Effect of Autofrettage Pressure on Stress Distribution and Operating Pressures of Thick - Walled Cylinders

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***Abstract* - Both of *von Mises* and *Tresca* yield criteria are used to evaluate the effect of autofrettage pressure on stress distribution, maximum and minimum autofrettage pressures, operating pressure limits and optimum autofrettage pressure of thick – walled cylinders. The finite element simulation is carried out on *ABAQUS* ver. 6.9. It has been observed that the minimum autofrettage pressure has to be bigger than minimum pressure that needed to yield the inner surface of non – autofrettage pressure (P_{Yi}) and also bigger than operating pressure, while the maximum autofrettage pressure's value has to be less than value of (P_{Yo}). Furthermore, it is reveal, the autofrettage pressure plays the main role to determine the location of maximum von Mises & Tresca stresses throughout the cylinder's wall, which it means, the autofrettage radius, R_a .**

On other hand, the optimum autofrettage pressure is depending on operating pressure for both von Mises & Tresca criteria. Also, the percentage reduction of maximum *von Mises* and *Tresca* stresses increase as both of operating and autofrettage pressures' values increase. It was found equal to (23.193 %) & (23.97 %) for *von Mises* & Tresca yield criteria respectively.

Key words : Maximum And Minimum Autofrettage Pressure, Optimum Autofrettage Pressure , Allowable Internal Pressure , Radial And Hoob Stresses, Von Mises And Tresca Yield Criterions, Autofrettage Radius.

I. INTRODUCTION

Among others, the autofrettage process is considered as one of the most methods have been used to strenthen the thick – walled cylinders. It is a process that a cylinder is subjected to sufficient high internal pressure before a cylinder is put into use. Then the internal pressure removes, thereby residual stresses are created in the wall of the cylinder. This residual stresses will serve to reduce the maximum stresses induced as a result of subsequent application of an operating pressure [1, 2]. Noraziah Wahi et al. [3] used FE analysis to propose the predicting of the required autofrettage pressure for various levels of allowable pressure to achieving maximum fatigue life. They presented that ; optimum autofrettage pressure of the cylinder is suitable when a minimum equivalent stress at elastic-plastic junction is to be minimized and maximum fatigue life are also to be achieved. Md. Tanjin Amin et al. [4] determined the optimum autofrettage pressure and optimum autofrettage radius by using von Mises yield criterion , then they have been compared with Zhu and Yang's model [5]. Also they observed that the percentage of maximum *von Mises* stress reduction increases as value of radius ratio (K) and working pressure increase. Amran Ayob and M. Kabashi Elbasheer [6], used von Mises and Tresca yield criteria to develop a procedure for determinding the autofrettage pressure analytically, resulting in a reduced stress concentration. Then they compared the analytical results with FEM results. They concluded that, the autofrettage process increases the maximum allowable internal pressure but it cannot increase the maximum internal pressure to case whole thickness of the cylinder to yield.

Zhong Hu and Sudhir Puttagunta [7] investigate the residual stresses in the thick-walled cylinder induced by autofrettage pressure, also they found the optimum autofrettage pressure and the maximum reduction percentage of the von Mises stress under elastic–limit working pressure. They mentioned that the optimum autofrettage pressure is to be about (1.5) times the elastic limit of working pressure and the maximum reduction percentage of maximum *von Mises* stress was about (28 %) under the elastic limit of working pressure. k. Ruilin Zhu and Jinlai Yang [5], by using both of yield criteria *von Mises* and *Tresca*, presented an analytical equation for optimum radius of elastic-plastic junction in autofrettage cylinder, also they studied the influence of autofrettage pressure on stress distribution and load bearing capacity. They concluded , to achieve optimum radius of elastic-plastic junction, an autofrettage pressure a bit larger than operating pressure should be applied before a pressure vessel is put into use. Noraziah et al. [8] presented an analytical autofrettage procedure to predict the required autofrettage pressure of different levels of allowable pressure and they validate their results with FEM results. They found three cases of autofrettage in design of pressurized thick walled cylinders.

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II. MATERIALS AND FINITE ELEMENT SIMULATION MODELS

In this investigation , an open end cylinder with inner radius of (a = 100 mm) and outer radius of (b = 200 mm) has been considered for both finite element and theoretical study. The FE model is illustrated in figure (1).The numerical simulation carried out on ABAQUS ver. 6.9 [9]. The testing cases are consider as 2D – planner problems and quadratic element CPS8R – with 8 – nodes have been used. The number of elements generated are (940) elements. The model's material used is carbon steel which has young's modulus of (E = 208 GPa), Poission ratio of (γ = 0.3) and yield stress of (σy = 485 MPa) [10]. The material is consider as homogeneous and isotropic material. The investigated cylinder is subjected to internal pressure (P_i).

