Dynamic Parameter Estimation of A Bend Rotor Bearing System

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***Abstract*—The vibrational characteristics due to shaft bow effect in the rotor are different from the vibrational characteristics due to unbalance. From designing point of view it is important to know the cause of vibration is unbalance or rotor bow. Present study is mainly focused on the estimation of rotor bow parameters of a simple Jeffcott rotor model. In this model, a flexible shaft is mounted on the two bearings and a rigid disc is mounted on the shaft at mid–span. Equation of motion is developed with the help of finite element method to determine the frequency response of the system. An identification algorithm is developed in the frequency domain for simultaneous estimation of the dynamic bearing parameters, residual unbalances and shaft bow. This identification algorithm is based on the least square technique. The present study pays attention to the simultaneous estimation of bearing dynamic parameters, residual unbalances and residual shaft bow parameters. The model is tested against different measurement noise and found to be robust.**

Keywords—Shaft Bow; Residual Unbalances; Finite Element Method (FEM); Least Squares Fit;

I. INTRODUCTION

Rotor dynamics is the branch of engineering that studies the lateral and torsional vibrations of rotating shafts for predicting the rotor vibrations and containing the vibration level under an acceptable limit. The principal components of a rotor-dynamic system are the shaft or rotor with disk, the bearings, coupling and seals. Flexible rotors are used in many industries is based on the design criteria. To maximizing the space available for other components such as impeller and seal, the shaft is relatively long shaft and thin geometry. Due to flexibility of rotor, bow effect arises in rotor. The bow effect is one kind rotor fault, which results in thermal defects and gravity sag. Vibration characteristics of a rotor bearing system having bowed shaft is different from vibration due to unbalance. For balancing point of view, it is important to know whether the cause of vibration is unbalance or shaft bow. The shaft bow in the large rotating machinery can lead to serious accident. Detail analysis of rotor response with bowed shaft has drawn attention of several researchers. In article [1] authors proposed a theoretical analysis of a single mass rotor with bowed shaft. The shaft is analyzed both theoretical and experimentally for balancing of the system. Reference [3] distinguished about the different types of rotor bows in rotating machines namely elastic bow, temporary elastic bow and permanent bow. Authors of [5] developed an identification method for getting the response characteristics of flexible rotor model with shaft bow. Reference [6] developed an identification algorithm for multi-degree-of-freedom rotor-bearing system for simultaneous estimation of the residual unbalance and bearing dynamic parameters. In article [7] authors developed a transient response analysis method for rotor transverse surface crack with a bow to estimate the effect of residual bow. During the study, the effect of residual bow is varying with the stiffness characteristics of rotating cracked shaft and dynamics of the cracked rotor. Reference [9] investigated the dynamic behavior of a geared-rotor system with viscoelastic supports under the effects of eccentricity, the transmission error of the gear mesh and the residual shaft bow is investigated under consideration of gear eccentricity, excitation of the gear transmission error and the residual shaft bow. Authors of [11-15 & 17] proposed an algorithm to quantitatively estimate misalignment both numerically and experimentally. Ref. [16] used active magnetic bearing to suppress the excessive vibration generate due to misalignment. Authors of [18] developed algorithm to estimate speed dependent bearing and coupling misalignment parameters. The present study is focused on the estimation of rotor bow parameters. Developed an equation of motion incorporated with bow effect for of a simple rotor bearing system. A solution of this equation of motion gives the response of the given system. An identification algorithm prepared for estimating the parameters like bearing dynamic parameters, residual unbalances and rotor bow. The obtained parameters from the developed identification algorithm are comparing with assumed parameters.

II. SYSTEM MODEL

A simple rotor-bearing model consists of a rigid disc attached at the mid span and it is supported by two bearings at the end. The rotor shaft is considered as flexible with mass m_s and diametral mass moment of inertia I_s. The rotor-bearing model shown in Fig. 1 respectively, and in this study; the bearing is modeled as equivalent to spring and dampers system. The rotor is modeled as the Timoshenko beam element [2] for the brevity of this study; we are not paying attention to the gyroscopic effect. Here, B_1 and B_2 are the bearing. C is center of rotation the shaft, G is the center of gravity of the shaft and e is the eccentric of the rotation to the center of gravity of the shaft. Four displacements coordinates, two orthogonal translational and rotational coordinate is associated with each node ($x, y, \varphi_x, \varphi_y$). The disc has mass m_d and diametral mass moment of inertia is I_d, the residual unbalance U_i. The bearings are modeled by eight linearized damping $c_{ij}^{B_n}$ and stiffness $k_{ij}^{B_n}$ coefficients (all bearings has different direct as well as cross-coupled terms, i.e. $i, j = x, y$ and n=1,2 for two bearings). Since the main aim of this

