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Effect of body acceleration on pulsatile flow of Herschel –Bulkley fluid through an inclined mild stenosed artery

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"Abstract"

The pulsatile flow of Herschel-Bulkley fluid through an inclined stenosed artery under the influence of body acceleration is studied. The non linear equations governing the flow are solved using perturbation method. The effects of pulsatility, inclination of artery and body acceleration of blood on velocity, flow rate, wall shear stress are discussed. It is noticed that the effect of the stenosis is to reduce flow rate. The impact of body acceleration and inclination of the artery is to enhance the velocity of the blood flow.

Key words : Pulsatile flow, Body acceleration, Herschel-Bulkley fluid, Stenosed artery.

1.Introduction.

Human body many a times get disturbed by external accelerations .Due to this, in the long run, health problems such as loss of vision, head ache , increase in pulse rate causes, on account of disturbances in blood flow [Burton et al ,1974[3];Hialt ct al .,1969[20]]. Therefore , for long and short term exposures of human body to such acceleration, it is desirable to set a standard..If the response of the human body to such accelerations is understood properly, the controlled accelerations can be used for therapeutic treatments, development of new diagnostic tools and for better designing of protective pads (Amtzenius et al,1972[1];Verdouw et al.,1973[33]).

There are many evidences that vascular fluid dynamics plays a major role in the development and progression of arterial stenosis. It is quite common to find arteries are narrowed by the development of atherosclerotic plaques that protrude into the lumen, resulting arterial stenosis. When an obstruction developed in an artery, one of the most serious consequences is the increased resistance and the associated reduction of the blood flow to the particular vascular bed supplied by the artery. Thus the presence of a stenosis leads to the serious circulatory disorder. A knowledge of the flow characteristics in the vicinity of a stenosis may help to further understand some major complications which can arise such as , an in- growth of tissue in the artery, the development of a coronary thrombosis, the weakening and bulging of the artery downstream from the stenosis, etc. Investigation of the role of hydrodynamic factors in the development of the above complications provides relevance to the analysis of flow through a modeled arterial stenosis.

Due to physiological importance of body acceleration many theoretical investigations have been carried out for the flow of blood under the influence of body acceleration with and without stenosis. Sud and Sekhon (1985)[34] studied the pulsatile flow of blood through a rigid circular tube subject to body acceleration, treating blood as a Newtonian fluid. Misra and Sahu (1988)[24] analyzed the flow of blood through large arteries under the action of periodic body acceleration. Belardinelli et al.(1989)[2] proposed mathematical models for various forms of body acceleration. Usha and Prema (1999) [32] studied the pulsatile flow of particle-fluid suspension model of blood under the presence of periodic body acceleration. Using Laplace and Henkel tansforms Elshehawey et al. (2000)[16] studied the effect of body acceleration on pulsatile flow of blood through a porous medium by treating blood as a Newtonian fluid .Later El-shahed (2003)[15] extended this study for a stenosed porous medium.

In all these investigations blood is modeled as a Newtonian fluid. It is reported that the rheological properties of blood and its flow behavior through tubes of varying cross section play an important role in understanding the diagnosis and treatment of many cardiovascular diseases (Fry, 1968[18], Dimenfass, 1977[11];Caro,1981[4]).It is well known that blood being a suspension of cells, behaves as a non Newtonian fluid at low shear rates and during its flow through small blood vessels, especially in diseased states when clotting effects in small arteries are present. Experiments conducted on blood (Scott Blair,1959[19];Cokelet et al.,1965[6]) with varying heamatocrit, anticoagulants, temperature, etc suggested that the behavior of blood at low shear rates can be best described by Casson model(Charm and Kurland ,1965[5]; Merill and Pelletier,1967[23]).Chaturani and Palaniswamy (1990a:1990b)[8],[9] analyzed the pulsatile flow of blood under the influence of periodic body acceleration by assuming blood as a Casson fluid and also a power law fluid by using finite difference scheme. Sarojamma and Nagarani (2002)[27] studied the flow of a Casson fluid in a tube filled with porous medium under periodic body acceleration with application to artificial organs. Mandal et al,.(2007)[22] developed a two dimensional mathematical model to study the effect of extremely imposed periodic body acceleration on non Newtonian blood flow through an elastic stenosed artery where the blood is characterized by the generalized power-law model. P.Nagarani and G.Sarojamma (2008)[28] analyzed the effect of body acceleration on pulsatile flow of Casson fluid through a mild stenosed artery. D.S.Sanker and Ahmad Izani Md.Ismail (2009)[12] did a comparative study on Two-fluid mathematical models for blood flow in stenosed arteries by considering Casson as well as Herschel-Bulkley fluids.