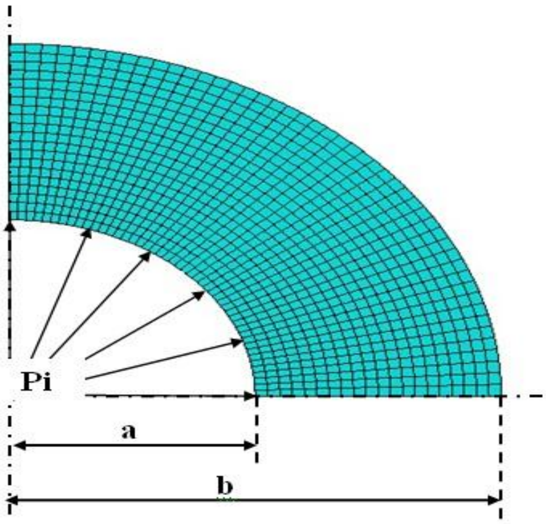

Figure (1) : Geometry of the FE model

III. VALIDATION OF FINITE ELEMENT SIMULATION

Validation of finite element simulation with theoretical's calculation results that can be obtained by solution of equations or results available in literatures play the main role for acceptance of FE model used in computation [11]. In the present investigations, the validation has been done by comparing the results of numerical simulation using ABAQUS ver. 6.9 with analytical results obtained by solutions of equations which are available in literatures [3,5,6,8].

Figure (2) demonstrates that, the radial, hoob and both of maximum *von Mises* and *Tresca* stresses which induced by applying different internal pressure, are compatible and overlap each other. Also, the static analysis reveals the percentage of deviation error between the analytical and numerical results are less than (0.5 %). This low percentage affirms well correlation between them in terms of magnitude of stresses. Thereby, FE models using ABAQUS ver. 6.9 can be used to investigate the stress distributions , operating pressure limits and optimum autofrettage pressure of thick – walled cylinder.

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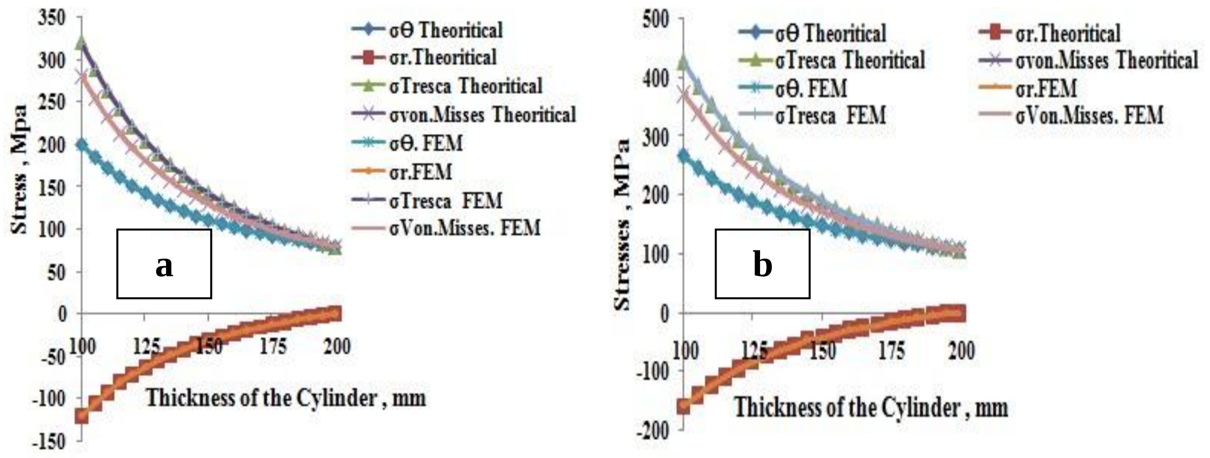

Figure 2 : Valdation of analytical and FE simulation results at different operating pressure;
a- operating pressure = 120 MPa, b- operating pressure = 160 MPa

IV. ANALYTICAL CALCULATIONS

5.1. Stress Components of Non – Autofrettage Cylinder

The stress components in non autofrettage thick – walled cylinder induced by internal pressure, Pi , are given by Lame's formulation as following [1, 2, 3, 6, 7, 8].

$$\sigma_r = \frac{Pi}{(K^2-1)}\left(1-\frac{b^2}{a^2}\right) \qquad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$\sigma_\Theta = \frac{Pi}{(K^2-1)}\left(1+\frac{b^2}{a^2}\right) \qquad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Where ; σ_r *: radial stress ;* σ_Θ *: hoob stress*
k : radius ratio = a / b, a & b : inner and outer bores of the cylinder respectively.
Pi : *internal pressure applied on the cylinder*.

From figure (2), it is observed that, the radial stress (σ_r), the hoob stress (σ_Θ) and maximum *von Mises* (σ_{vM}) and *Tresca* (σ_{Tr}) stresses are maximum at the inner surface of the non – autofrettage cylinder. On other hand, the radial stress is always compressive stress while the hoob stress is always tensile stress. Also, it is revealed, the hoob stress is greater than the radial stress.