present study is to analyzed the effect of shaft bow on the rotor system and under the general shaft bow condition (i.e. shaft bow rises from
gravity sagging) the actual shaft centerline and the theoretical shaft centerline will not be coincide which otherwise happens in shaft bow conditions. Hence, due to the self-weight of the shaft takes place which leads to bending in the shaft to the bending plane carrying a rotor bow effect on the rotor and the bearing.

A. Shaft Modelling

The equations of motion of the shaft can be given as (12 degrees of freedom).

$$[M]_s\{\ddot{\eta}\}_s+[C]_s\{\dot{\eta}\}_s+[K]_s\{\eta\}_s=\{f(t)\}_s \tag{1}$$

Where, $\{\eta\}_s$ is the nodal displacement vector, $\{f(t)\}_s$ is the force vector and mass matrix $[M]_s$, stiffness matrix $[K]_s$ and damping matrix $[C]_s$ of the shaft are given in [10].

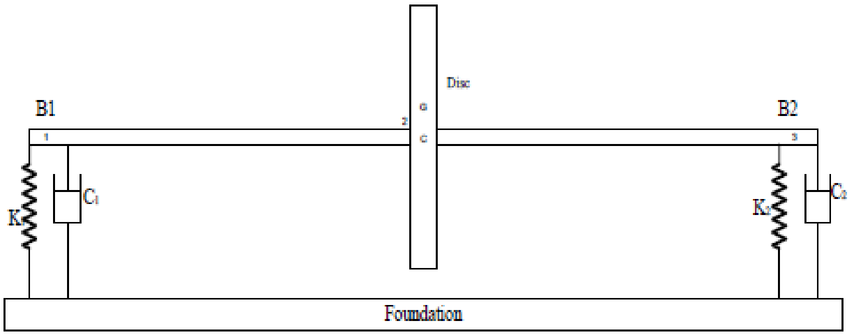

Fig. 1. *Simple rotor bearing model*

B. Rigid Disc Modelling

The disc for analysis is assumed axisymmetric and rigid with rotational and translational motion. In the mass matrix of the disc, the mass of the disc and mass moment of inertia of the disc in terms are used to corresponding nodes. The equation of motion of the disc is discussed in [12], which is given below,

$$[M]_d\{\ddot{\eta}(t)\}_d=\{f(t)\}_d \tag{2}$$

Where vectors $\{\eta\}_d$ and $\{f(t)\}_d$ are the disc nodal displacement and force vectors respectively. Details of the $[M]_d$ elemental disc matrix are given in [10].

C. Bearing Modelling

Modelling of the bearing is given by the eight linearized damping and stiffness coefficient (all bearings have distant direct as well as cross-coupled coefficients). Equation of motion for bearing is discussed in [12], which is given below as,

$$[C_B]\{\dot{\eta}\}_B+[K_B]\{\eta\}_B=\{f(t)\}_B \tag{3}$$

Where $\{\eta\}_B$ and $\{f\}_B$ are the bearing nodal displacement and force vectors. Details of elemental bearing are given in [10].

D. Development of Unbalance Force

Unbalance force in the rotor arises due to eccentricity between the center of gravity of the disc and center of rotation of the shaft. When the shaft rotates then due to the eccentricity a centrifugal forces is generated which is the cause of unbalance force, which is given by the following formula.

$$\{f_{unb}(t)\}=\{F_{unb}\}e^{j\omega t} \tag{4}$$

$$\{F_{unb}\}=me\omega^2 e^{j\phi} \tag{5}$$

Where $\{F_{unb}\}$ is the residual unbalance force vector, which has phase and amplitude information an in generally the element of the unbalance force vector are complex, mass of disc m and eccentricity of the rotor from the center of rotation of the shaft to the geometric center of the shaft e and the residual unbalance $U_{Res}=me$. When rotor rotate in counter clockwise direction the relation between unbalance force of the rotor in y direction and x direction is given by the following formula that can be written as,

$$(F_{unb})_y=-j(F_{unb})_x \tag{6}$$

E. Shaft Bow Force

The bow force of bent rotor bearing system is given by the formula, which is discussed in [1]. The bow force depends on the stiffness of the shaft, the amount of bow and phase angle between the residual unbalance.

$$F_b=[K_r]\{B\}e^{j(\omega t+\alpha)} \tag{7}$$

Where K_r is the rotor stiffness matrix, B is the rotor bow vector, α is the location of the bow with respect to eccentricity, μ is the angular speed of the shaft.