In view of the above, a mathematical model is developed to study the pulsatile flow behavior of blood in an inclined artery under stenotic condition subject to both the pulsatile pressure gradient due to normal heart action and of periodic body acceleration . Blood is modeled as a Herschel-Bulkley fluid by properly accounting for yield stress of blood .The combined effect of pulsatality, stenosis, body accelertion, inclination of the artery on the flow parameters is illustrated graphically in results and discussion section.

2.Mathematical formulation

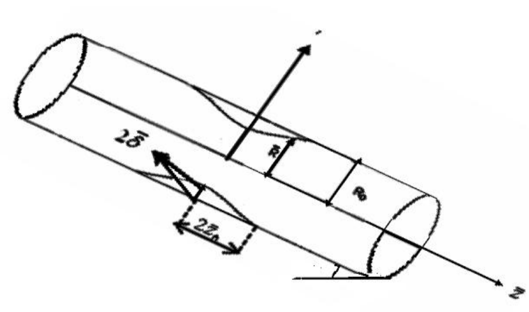

Fig 1.Geometry of the inclined stenosed artery

Consider the pulsatile flow of blood in presence of externally imposed periodic body acceleration in an inclined artery with mild stenosis. We consider the flow is axially symmetric, laminar, fully developed where the flowing blood is modeled as a Herschell-bulkley fluid .Following Young (1968) , the stenotic protuberance is assumed to be an axisymmetric surface generated by a cosine curve. The geometry of the stenosis is as shown in the figure 1 and is given by

$$\bar{R}(\bar{z}) = \begin{cases} R_0 - \bar{\delta}\left(1+\cos\dfrac{\pi \bar{z}}{2z_0}\right) & for \quad \bar{z}=-2z_0 \quad to \quad \bar{z}=2z_0 \\ R_0 & otherwise \end{cases} \quad (1)$$

where $4\bar{z}_0$ is the length of the stenotic region, $2\bar{\delta}$ is the maximum proturberance of the stenotic form of the artery wall and R_0 is the radius of the normal artery.

The periodic body acceleration F($\bar{t}$) in the axial direction is given by

$$F(\bar{t}) = a_0 \cos(w_b \bar{t} + \phi), \quad (2a)$$

where a_0 is its amplitude, $w_b = 2\pi f_b$, f_b is its frequency in Hz, ϕ the lead angle of F($\bar{t}$) with respect to the heart action. The frequency of body acceleration f_b is assumed to be small, so that wave effects can be neglected .The pressure gradient at any $\bar{z}$ may be represented as follows.

$$\frac{\partial \bar{p}}{\partial \bar{z}} = A_0 + A_1 \cos(w_p \bar{t}), \quad (2b)$$

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where A_0 is steady component of the pressure gradient, A_1 is amplitude of the fluctuating component and $w_p = 2\pi f_p$, f_p is the pulse frequency.Both A_0 and A_1 are functions of $\bar{z}$.It can be shown that the radial velocity is very small in magnitude so that it may be neglected for problem with mild stenosis.