5.2. Maximum Allowable Internal (Operating) Pressure of Non – Autofrettage Cylinder

It is vital to calculate the value of internal pressure required to yield the inner surface of the non-autofrettage cylinder (P_{Yi}). It can be calculated for both *von Mises* and *Tresca* criteria from equations (3 & 4) [1, 2, 3, 6, 8].

$$P_{Yi} = \frac{\sigma y\,(K^2-1)}{\sqrt{3K^2}} \quad \text{…. According to von Mises criterion} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

$$P_{Yi} = \frac{\sigma y\,(K^2-1)}{2K^2} \quad \text{…. According to Tresca criterion} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Where ; k = radius ratio = a / b,a & b : inner and outer bores of the cylinder respectively
σy = yield stress of cylinder's material.

By computing the value of (P_{Yi}) from the above questions, it was found equal to (210 MPa) for *von Mises* criterion and equal to (181.875 MPa) for *Tresca* criterion. These values represent the maximum allowable operating (internal) pressure, $P_{oprating}$, can be safely applied on non – autofrettage cylinder without any damage.

5.3. The relationship between the Operating pressure and Optimum autofrettage Pressure

The optimum autofrettage pressure can be calculated from equations (5 & 6) [3, 6, 8].

$$P_{opt.\ aut.} = \frac{\sigma y}{2}\left[1 - \frac{e^{\sqrt{3}n}}{K^2} + \sqrt{3}n\right] \text{......according to } von\ Mises \text{ criterion} \qquad ....\ (5)$$

$$P_{opt.\ aut.} = \frac{\sigma y}{2}\left[1 - \frac{e^{2n}}{K^2} + 2n\right] \text{........according to } Tresca \text{ criterion} \qquad ....\ (6)$$

Where ; n = operating pressure ($P_{oper.}$) */ yield stress of cylinder's material* (σ_y).

VI. RESULTS AND DISCUSSIONS

6.1. Effect of Autofrettage Pressure on Stress Components and Maximum von Mises and Tresca Stresses

Figure (3) illustrates that, the autofrettage pressure leads to reduce the resultant hoob stress (σ_Θ) obviously, and the maximum value occurs at somewhere through the thickness of the cylinder rather than inner bore. While there is no significant effect induced by the same autofrettage pressure on radial stress (σ_r). Furthermore, it is clear that, both values of resultant hoob stress and its location strongly depends on the value of autofrettage pressure in such a manner, as the autofrettage pressure increases, the resultant hoob stress value decreases, and it will be more far away from the inner bore of the cylinder.

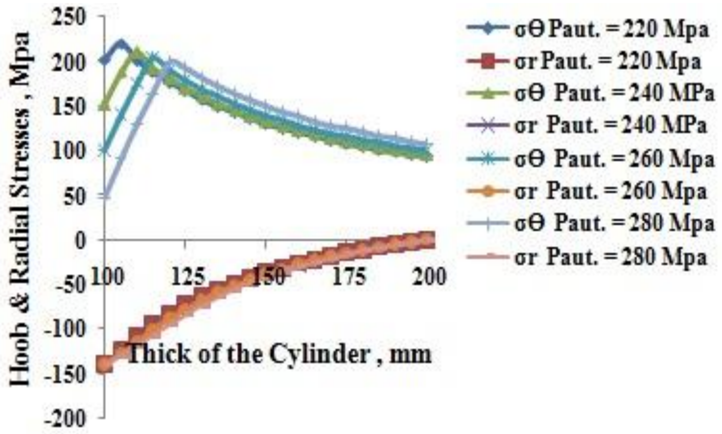

Figure (3) : FE results of Hoob & Radial Stress distribution at different autofrettage pressure Pi = 140 MPa.

The FE relationships between the autofrettage pressure and maximum *von Mises* and *Tresca* stresses have been graphically represented in figure (4). It is obvious, they are inverse relationships. It means, the maximum stresses decrease with increasing the autofrettage pressure, and the maximum stresses move towards the outer surface of the cylinder as shown obviously in figure (5 & 6). These new locations of maximum *von Mises* and *Tresca* stresses called *Autofrettage Radius* (*Ra*). This results correlate well with results had been found by [2, 4, 14]. Furthermore, the FE simulations revealed, the percentage reduction in maximum *von Mises* and *Tresca* stresses are depending on operating pressure which applied on the autofrettage cylinder after release the autofrettage pressure. It was found, according to *von Mises* criterion, and for operating pressure of ($P_{operating}$ = 100 MPa) , it is varying from (7.11 % at $P_{autofrettage}$ = 220 MPa) to (16.64 % at $P_{autofrettage}$ = 280 MPa), while for operating pressure of ($P_{operating}$ = 200 MPa), it changes from (5.29 % at $P_{autofrettage}$ = 220 MPa) to (23.193 % at $P_{autofrettage}$ = 280 MPa). On other hand, according to *Tresca* criterion, and for operating pressure of ($P_{operating}$ = 100 MPa) , it is varying from (7.6 % at $P_{autofrettage}$ = 220 MPa) to (17.5 % at $P_{autofrettage}$ = 280 MPa), while for operating pressure of ($P_{operating}$ = 200 MPa), it changes from (5.02 % at $P_{autofrettage}$ = 220 MPa) to (23.97 % at $P_{autofrettage}$ = 280 MPa). Also, it is obvious, there is no significant differences in percentage reduction between maximum *von Mises* and *Tresca* stresses for different autofrettage and operating pressures.