F. Assembled Equation of Motion of the System

Combining the Eq. (1)-(7), the assembled equation of motion of the rotor bearing system is represented as

$$[M]\{\ddot{\eta}\}+[C]\{\dot{\eta}\}+[K]\{\eta\}=\{F(t)\}_{unb}+\{F(t)\}_{bow} \tag{8}$$

Where M, C and K are the global mass, damping and stiffness matrices respectively. $\{F(t)\}_{unb}$, $\{F(t)\}_{bow}$ and $\{\eta\}$denote the vectors of generalized forces and displacements. The global rotor mass matrix M, rotor stiffness K_r and rotor displacement C_r. The damping C and stiffness K matrices combine the effects of the rotor and the bearing supports, i.e. K and C matrices for the system can be represented as,

$$K=K_r+K_B \text{ and } C=C_r+C_B$$

where K_B and C_B are the bearing stiffness and damping matrices.

$$\{F(t)\}^{unb}=\omega^2\{U_{Res}\}e^{j(\omega t+\phi)} \tag{9}$$

$$\{F(t)\}^{Bow}=[K_r]\{B\}e^{j(\omega t+\alpha)} \tag{10}$$

Where $\{U_{Res}\}$ and $\{B\}$ are the residual unbalance and residual shaft bow vector. These vectors contain the magnitude and phase information of residual unbalance and residual shaft bow, i.e. the element of these vectors is complex.

III. METHODOLOGY

This study is based on the parametric estimation of rotor bow of rotor in rotor bearing system. This has followed the steps

Step 1: A system model is developing which consist a flexible rotor and this rotor is supported by two bearings at the end. A rigid disc is mounted on the midpoint of the shaft. Degree of freedom at each node is four and total degree of freedom of this system is 12.

Step 2: An equation of motion has been developed for this system. Solution of this equation of motion gives information of the displacements at each node of the shaft. Response of the system is given the solution of the equation of motion.

Step 3: For balancing of this system an identification algorithm has been developed which is based on the least square fit techniques. The regression equation is use to prepare the identification algorithm. A regression matrix is prepared which contained the responses and a regression vector, which contained the forces of the equation of motion.

Step 4: After the estimation of the parameters. These parameters are used for developing the response and a draw a plot of response with assumed parameters and estimated parameters.

Step 5: In this study, parameters estimated, when rotor rotates in counter clockwise direction.

TABLE 1, represents the physical property of the bent rotor bearing system considered. These properties are used to develop the response, and further used in identification algorithm. The equation of motion for the bend rotor bearing system is taken from [1], after the getting equation of motion we have to calculate the solution of this equation, the solution is comes in the vector from because instead of all the constant here used only the matrices. The solution has the contained the complex numbers, the modulus of the complex numbers and direction of the complex numbers plotted with angular speed of the shaft is shown in Fig. 2. In Fig. 2, first two represents the horizontal and vertical displacement variation with respect to angular speed. The amplitude of the horizontal and vertical displacement are consider in these plots and last two represents the angular displacement in horizontal and vertical direction with respect to angular speed variation.

TABLE 1. PROPERTIES OF THE FLEXIBLE ROTOR SYSTEM

Parameters	Values (Units)
Mass of the disc	$m_d = 5$ kg
Diametral mass moment of inertia ($A=\pi d^4/64$)	$I_d = 0.014$ kg-m^2
Length of the shaft	L =2.5m
Diameter of the shaft	d = 0.01m
Modulus of Elasticity of shaft material (i.e. Steel)	E = 210 GPa
Modulus of rigidity of shaft material (i.e. Steel)	G = 80.8 GPa
Density of the shaft material	ρ = 7850 kg/m^3
The Residual Unbalance U_{Res}= m$_d$×e , (U_{yres} = -JU_{xres}) where 'e' is the eccentricity of the unbalance	U_{XRes}= 0.006 kg-m.