The specified momentum equation for the flow in cylindrical coordinate system is given by

$$\bar{\rho}\frac{\partial \bar{u}}{\partial \bar{t}} = -\frac{\partial \bar{p}}{\partial \bar{z}} + \frac{1}{\bar{r}}\frac{\partial}{\partial \bar{r}}(\bar{r}\,\bar{\tau}_{\bar{r}\bar{z}}) + F(\bar{t}) + \bar{\rho}\, g \sin al \tag{3a}$$

$$\frac{\partial \bar{p}}{\partial \bar{z}} = 0 \tag{3b}$$

where $\bar{r}$ and $\bar{z}$ denote the radial and axial coordinates respectively and $\bar{\rho}$ denotes density, $\bar{u}$ axial velocity of blood, $\bar{t}$ time, $\bar{p}$ pressure and $\bar{\tau}$ the shear stress and al be the small angle of inclination.For Herschell-bulkley fluid the relation between shear stress and shear rate is given by (Herschell-Bulkley 1929),

$$\bar{\tau} = \bar{\mu}_H^{1/n}\left(-\frac{\partial \bar{u}}{\partial \bar{r}}\right)^{1/n} + \bar{\tau}_H \text{ if } \bar{\tau} \geq \bar{\tau}_H \tag{4a}$$

$$\frac{\partial \bar{u}}{\partial \bar{r}} = 0 \;\; if\; \bar{\tau} \leq \bar{\tau}_H \tag{4b}$$

Where $\bar{u}$ is the total velocity, $\bar{p}$ is the pressure, $\bar{t}$ is the time, $\bar{\tau}_H$ is the yield stress and $\bar{\mu}_H$ is the coefficient of viscosity for Herschell-bulkley fluid .Equation (5b) corresponds to the vanishing of the velocity gradient in the region where the shear stress is less than the yield stress which impliesa plug flow whenever $\bar{\tau} \leq \bar{\tau}_H$.However ,the fluid behavior is indicated whenever $\bar{\tau} \geq \bar{\tau}_H$.

The boundary conditions appropriate to the problem under study are the no slip condition

(i) u=0 at $\bar{r} = \mathrm{R}(\bar{z})$ (5a)

and(ii) $\bar{\tau}$ and is finite at $\bar{r} = 0$ (5b)

Introducing the non-dimensional variables,

$$u = \frac{\bar{u}}{A_0 R_0^{\,2}/4\mu 0},\quad z = \frac{\bar{z}}{R_0},\quad t = w_p.\bar{t},\quad \delta = \frac{\bar{\delta}}{R_0},$$

$$\tau = \frac{\bar{\tau}}{A_0 R_0/2},\quad \theta = \frac{\bar{\tau}_H}{A_0 R_0/2},\ R(z) = \frac{\bar{R}(\bar{z})}{R_0},\ r = \frac{\bar{r}}{R_0},$$

$$a = \frac{a_0}{A_0},\ e = \frac{A_1}{A_0},\ w = \frac{w_b}{w_p},\ \bar{\mu}_0 = \bar{\mu}_H (2/R_0 A_0)^{n-1}$$

$$F = 4\bar{\mu}0 / g A_0 R_0^{\,2} \tag{6}$$

Here A_0 is steady component of the pressure gradient, R_0 is the radius of the normal artery, w_p is the frequency of oscillation of the pulsatile flow and then $\frac{A_0 R_0^{\,2}}{4\mu}$ represents the central line velocity in a Poiseuille flow.

The non dimensional momentum equation 3(a)becomes

$$\alpha^2 \frac{\partial u}{\partial t} = 4(1+e\cos t) + 4a\cos(wt+\phi) + \frac{\sin al}{F} + \frac{2}{r}\frac{\partial}{\partial r}(r\tau_{rz}) \tag{7}$$

Where $\alpha^2 = \frac{\omega_0 R_0^{\,2}}{\left(\mu / p\right)}$, α is Womersley frequency parameter.

Equation (4) can be rewritten as

$$\tau = \left(-\frac{1}{2}\frac{\partial u}{\partial r}\right)^{1/n} + \theta \qquad if \quad \tau \geq \theta, \tag{8a}$$

$$\frac{\partial u}{\partial r} = 0 \qquad if \quad \tau \leq \theta \tag{8b}$$

The boundary conditions (5a,5b) reduce to

i) u= o at r=R(z) (9a)

ii) τ isfiniteatr=0 (9b)

The geometry of the stenosis in non-dimensional form is given by

$$R(z)=\begin{cases}1-\delta\left(1+COS\dfrac{\pi z}{2z_0}\right) & for\ z=-2z_0\ to\ z=2z_0\\ 1 & otherwise\end{cases} \quad (10)$$