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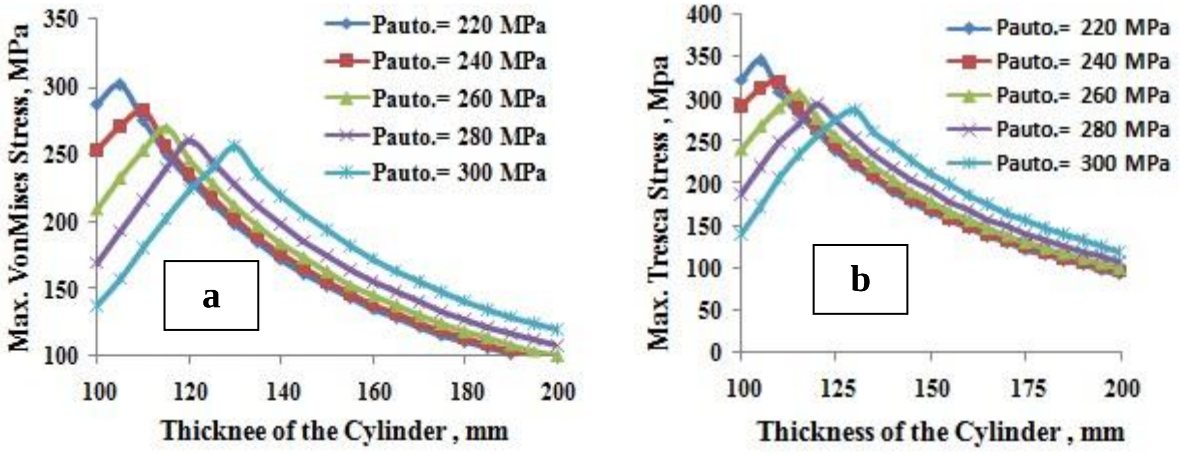

Figure (4) : Simulation solution results of variation of maximum stresses with autofrettage pressure at operating pressure of (Popr. = 140 MPa) ; a) *vonMises* and b) *Tresca.*