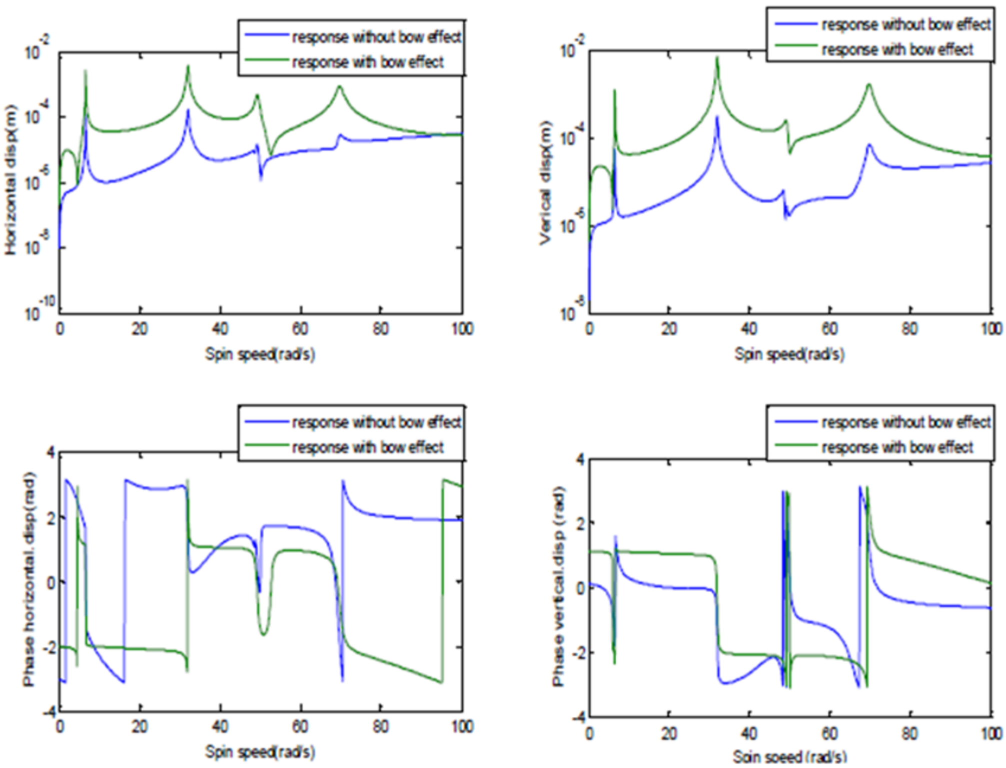

Fig. 2. Semi-log representation of horizontal and vertical displacement with and without bow and Phase variation with the speed of the rotor

IV. DEVELOPMENT OF IDENTIFICATION ALGORITHM

The present section deals with the response of the system to develop the identification algorithm. The generalized form of equation of motion of the system is discussed in [1], which is given below

$$[M]\{\ddot{\eta}\}+[C]\{\dot{\eta}\}+[K]\{\eta\}=\{F(t)\}_{unb}+\{F(t)\}_{bow} \quad (11)$$

Assume the initial solution of the equation of motion is $\{\eta\}=\{\eta\}e^{jwt}$ where $\{\eta\}$ contained the magnitude and phase information. Substitute the values of $\{\eta\}$, $\{F(t)\}_{unb}$ and $\{F(t)\}_{bow}$ in the Eq.(11).

$$\left(-\omega^2[M]+j\omega[C]+[K]\right)\{\eta\}= \omega^2\{U_{\mathrm{Res}}\}e^{j\phi}+[K_r]\{B\}e^{j\alpha} \quad (12)$$

The development of identification algorithm can be arranged such that the all known quantity (i.e. rotor model parameters and measurable response) on the right hand side and all of the unknown quantities (i.e. bearing dynamic parameters, residual unbalance and residual shaft bow parameters) are placed on the left hand side. After rearranging the regression equation can be written as,

$$\left[A(\omega_i)\right]\{X\}=\left[B(\omega_i)\right] \quad (13)$$

V. RESULTS AND DISCUSSION

When rotor is runs in counter clockwise direction, the estimated bearing stiffness, bearing damping, shaft bow and residual unbalance parameters are shown in

TABLE 2. From

TABLE 2, the values of all the parameters without noise and with 1%, 2% and 5% noises is estimated and compared with the assumed values. From Fig. 3-Fig. 5, shows the comparison of all the parameters in with 1%, and 5% with the assumed parameter values. From Fig. 5, the shaft bow parameters w.r.t, 5% noise the maximum error is found in imaginary part of the residual shaft bow in x direction is58% error and rest of the parameters are below this limit. The estimation of error in assumed and estimated parameters by adding of the some percentage of noise in response , therefore, the variation of assumed parameters to the estimated parameters with 1% and 5% of noise adding in the response. The variations of responses in all estimated parameters are represented in Fig. 6, which shows the variation of error in estimated parameters to the assumed parameters, and plot between percentage errors corresponding to the parameters.