3. Method of solution

On using perturbation method, the velocity u, shear stressτ, plug core radius Rp and plug core velocity u_p are expanded as follows in terms of α^2 (where α^2 <<1)

$$u(z,r,t)=u_0(z,r,t)+\alpha^2 u_1(z,r,t)+.................. \quad (11a)$$

$$\tau(z,r,t)=\tau_0(z,r,t)+\alpha^2\tau_1(z,r,t)+.................. \quad (11b)$$

$$u_p(z,r,t)=u_{0p}(z,r,t)+\alpha^2 u_{1p}(z,r,t)+.........(11c)$$

$$R_p(z,r,t)=R_{0p}(z,r,t)+\alpha^2 R_{1p}(z,r,t)+.......... \quad (11d)$$

Substituting (11a)and (11b) in Equation (7) and equating the constant term and α^2 term we get

$$\frac{\partial}{\partial r}(r\tau_0)=-2r\left(1+e\cos t+a\cos(\omega t+\phi)-\frac{\sin al}{4F}\right) \quad (12)$$

$$\frac{\partial u_0}{\partial t}=\frac{2}{r}\frac{\partial}{\partial r}(r\tau_1) \quad (13)$$

Integrating Equation (12) and using boundary condition (9b) we obtain

$$\tau_0=-f(t)\,r \quad (14)$$

Where

$$f(t)=\left(1+e\cos t+a\cos(\omega t+\phi)-\frac{\sin al}{4F}\right) \quad (15)$$

Substituting (11a) and (11b) in (8) we get

$$-\frac{\partial u_0}{\partial r}=2\tau_0^{\;n-1}(\tau_0-n\theta) \quad (16)$$

$$-\frac{\partial u_1}{\partial r}=2\,n\,\tau_0^{\;n-2}\,\tau_1(\tau_0-(n-1)\theta) \quad (17)$$

Integrating Equation(16),using the relation (14) and the boundary condition (9a) we obtain

$$u_0=A(R^{n+1}-r^{n+1})+B(R^n-r^n) \quad (18)$$

Where $A=\dfrac{2(-1)^{n-1}f(t)^n}{n+1}$,

$B=2(-1)^{n-1}f(t)^{n-1}\vartheta$

The plug core velocity u_{0p} can be obtained from equation (18) as

$$u_{0p}=A(R^{n+1}-R_{0p}^{\;n+1})+B(R^n-R_{0p}^{\;n}) \quad (19)$$

Neglecting the terms of O(α^2) and higher powers of α in equation (11c) R_{0p} can be obtained from (14) as

$$R_{0p}=\vartheta/f(t)=k^2 \quad (20)$$

Using Equation (13) we get the solution for τ_1 as,

$$\tau_1=a_7\,r-a_6\,r^{n+1}-a_3\,r^{n+2} \quad (21)$$

Where

$a_1=(-1)^{n-1}f(t)^{n-1}f^1(t)$,

$a_2=\dfrac{n\,a_1R^{n+1}}{2(n+1)}$, $a_3=\dfrac{n\,a_1}{(n+1)(n+3)}$,

$a_4=\dfrac{(n-1)\theta}{f(t)}$, $a_5=\dfrac{a_1a_4R^n}{2}$,

$a_6=\dfrac{a_1a_4}{n+2}$, $a_7=a_2+a_5$,

Similarly using Equations (17) and (18)we can obtain the solution for , u_1 and u_{1p} as

$$u_1=\frac{a_{16}}{2n+1}(R^{2n+1}-r^{2n+1})+\frac{a_{15}}{2n+2}(R^{2n+2}-r^{2n+2})+\frac{a_9}{n}(R^n-r^n)+\frac{a_{13}}{n+1}(R^{n+1}-r^{n+1}) \quad (22)$$

$$u_{1p}=\frac{a_{16}}{2n+1}(R^{2n+1}-R_{0p}^{\;2n+1})+\frac{a_{15}}{2n+2}(R^{2n+2}-R_{0p}^{\;2n+2})+\frac{a_9}{n}(R^n-rR_{0p}^{\;n})+\frac{a_{13}}{n+1}(R^{n+1}-R_{0p}^{\;n+1}) \quad (23)$$

where $a_8=2n(-f(t)^{n-2}(n-1)\theta)$, $a_9=a_7a_8$,

$a_{10}=-a_6a_8$, $a_{11}=-a_3a_8$,

$a_{12}=-2n(-f(t)^{n-1})$, $a_{13}=a_7a_{12}$,

$a_{14}=-a_6a_{12}$, $a_{15}=-a_{12}a_3$, $a_{16}=a_{11}+a_{14}$.