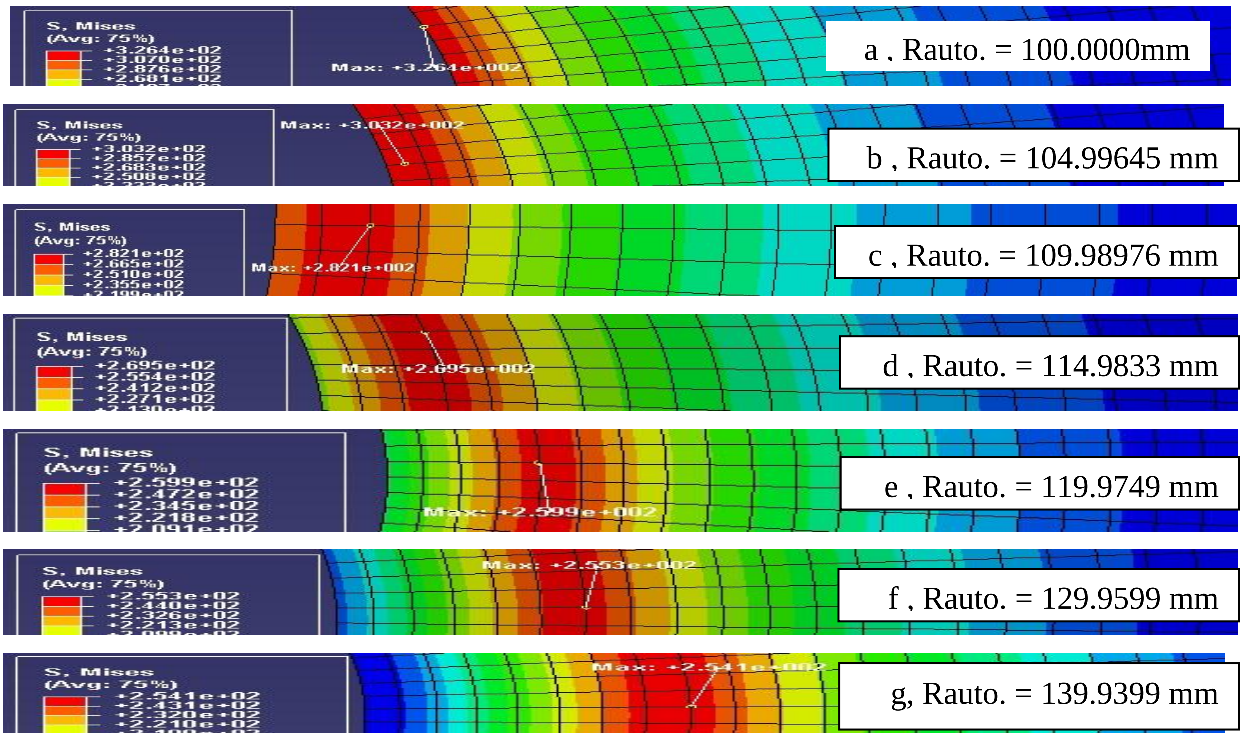

Figure (5) : FE simulation results of location of maximum vonMises stresses with and without autofrettage pressure at operating pressure of (Popr. = 140 MPa) ; a) Without autofrettage,
b) Pauto. = 220 Mpa , c) Pauto. = 240 Mpa, d) Pauto. = 260 Mpa , e) Pauto. = 280 Mpa,
f) Pauto. = 300 Mpa, , g) Pauto. = 320 Mpa.

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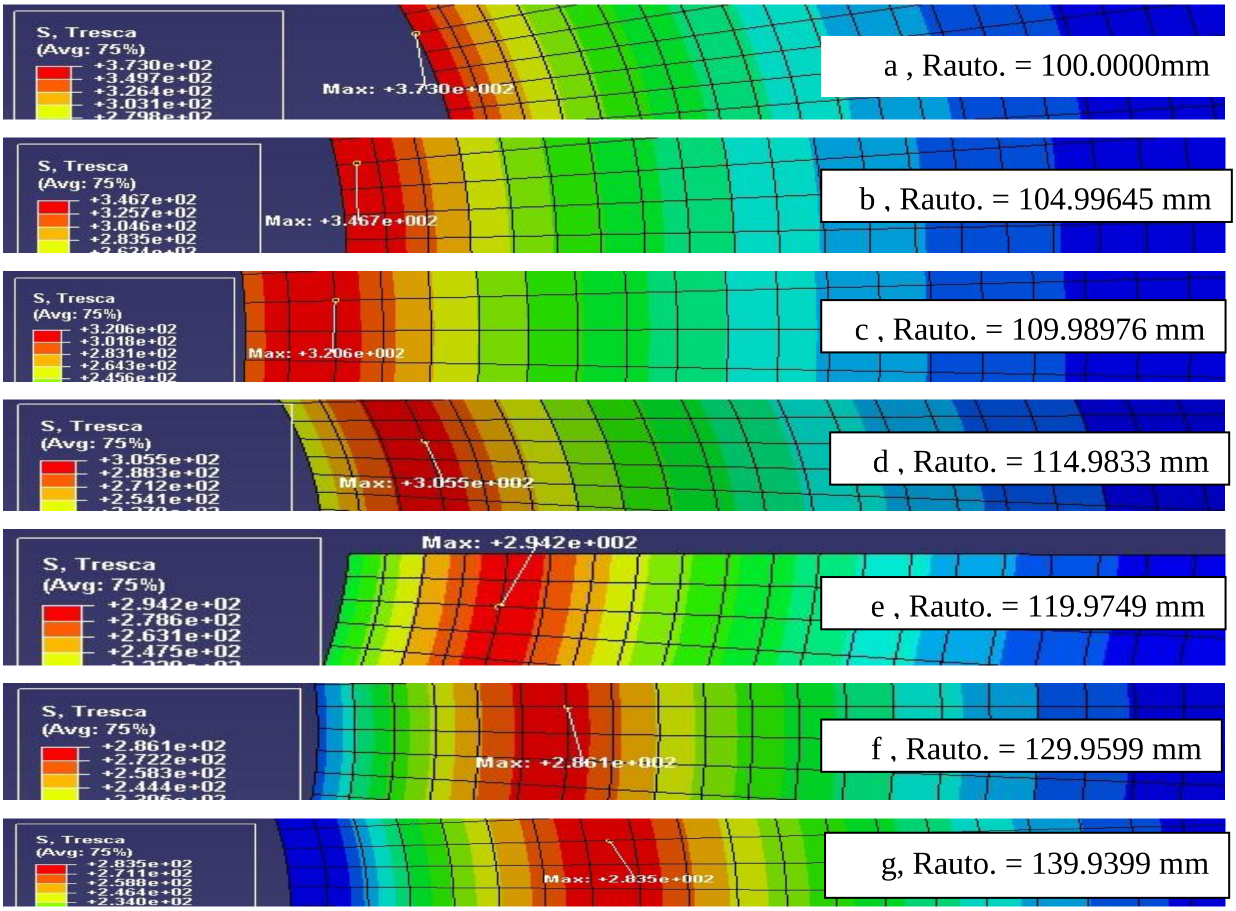

Figure (6) : FE simulation results of location of maximum Tresca stresses with and without autofrettage pressure at operating pressure of (Popr. = 140 MPa) ; a) Without autofrettage,
b) Pauto. = 220 Mpa , c) Pauto. = 240 Mpa, d) Pauto. = 260 Mpa , e) Pauto. = 280 Mpa,
f) Pauto. = 300 Mpa, g) Pauto. = 320 Mpa.

