding of pipe joints under torsion loads:a model for arbitrary nonlinear cohesive laws". *International Journal of Fracture*,(155):19–31, 2009.

[8] Hiroshi Kawamuraa and Toshiyuki Sawab. "Effect of fitted position on stress distribution and strength of a bonded shrink fitted joint subjected to torsion". *International Journal of Adhesion and Adhesives*, (23):131–140, 2003.

[9] O. Nemes and F.Lachaud. "Contribution to the study of cylindrical adhesive joining". *International Journal of Adhesion and Adhesives*, 26:474–480, 2006.

[10] Je Hoon Oh. "Nonlinear analysis of adhesively bonded tubular single-lap joints for composites in torsion". *Composites Science and Technology*, 2006.

[11] Avinash Parashar, Pierre Mertiny. "Adhesively bonded composite tubular joints: Review". *International Journal of Adhesion & Adhesives* (38): 58–68, 2012

[12] M.M. Abdel Wahab "Fatigue in Adhesively Bonded Joints: A Review". *International Scholarly Research Network , ISRN Material science, Article ID* 746308, 2012