Using equation (11),the total velocity distribution and shear stress can be written as

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$$u = A(R^{n+1} - r^{n+1}) + B(R^n - r^n) + \alpha^2 \left(\frac{a_{16}}{2n+1}\left(R^{2n+1} - r^{2n+1}\right) + \frac{a_{15}}{2n+2}\left(R^{2n+2} - r^{2n+2}\right) + \frac{a_9}{n}\left(R^n - r^n\right) + \frac{a_{13}}{n+1}\left(R^{n+1} - r^{n+1}\right) \right) \quad (24)$$

$$\tau_\omega = \left(\tau_0 + \alpha^2 \tau_1\right)_{r=R} = - f(t)R + \alpha^2 \left(a_7 R - a_6 R^{n+1} - a_3 R^{n+2}\right) \quad (25)$$

The second approximation plug core radius R_{1p} can be obtained by neglecting terms of $0(\alpha^4)$ and higher powers of α in Equation (11c)as

$$R_{1p} = \frac{-\left(\tau_1\right)_{R_{0p}}}{f(t)} \quad (26)$$

With the help of Equations (26),(20) and (11c), R_p can be given by

$$R_p = k^2 + \frac{\alpha^2}{f(t)}\left(a_7 R_{0p} - a_6 R_{op}^{n+1} - a_3 R_{0p}^{n+2}\right) \quad (27)$$

The volumetric flow rate Q is given by

$$Q(t) = 4 \int_0^{R(z)} r\, u(z,r,t)\,dr \quad (28)$$

4.Results and discussion

The wall shear stress τ_ω is a physiologically important quantity which plays an important role in determining aggregate sites of platelets and is given by

It is generally observed that the typical value of n for blood flow is taken to be lies between 0.9 and 1.1 and in this paper we have taken typical value of n as 0.95.Since the typical values for non-dimensional yield stress θ for blood are between 0.02 and 0.04, the value of yield stress has been takes as 0.04 for Herschell-Bulkley fluid.Thestenosisheight δ_s =0.2, Womersley parameter α = 0.1and as f = 2.0

Axial velocity profiles at the centre of the stenosis z = 0 with body acceleration a, constant pressure gradient e and inclined angle al are shown in Fig 2. It is observed that body acceleration parameter a brings qualitatative as well as quantitative changes in velocity profiles.(Fig 2 (a)).

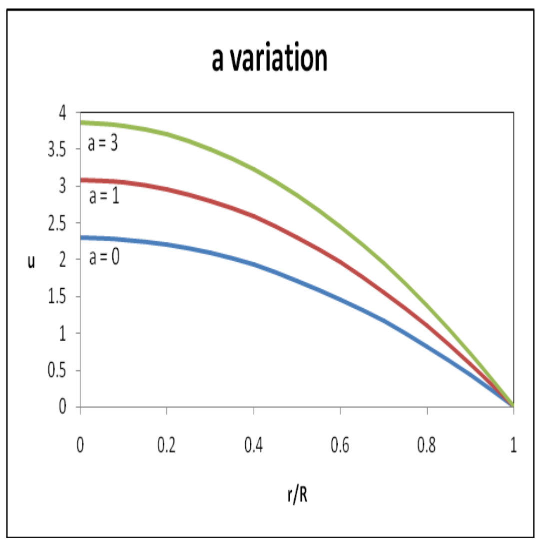

Fig 2.(a) Variation of velocity for different values of body acceleration parameter a.