6.2. Effect of Autofrettage pressure on Autofrettage Radius

The FE simulation results of autofrettage pressure's effect on autofrettage radius at different operating pressure are demonstrated in figure (7). It is clear that, the autofrettage radius; $R_{aut.}$, increases with increasing of autofrettage pressure (see figure 5, 6 & 8). This is because of the autofrettage process relocates the maximum stresses from the inner bore of the cylinder to somewhere through the cylinder thickness. On other hand, it is revealed, the operating pressure does not affect on the autofrettage radius. Thereby, the autofrettage pressure , not operating pressure, play the main role to determine the location of maximum stresses throughout the wall of the cylinder, which it means, the *autofrettage radius*, $R_{aut.}$.

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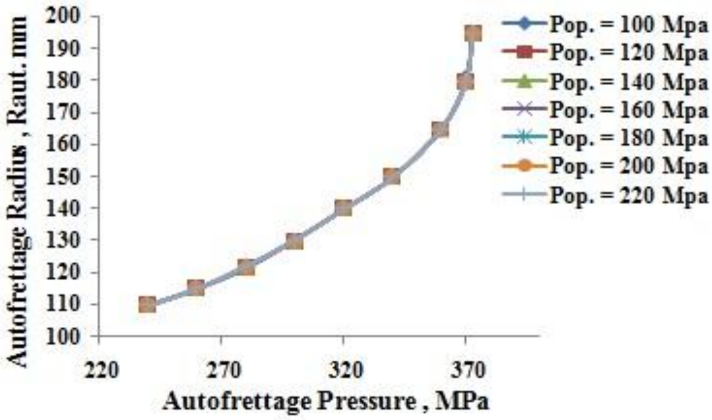

Figure (7) : FE simulation results of autofrettage pressure's effect on autofrettage radius at different operating pressure for both *vonMises* & *Tresca* criteria.

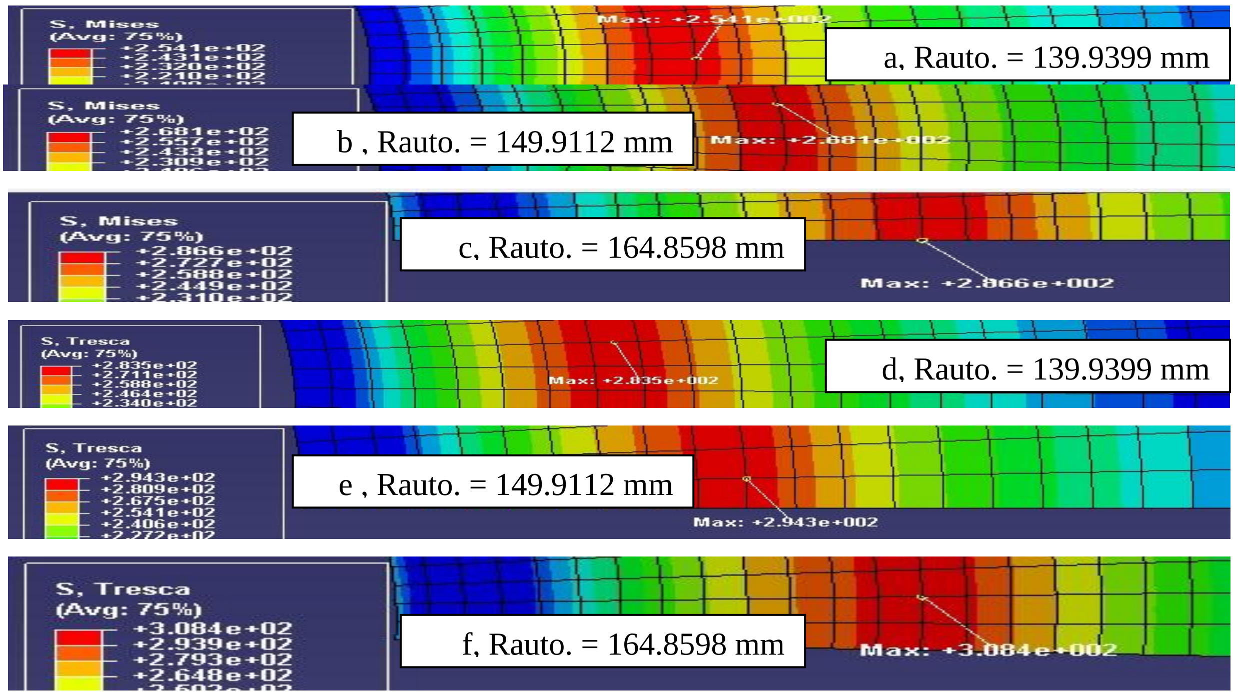

Figure (8) : FE simulation results of location of maximum vonMises (a, b, c) & Tresca (d, e f) stresses at operating pressure of (Popr. = 140 MPa) & different autofrettage pressure; a & d) Pauto. = 320 Mpa , b & e) Pauto. = 340 Mpa, c & f) Pauto. = 360 Mpa .