TABLE 2. ESTIMATED PARAMETERS WITHOUT NOISE AND WITH 1%, AND 5% NOISE ADDED IN RESPONSE

Parameters	**Assumed values**	**Estimated parameters**		
		Without Noise	*With 1%noise*	*With 5% noise*
K_{xx1} (N/m)	2.50×10^5	2.46×10^5	2.48×10^5	2.56×10^5
K_{xy1} (N/m)	1.20×10^5	1.22×10^5	1.23×10^5	1.27×10^5
K_{yx1} (N/m)	1.35×10^5	1.31×10^5	1.29×10^5	1.21×10^5
K_{yy1} (N/m)	2.75×10^5	2.77×10^5	2.76×10^5	2.72×10^5
K_{xx2} (N/m)	2.75×10^5	2.71×10^5	2.69×10^5	2.61×10^5
K_{xy2} (N/m)	1.46×10^5	1.48×10^5	1.47×10^5	1.43×10^5
K_{yx2} (N/m)	1.39×10^5	1.37×10^5	1.37×10^5	1.42×10^5
K_{yy2} (N/m)	2.82×10^5	2.84×10^5	2.85×10^5	2.87×10^5
C_{xx1} (Ns/m)	300	304.79	303.40	299.06
C_{xy1} (Ns/m)	20	23.75	17.72	11.95
C_{yx1} (Ns/m)	50	54.79	51.40	38.92
C_{yy1} (Ns/m)	399	402.75	399.41	386.46
C_{xx2} (Ns/m)	315	319.99	317.53	307.39
C_{xy2} (Ns/m)	59	62.79	59.38	48.98
C_{yx2} (Ns/m)	79	83.51	82.84	77.11
C_{yy2} (Ns/m)	300	303.19	297.42	274.67
U^r_{xResr} (kg-m)	0.0052	0.0054	0.0057	0.0075
U^i_{xResi} (kg-m)	0.0030	0.0031	0.0035	0.0046
B_x (mm)	0.0050	0.0045	0.00473	0.002188
B_y (mm)	0.0050	0.0055	0.00295	0.001254

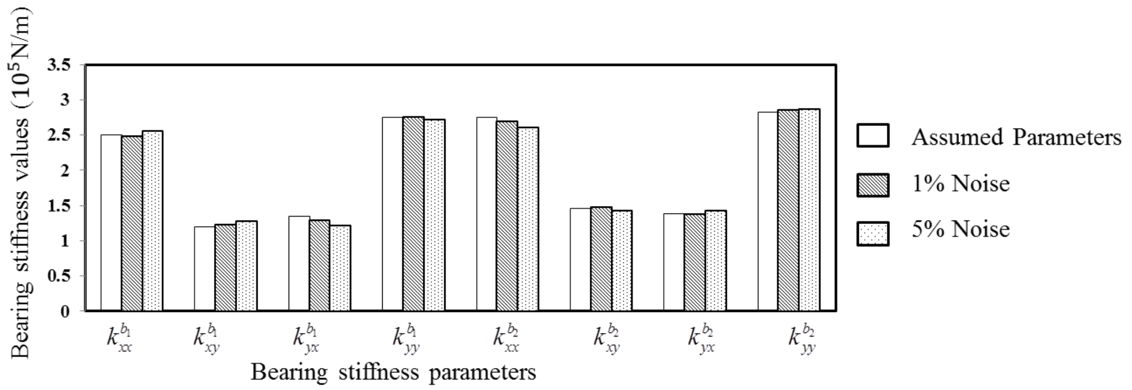

Fig. 3. *Bearing stiffness parameters with 1% and 5% noise in simulated response*