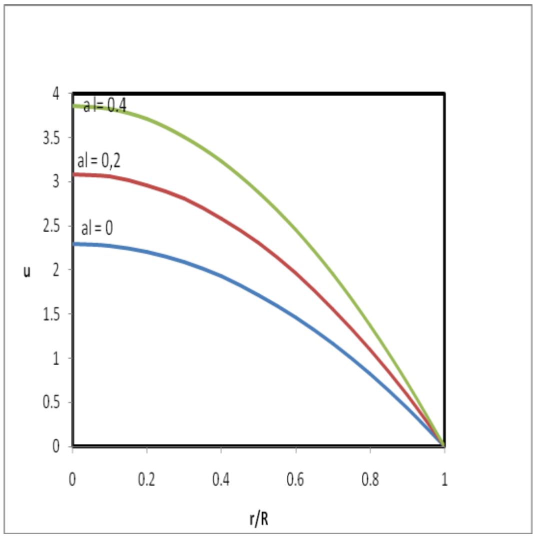

Fig 2.(b).Variation of velocity for different values inclinationangle al.

Magnitude of velocity increases with inclination angle al increases.(Refer Fig 2 (b)), but it decreases with increases in pressure gradient parameter e.(Refer Fig 2.(c)).

Fig 3 (a), (b) and (c) as well as Fig 4 (a), (b), (c) depicts plug core velocity profiles and wall shear stress profiles with axial difference..We have seen symmetry at the centre of the stenosis in both the profiles.

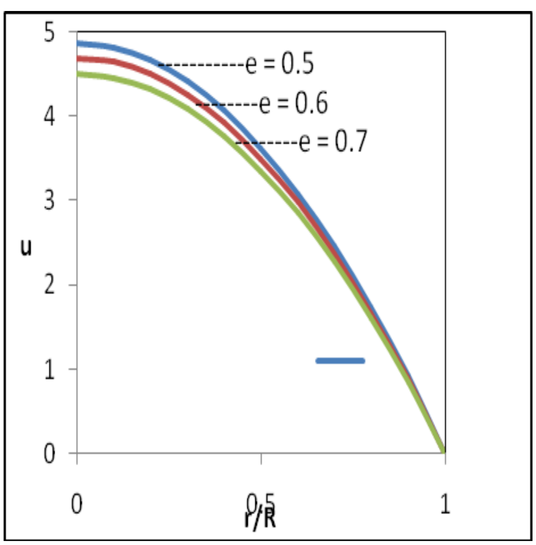

Fig 2.(c). Variation of velocity for different values of pressure gradient parameter e.

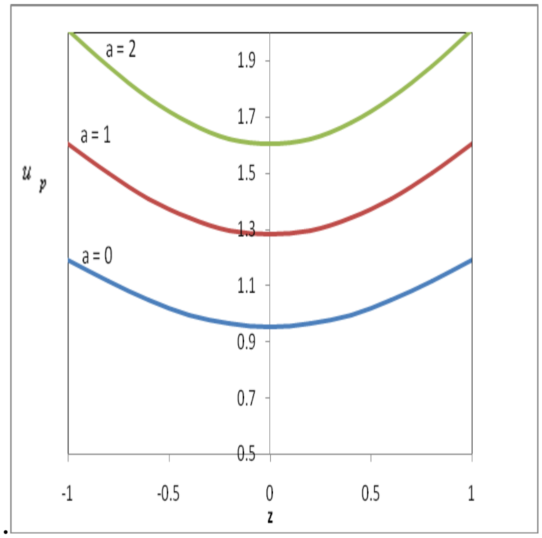

Fig 3 (a). Variation of plug core velocity for different values of body acceleration parameter a.