6.3. Minumum and Maximum Autofrettage pressure

Figure (9) illustrates both maximum *von Mises* and *Tresca* stresses remain constant without any change as long as the autofrettage pressure value does not exceed the value of minimum pressure that needed to yield the inner surface of non – autofrettage cylinder (P_{Yi}). It means, for any operating pressure's value less than value of (P_{Yi}), the autofrettage pressure has to be bigger than P_{Yi}'s value.On the other hand, when the operating pressure bigger than (P_{Yi}) as shown in figure (10) and in spite of ($P_{autofrettage} > P_{Yi}$) but the autofrettage pressure does not affect on both maximum *von Mises* and *Tresca* stresses even it exceeds the value of operating pressure. Therefore and to satisfy the aim of autofrettage process, the autofrettage pressure has to be bigger than the minimum pressure that needed to yield the inner surface of non – autofrettage cylinder (P_{Yi}) and operating pressure ($P_{operating}$) too. Also, it is clear, the FE simulation results of minimum autofrettage presure's value is about (210 - 220 MPa) which is more close to value getting from equation (3) according to von Mises criterion than that gets from equation (4) according to Tresca criterion.

Also, it observed, the decreasing of maximum *von Mises* and *Tresca* stresses with increasing of autofrettage pressure will continue even reaches a certain value then will increase again. Subsequently, as autofrettage pressure

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increasing, the maximum von Mises and Tresca stresses either decreases gradualy or remain constant then decreases as can be seen in figure (11). This results agree with results had been get by Abu. Rayhan Md. el at. [14]. The autofrettage pressure's value that leads to get the minimum value of the maximum stresses is called *optimum autofrettage pressure*
($P_{opt.\ auto.}$), and the the radius that the minimum value of maximum stresses occurs, is called *optimum autofrettage radius* ; $R_{opt.auto.}$

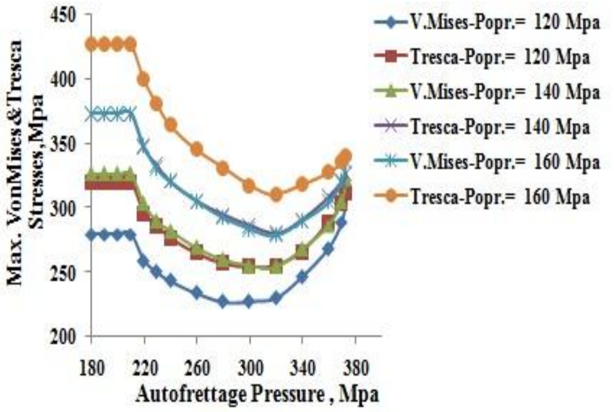

Figure (9) : FE simulation results of autofrettage pressure's effect on maximum vonMises & Tresca stresses at different autofrettage pressure and ($P_{operating} < P_{Yi}$)

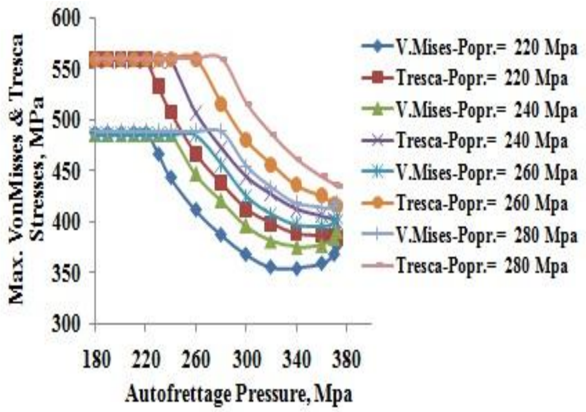

Figure (10) : FE simulation results of autofrettage pressure's effect on maximum vonMises & Tresca stresses at different autofrettage and pressure ($P_{operating} > P_{Yi}$)