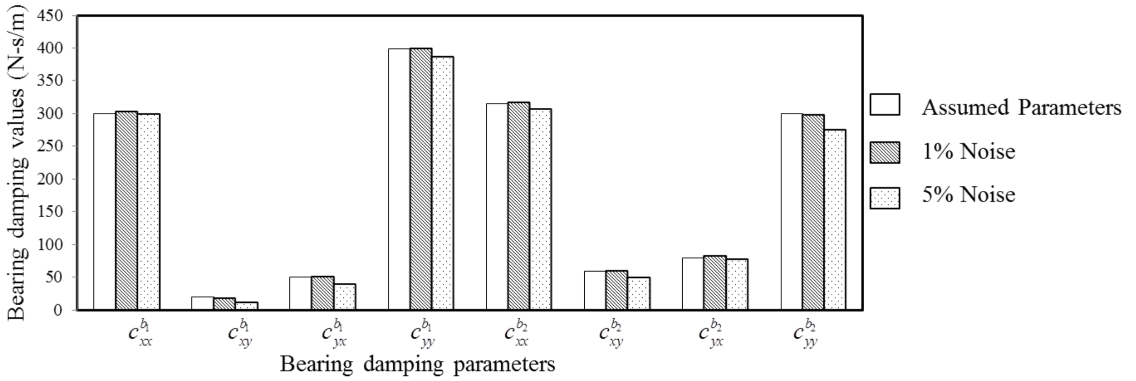

Fig. 4. *Bearing damping parameters with 1% and 5% noise in simulated response*

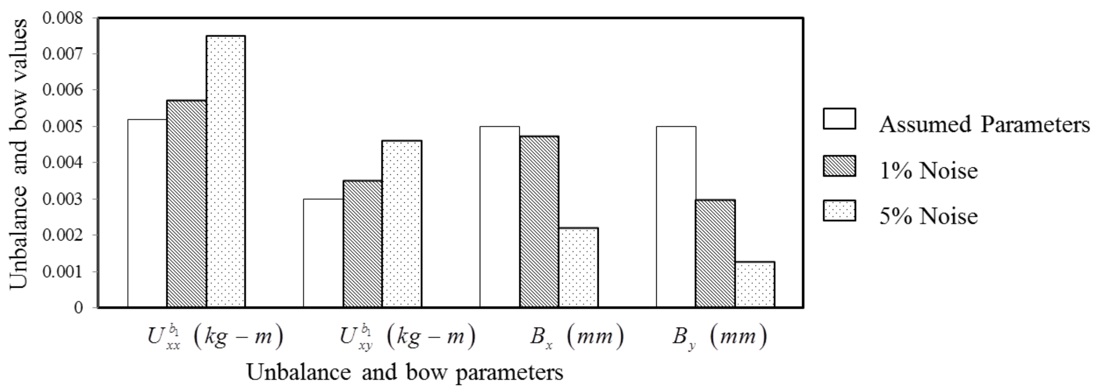

Fig. 5. *Rotor bow and unbalance parameters with 1% and 5% noise in simulated response*

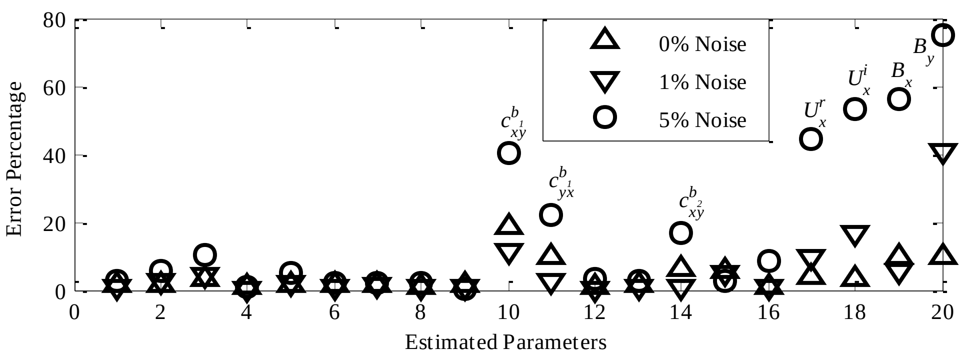

Fig. 6. V*ariation of error in estimated parameters*

VI. CONCLUSIONS

The main objective of the study is to estimate the parameters, which are responsible to deteriorate the efficiency of rotating machinery. There are various parameters in the rotating machines like bearing dynamic parameters, residual unbalances, shaft misalignment and residual bow, which are responsible for decreasing the efficiency. The present study is focused on the estimation of the bow effect of the rotor. Therefore, this study reveals the information related to parameters, which are obtained from the response with noise error. After that the concluding word is that the estimated parameters are from the presented algorithm is best because it helps to avoiding of the critical speeds consequently the main concern is fulfilled by this presented algorithm. The conclusion of this study is an estimation of parameters with help of presented identification algorithm give most suitable results. This algorithm gives the information of the appropriate parameters, which is using for the designing of the rotor bearing system.

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