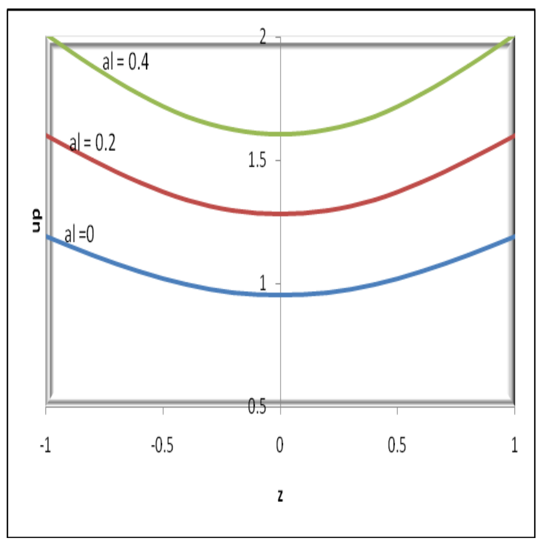

Fig 3 (b). Variation of plug core velocity for different values of inclination angle al.

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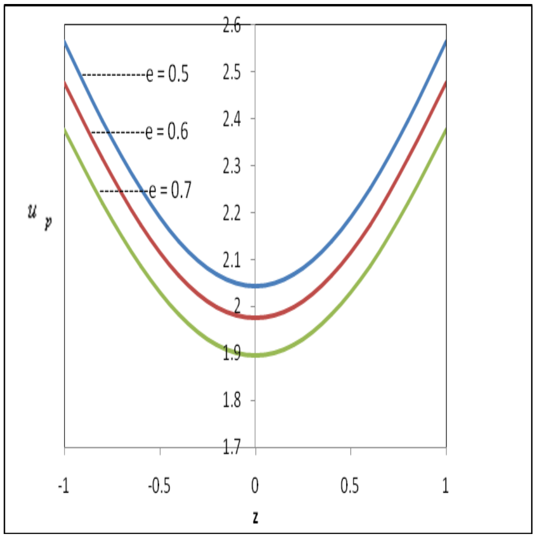

Fig 3 (c).Variation of plug core velocity for different values of pressure gradient parameter.

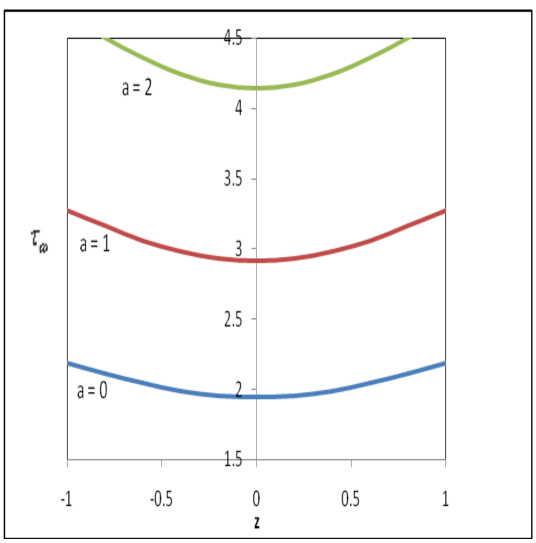

Fig 4(a).Variation of wall shear stress for different values of body acceleration parameter a

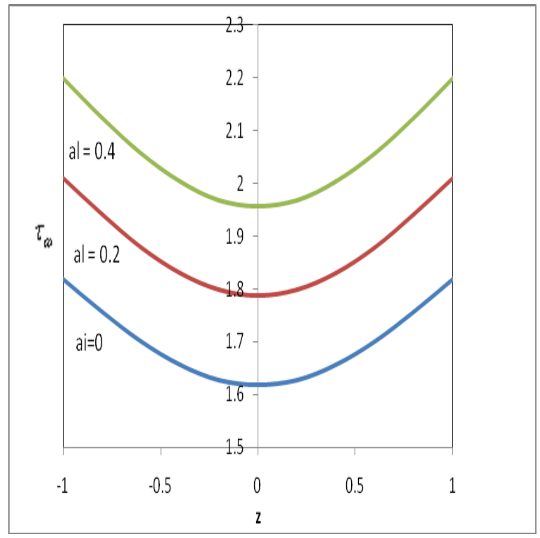

Fig 4 (b). Variation of wall shear stress for different values of inclination angle al.