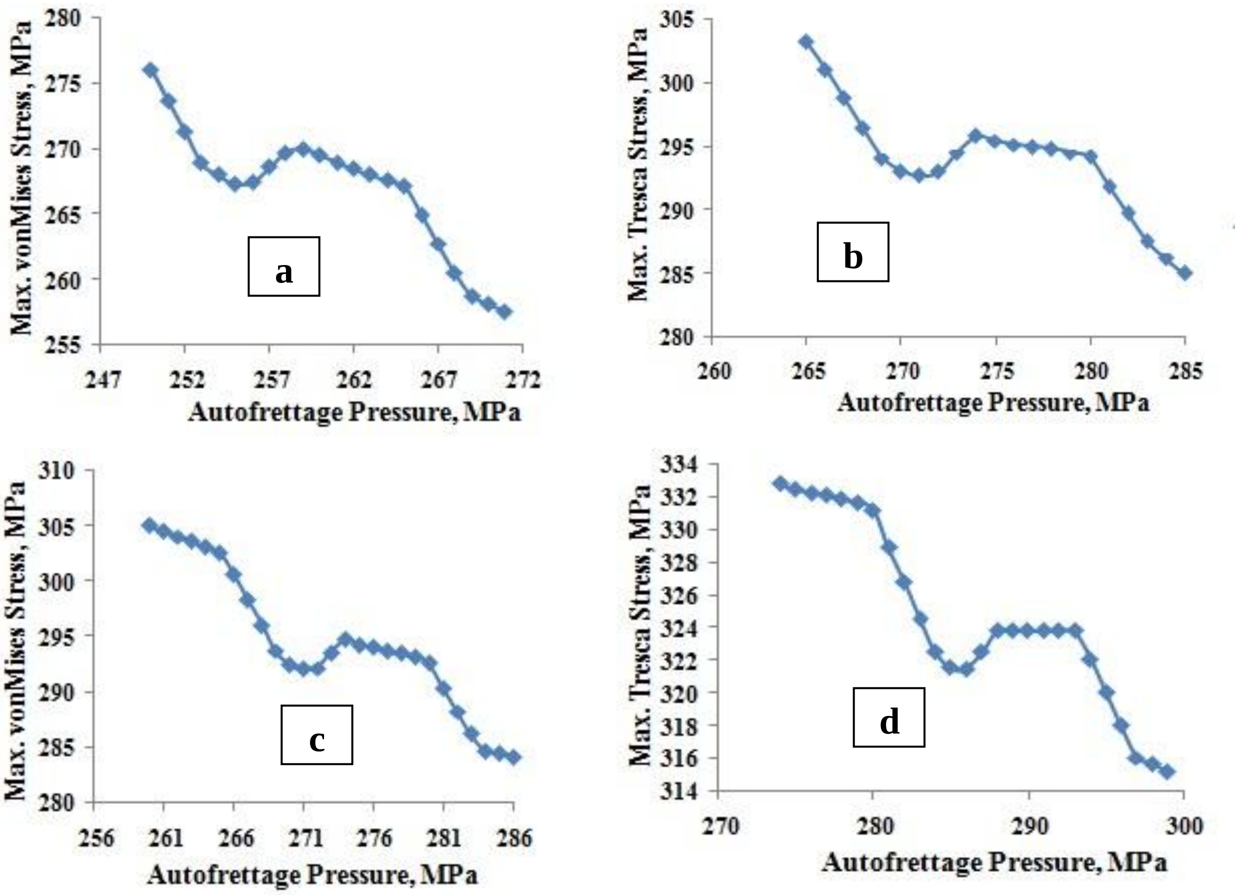

Figure (11) : FE simulation results of variation of autofrettage pressure on maximum von Mises & Tresca stresses at different operating pressure. a & b - 140 MPa , c & d = 160 MPa.

6.4. The relationship between the Operating pressure and Optimum autofrettage Pressure

The effect of operating pressure on optimum autofrettage pressure has been drawn in figure (12). It can be seen, for both *von Mises* and *Tresca* criterion, the optimum autofrettage pressure increases with increasing the operating pressure. Also, it is obvious that, for any value of operating pressure, the optimum autofrettage pressures' value according to *Tresca* criterion are bigger than the values according to *vonMises* criterion. On the other hand, there is no significant differences between the theoretical and FE simulation results. The deviation between the two values are less than 2 (1.8 % for *von Mises* and 1.6 % for *Tresca* criterions).

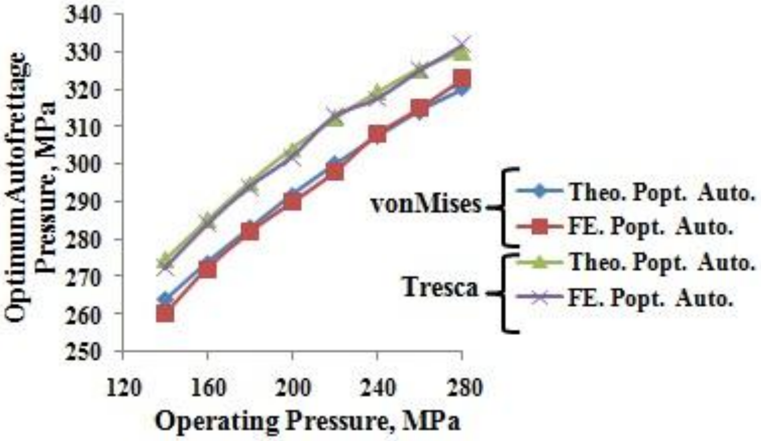

Figure (12) : FE simulation results of the relationship between the operating pressure and autofrettage pressure , Tresca & VonMises.

VI. CONCLUSION

The results of present investigation can be concluded as :-

1. The minimum autofrettage pressure has to be bigger than minimum pressure that needed to yield the inner surface of non – autofrettage pressure (P_{Yi}) and also bigger than operating pressure, while the maximum autofrettage pressure's value has to be less than value of (P_{Yo}).
2. The autofrettage pressure, not operating pressure, plays the main role to determine the location of maximum vonMises & Tresca stresses throughout the cylinder's wall, which it means, the autofrettage radius, R_a .
3. The optimum autofrettage pressure is depending on operating pressure for both von Mises & Tresca criteria.

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