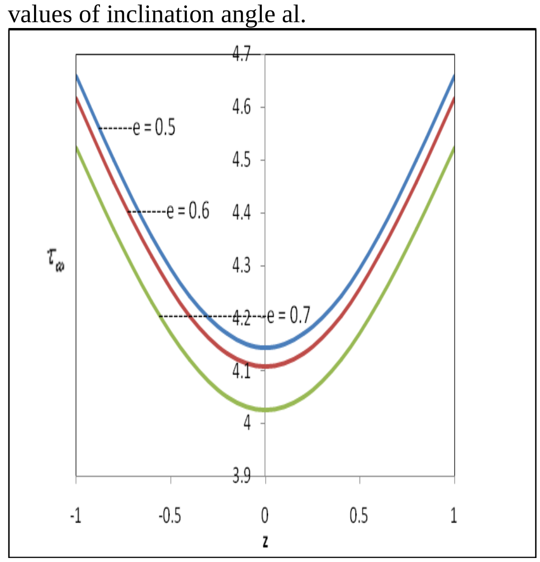

Fig 4 (c). Variation of wall shear stress for different values of pressure gradient parameter

The volumetric flow variation with the stenosis height δ_s, for for different values of a, al, e is shown in the tables 1 and 2.As e increases , the volumetric flow decreases.Also when the stenosis height δ_s increases, the volumetric flow quantitatively decreases which is true with the physiological condition.The inclination of the artery does not effect the flow rate at the vicinity of the stenosis.As the body acceleration parameter increases, the volumetric flow also increases, but the

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volumetric flow drastically decreases as stenosis height increases.

Table 1.

δ_s	e = 0-5	e=0.6	e =0.7
0	3.39791	3.280225	3.146342
0.1	1.412344	1.363609	1.308166
0.2	0.455994	0.440356	0.422565
0.3	0.092984	0.089834	0.08625
0.4	0.006223	0.00602	0.005789

Table 2

δ_s	a = 0	a = 1	a = 2
0	3.39791	3.435861	3.471279
0.1	1.412344	1.42806	1.442728
0.2	0.455994	0.461037	0.465744
0.3	0.092984	0.094	0.094948
0.4	0.006223	0.006289	0.00635

5. Conclusions

By using perturbation analysis assuming that the Womersley frequency parameter is small, the pulsatile flow of blood with periodic body acceleration under the presence of stenosis in an inclined artery is studied by modeling blood as a Herschell-bulkley fluid .It is observed that the body acceleration parameter, inclination and radius of stenosis are the strong parameters influencing the flow qualitatively and quantitatively..The presence and increase of the protuberance is found to reduce the magnitude of the velocity. The effect of stenosis is to reduce the flow rate and the presence of body acceleration and inclination of artery are to increase the flow rate. The body acceleration and inclination of artery are found to reduce the flow resistance.

List of symbols

‘–’ represents the dimenmsional quantities
a_0 amplitude of body acceleration.
A_0 steady component of pressure gradient
A1 amplitude of fluctuating component of presure gradient
A ratio of amplitude of body acceleration with steady component of pressure gradient given in Eq. (6)
e ratio of amplitude of fluctuating component with steady component of pressure gradient given in Eq. (6)
f(t) function defined in Eq. (15)
F(t) body acceleration function
f_b frequency of the body acceleration
f_p pulse frequency
P pressure
Q(t) volumetric flow rate
r radial coordinate
R(z) radius of the stenosed artery
R_0 radius of the normal artery
R_{0p} first approximation plug radius
R_{1p} second approximation plug radius
R_p radius of plug core velocity
T Time
u axial velocity of the fluid
u_0 first approximation velocity
u_{0p} first approximation plug velocity
u_1 second approximation velocity
u_{1p} first approximation plug velocity
u_p plug core velocity
z axial coordinate
$4z_o$ length of the stenotic region

Greek letters

φ lead angle of body acceleration function with respect to heart action
ρ density
λ flow resistance
θ non-dimensional yield stress of the fluid
ω ratio of ω_b with ω_p
τ Shear stress of the fluid
μ viscosity of the fluid
α Womersley frequency parameter
τ_0 first approximation shear stress

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τ_1 second approximation shear stress
ωb 2πfb
ωp 2πfp
τ_y yield stress of the fluid
2δ maximum protuberance of the stenotic form of the artery wall

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